On observability of the free core nutation

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Introduction

Problem:

— how one can determine RFCN from observations?

— what does it mean to determine RFCN?

— what are the limits?

Naive point of view:

1. VLBI measures nutation time series
2. there exists a precise theory of forced nutation
3. subtract 2 from 1 and that what you get is RFCN.

Unfortunately the nature is not that simple...

Units conversion: 1 ppb = 1 nrad ≈ 0.21 mas ≈ 14 μs

What does VLBI really measure?

…thermal noise at receivers!!

VLBI technology provides hardware and software for evaluation of the cross-correlation function and estimation of group delays.

Estimates of group delays and constraints are used for adjusting site positions, source coordinates, EOP, including nutation parameters, and 1–2 million other parameters.

Nutation offsets are **not measurements**, but *results* of a multi-step analysis procedure.

Nutation offsets are fitting parameters which depends on rotation of the idealized Earth **and** a subjective choice of parameterization, constraints, analysis strategies.

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Two approaches for EOP estimation:

1. Direct approach: (L. Petrov, AA, **467**, p. 359, 2007)

Perturbation of the Earth rotation vector as a function of time is expanded into a sum of the B-spline and Fourier basis functions. Coefficients are adjusted by the LSQ to group delays.

2. Traditional approach: (Herring et al., JGR, **91**, p. 4745, 1986)

- Simple estimation model over 24^h:

$$
\begin{aligned}
q_1(t) &= \boldsymbol{c(t)} \cos -\Omega t + \boldsymbol{s(t)} \sin -\Omega t + \boldsymbol{b_1}(t) + \boldsymbol{d_1}(t) * (t - t_o) \\
q_2(t) &= \boldsymbol{c(t)} \sin -\Omega t - \boldsymbol{s(t)} \cos -\Omega t + \boldsymbol{b_2}(t) + \boldsymbol{d_2}(t) * (t - t_o) \\
q_3(t) &= \boldsymbol{b_3}(t) + \boldsymbol{d_3}(t) * (t - t_o)
\end{aligned}
\tag{1}
$$

- Complicated a priori model for $\widehat{\mathcal{M}}_a(t)$:
  - — $\sim$ 1400 periodic terms
  - — Time series with 1^d step
- Filtering and smoothing time series of $c(t), s(t), b(t), d(t)$.
- Computation of interpolation spline for $c(t), s(t), b(t)$.

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Deficiency of the traditional approach:

- In fact, we have **three models:**
  - `a priori model`
  - estimation model
  - *post-processing model*

  They **contradict** each other.
- Estimation is **not optimal**: raw time series minimizes residuals in least square sense, the filtered and smoothed series **does not**.
- Since model of estimation is very simple => the a priori model should be **very complex** and very precise: at a level of 3–5 $\cdot\, 10^{-9}$ rad.
- Small changes in a priori model result in **changes in total** EOP adjustments
- Result of analysis, time series of $c(t), s(t), b(t)$, **is not usable directly**.
- It is **difficult to assess** errors of the interpolated $c(t), s(t), b(t)$ series. Correlations are lost, contribution of errors of the a priori EOP model is not taken into account.

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Do we have a theory of nutation?

Yes . . . and no.

Excitation * Transfer function = Nutation estimates

tidal torque **ocean tides** **non-tidal ocean** **atmosphere**	Earth's response	**empirical EOP spectrum**

Earths' response can be determined from

1. seismology observations + equations of the theory of Earth' dynamics
2. from a portion of the empirical EOP spectrum

Ultimate test of the theory: do these estimates agree?

