# Geodetic Relativistic Rotation of the Solar System Bodies

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**The angular velocity vector of the geodetic rotation for any body of the solar system is following:**

$$\bar{\sigma}_i = \frac{1}{c^2} \sum_{j \neq i} \frac{G m_j}{\left| \bar{R}_i - \bar{R}_j \right|^3} \left( \bar{R}_i - \bar{R}_j \right) \times \left( \frac{3}{2} \dot{\bar{R}}_i - 2 \dot{\bar{R}}_j \right).$$

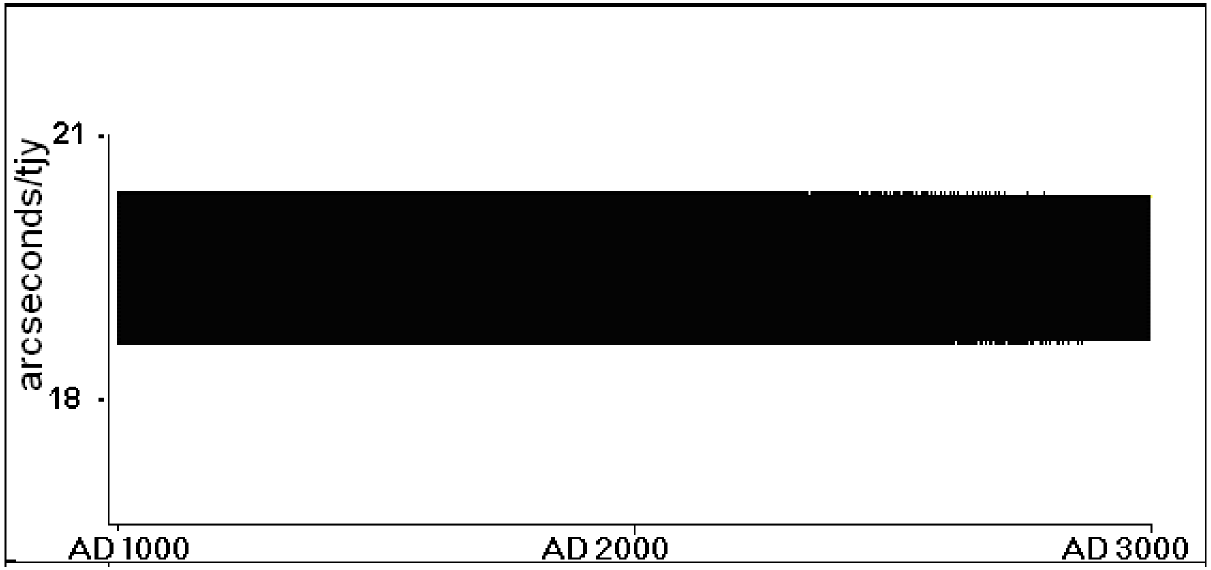
Here the subscripts $i$ and $j$ correspond to the Sun, the major planets, and the Moon; $G$ – the gravitational constant; $m_j$ – the mass of a body $j$; c – velocity of light in vacuum; $\bar{R}_i, \dot{\bar{R}}_i, \bar{R}_j, \dot{\bar{R}}_j$ – the vectors of the barycentric position and velocity of bodies $i$ and $j$; the symbol × means a vector product.

Since the mass of the Sun is dominant in the solar system then the main part of $\vec{\sigma}_i$ for the major planets and the Moon is a result of the orbital motion of these bodies. It means that vector $\vec{\sigma}_i$ is almost orthogonal to the plane of the heliocentric orbit. For the Moon and all planets (except Pluto) the vectors $\vec{\sigma}_i$ are practically directed to the north pole of the ecliptic.

The geodetic rotation of the Sun depends on the orbital motion of the major planets and the Moon. Only the component $\sigma^Z$, orthogonal to the plane of the fixed ecliptic J2000.0 is studied, because the geodetic rotation in the ecliptic plane is the most interesting phenomenon and the rest components are generally insignificant.
For each body the file of the values of the ecliptically component $\sigma^Z$ is formed over the time span from AD1000 to AD3000 with one day spacing. The secular and periodic components of the geodetic rotation vector are determined by means of the procedure which involves the least-squares method and spectral analysis methods.
The result is presented in the form

$$\sigma^Z = \sum_{k=0}^{6} \mathrm{T}^k \sum_{m=1} \left( A_{km} \sin\alpha_m + B_{km} \cos\alpha_m \right), \quad \alpha_m = \sum_{l=1}^{10} n_l \lambda_l .$$

The mean heliocentric longitudes of the planets $\lambda_1, \ldots, \lambda_8$ and the mean geocentric longitude of the Moon $\lambda_{10}$ with respect to the fixed equinox J2000, adjusted to DE404/LE404 ephemeris, are taken from Brumberg and Bretagnon (2002). The mean heliocentric longitudes of Pluto $\lambda_9$ with respect to the fixed equinox J2000.0 is determined by the least squares method. T means the Dynamical Barycentric Time (TDB) measured in thousand Julian years (tjy). The coefficients $A