Answer: **no** $\Longrightarrow$ the theory is **wrong**

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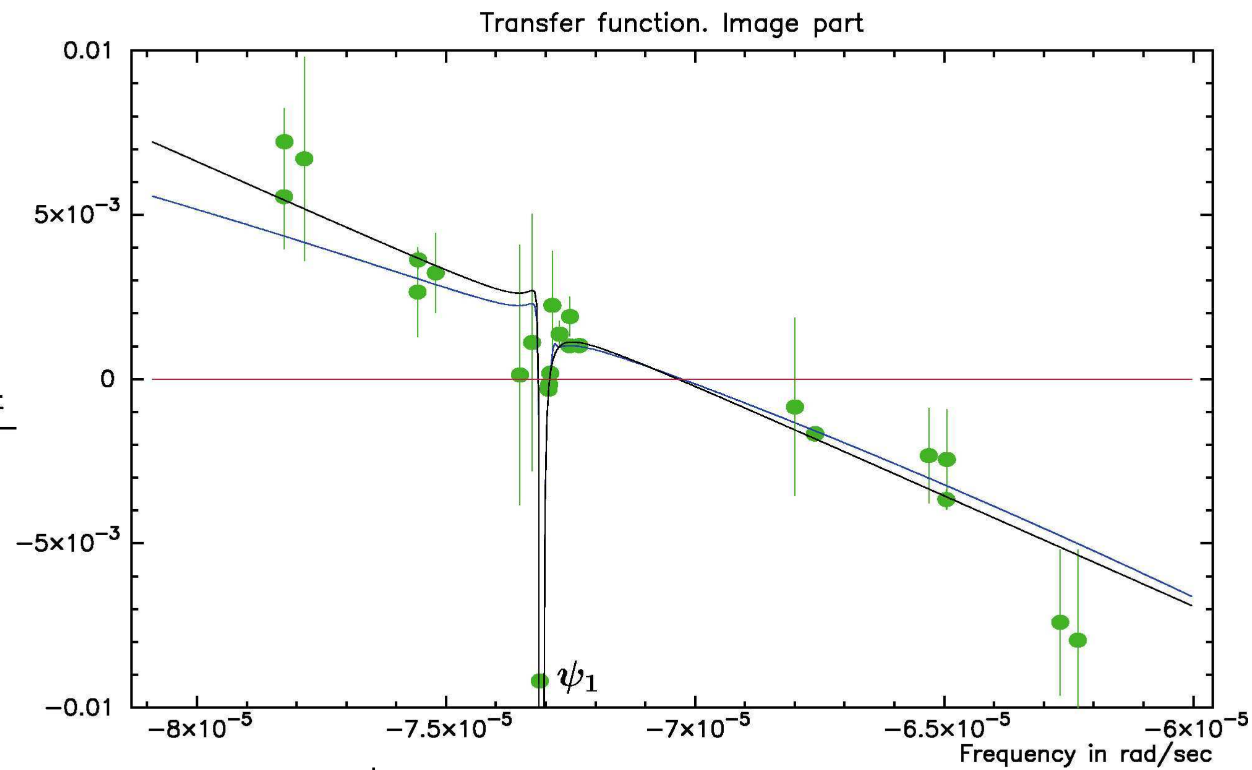
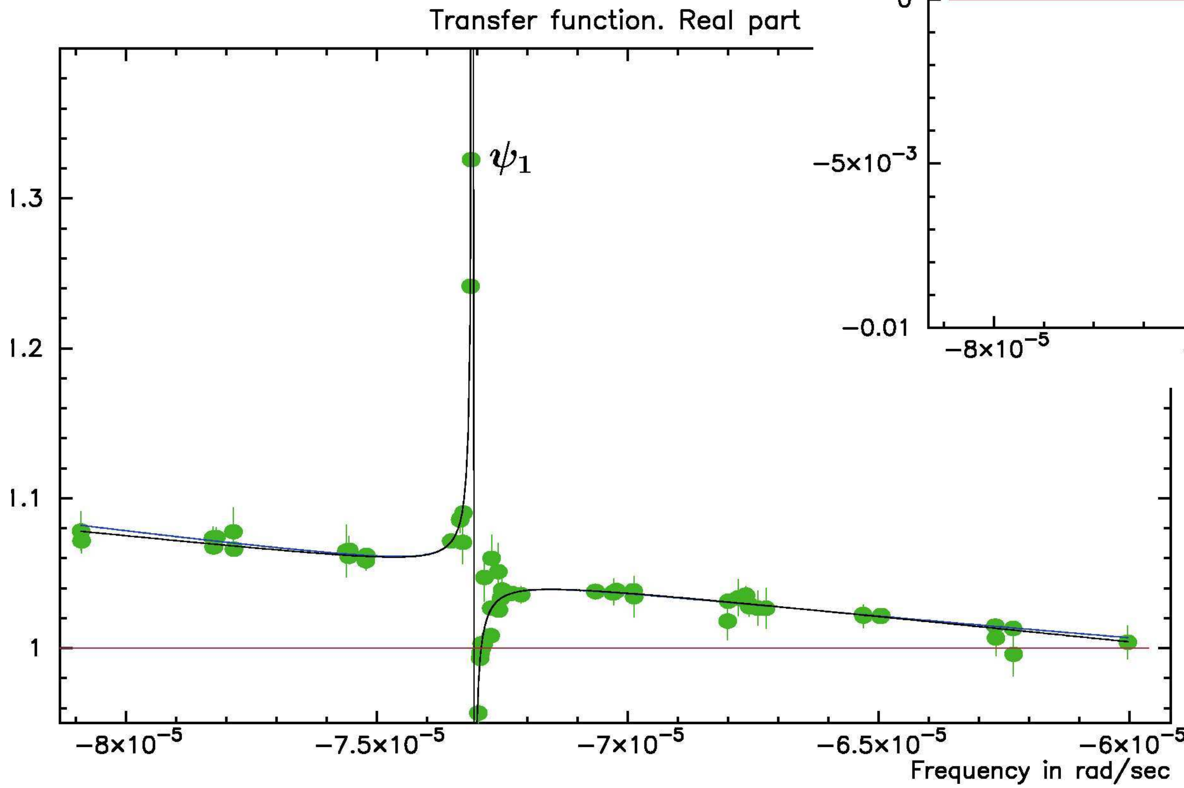
Empirical transfer function <u>directly</u> from VLBI group delays

green — estimates
blue — MHB transfer function
black — empirical transfer function
red — rigid Earth's response





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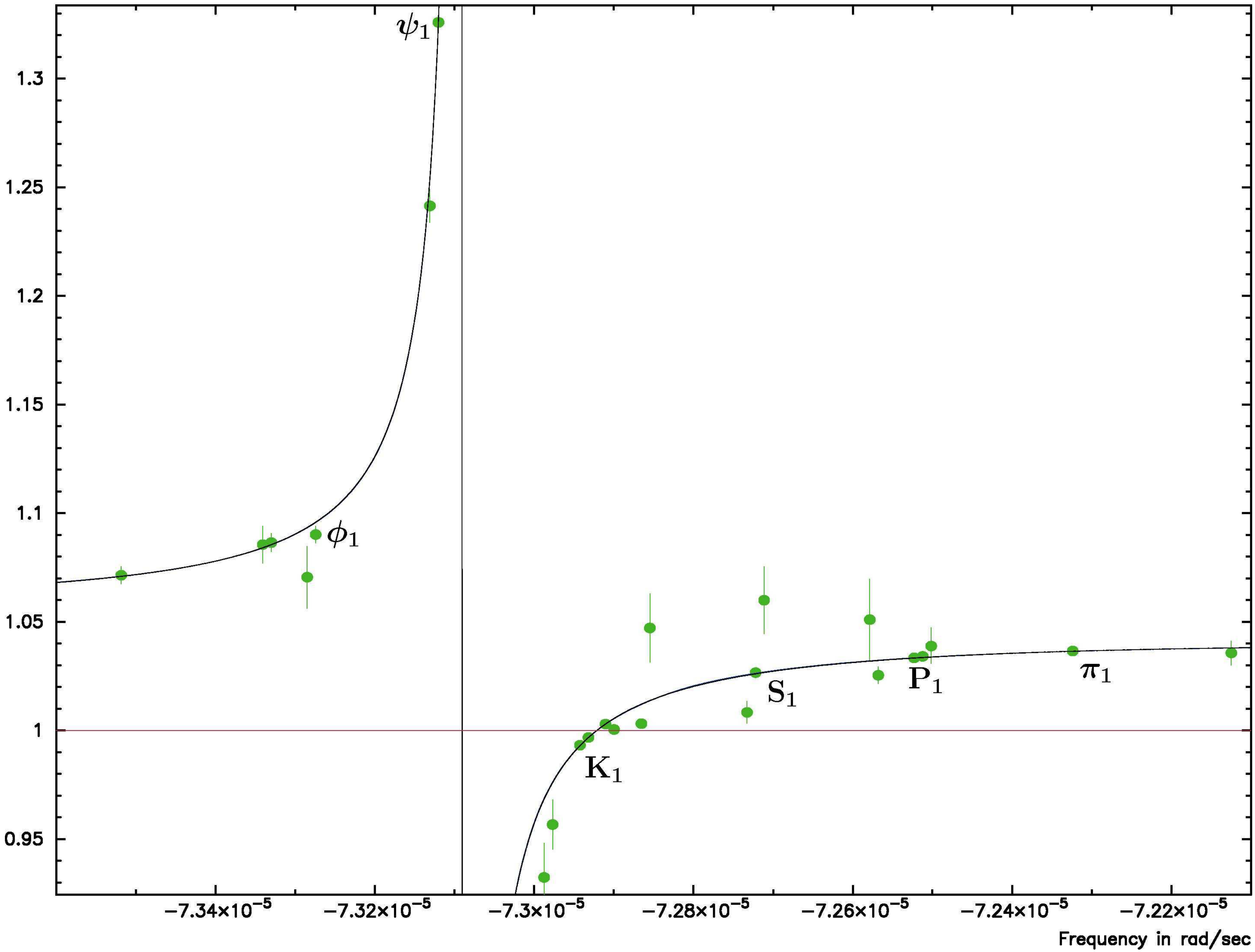
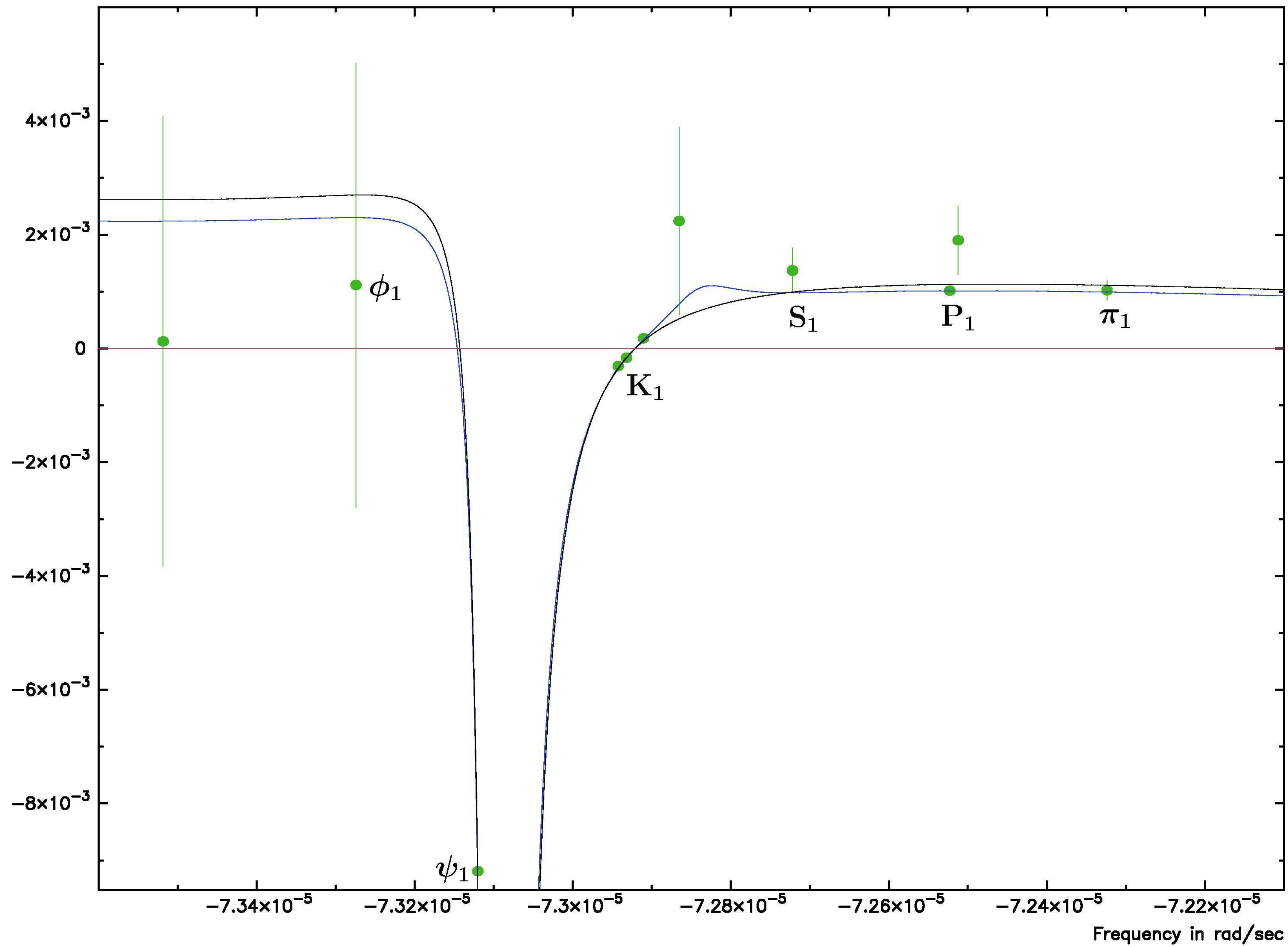
Transfer function. Real part



Units conversion: 1 ppb = 1 nrad $\approx$ 0.21 mas $\approx$ 14 μs

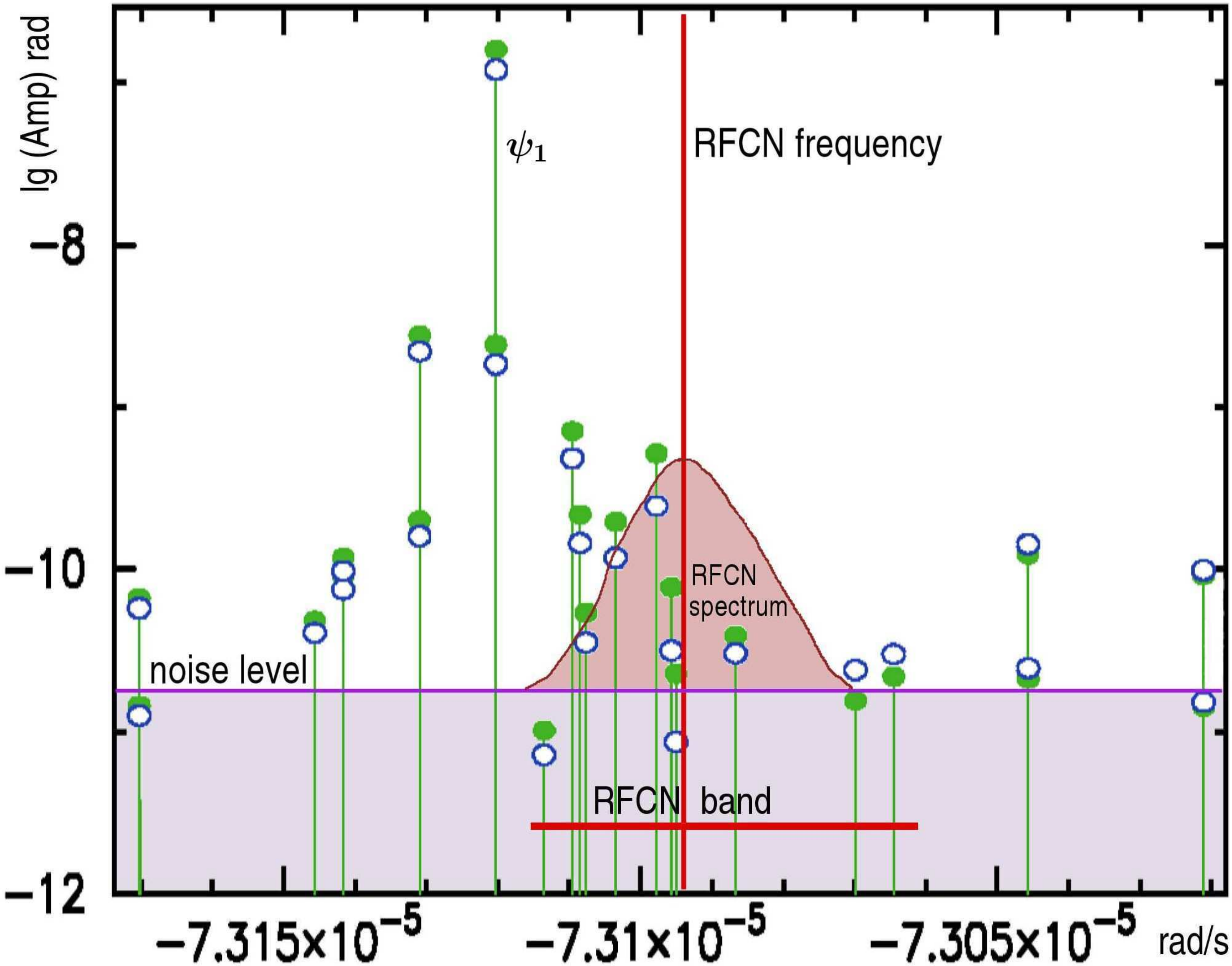
Transfer function. Image part



Units conversion: 1 ppb = 1 nrad $\approx$ 0.21 mas $\approx$ 14 μs

Amplitude spectrum of forced nutation near the RFCN band.

• **MHB2000** and ○ **rigid Earth** amplitude



Estimation of the RFCN as a filtration problem

Assumptions:

- RFCN is a band-limited process with known frequency band with unknown excitation
- forced nutations with known frequencies and known excitation can be evaluated independently from the RFCN

Then the spectrum of the RFCN can be separated from the spectrum of forced nutations.

The problems:

- How to estimate forced nutations within the RFCN band?
- How to determine the bandwidth of the RFCN?

How this can be done?

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Assuming

- triaxiality of the Earth's inertial ellipsoid is negligible
- The Earth consist of k layers
- The Earth's response is linear to external torque

we get that the transfer function is in the form

$$T(\omega) = \alpha\,(\omega - \beta) + \sum_{i=1}^{i=k+1} \frac{\gamma_i}{\omega - \delta_i}$$

where ω is the frequency; α, β, γ_i, δ_i are complex parameters.

Only one resonance, RFCN, noticeably affects nutation. We have 6 real value parameters that can be estimated from observations, provided the excitation is known.

Problems:

- Estimates of transfer function heavily depends on a small set of constituents, f.e. ψ_1, which is within the RFCN band.
- Estimation of the transfer function is a non-linear problem.
- Oceanic and atmospheric excitation is not well known.

Consequences:

1. Estimates of the RFCN spectrum have the **statistical uncertainties** due to noise in the data and the **constraint uncertainties** due to constituents separation.
2. Estimates of the RFCN spectrum depends on ocean and atmospheric models.
3. Constraint uncertainties exceed statistical uncertainties.
4. Estimates of the RFCN spectrum are not unique.

Examples:

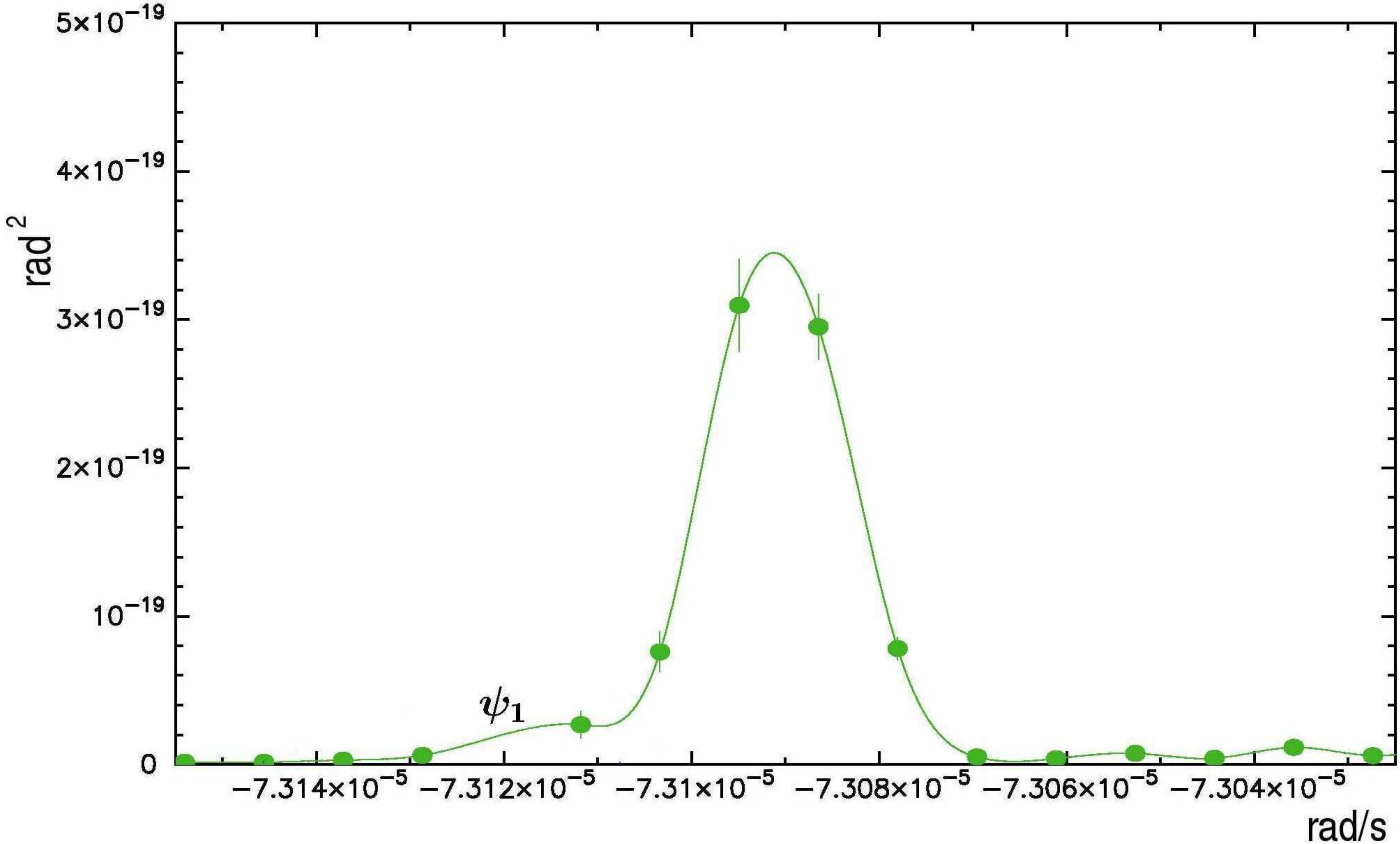
A. Empirical transfer function from ψ_1, K_1, P_1, O_1, etc was extrapolated to the RFCN band.

B. The same as A, but random noise commensurate to the model errors was added before computation of the empirical transfer function.

C. Transfer function = (1.0,0.0) (rigid Earth) was extrapolated to the RFCN band.

D. Forced nutations within the RFCN band were ignored and $\longrightarrow$ propagated to the RFCN spectrum.

Estimate of the RFCN power spectrum from VLBI time delays and resonance constraints of forced nutations



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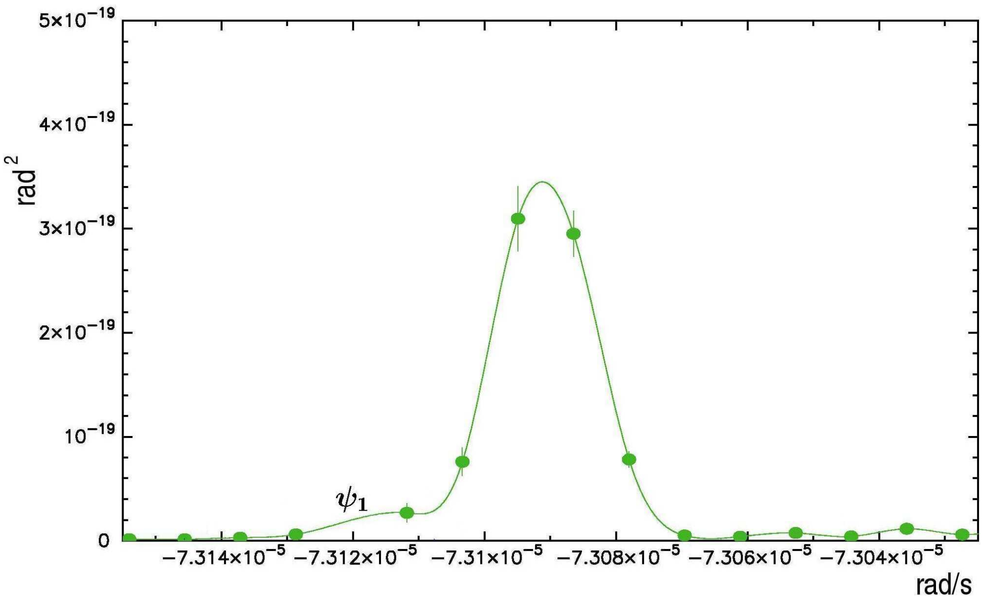
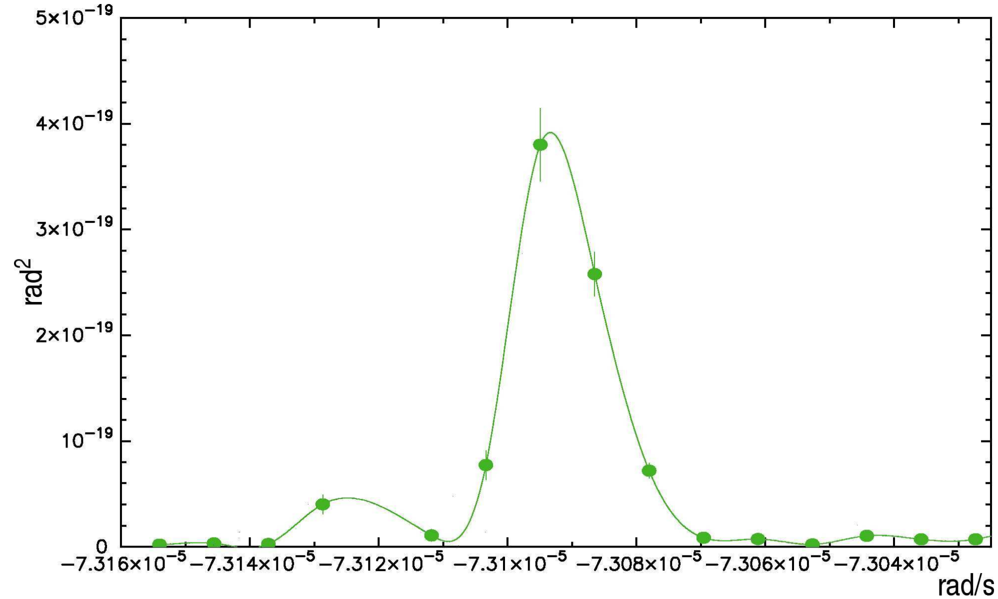
RFCN power spectra from examples a–d

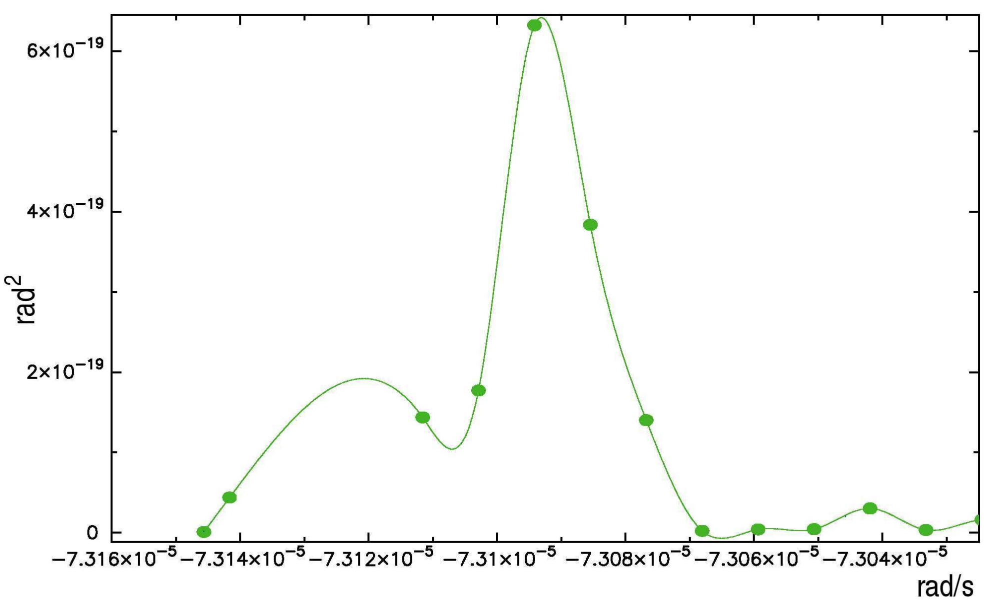
A)



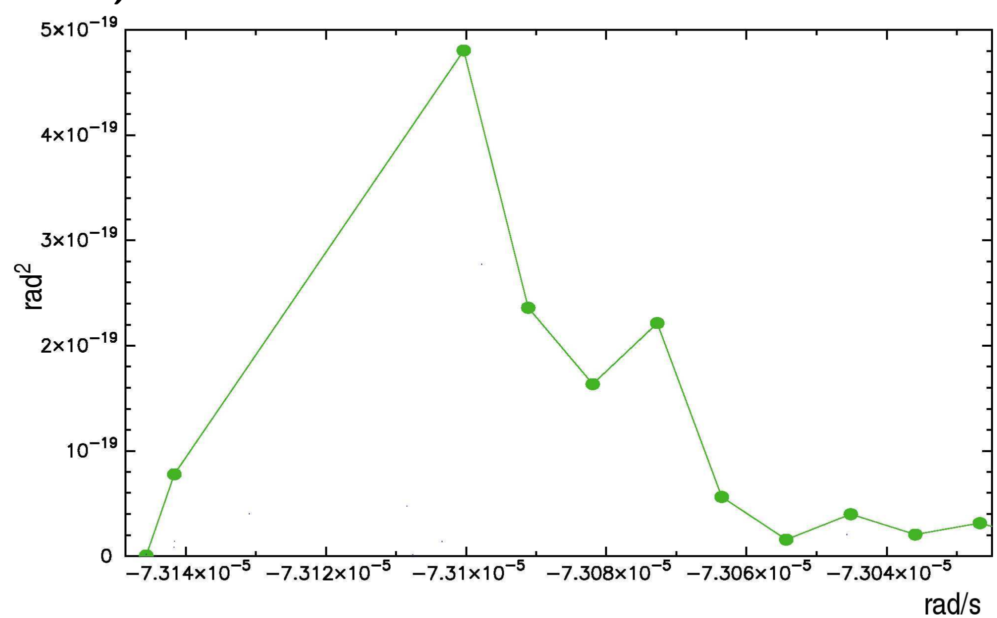
B)



C)



D)



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How much constraints on forced nutations affects estimates of RFCN spectrum?

Monte Carlo simulation:

1. added Gaussian noise to estimates of nutations at K_1, S_1, P_1 due to uncertainty of ocean contribution with σ 150 prad;
2. added Gaussian noise to estimates of nutations at ϕ_1, ψ_1 with σ 300 prad;
3. obtained new estimates of empirical transfer function;
4. computed new set of constraints;
5. ran a new LSQ solution for estimation of nutations from group delay;
6. repeated the procedure many times;
7. computed rms of estimates of the RFCN spectra for different runs.

Constraint uncertainty on RFCN spectra: **43 prad**
Formal uncertainty on RFCN from noise in group delays: **19 prad**

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Conclusions:

- Estimates of the RFCN depends on the mathematical model used for separation of the free and forced nutations.
- Separation of nutations results in the constraint uncertainty.
- Constraint uncertainty sets the limit of accuracy of the RFCN estimates.
- Longer series, more precise observations will not result in improving of accuracy. (But improvement of ocean and atmospheric models may improve the accuracy.)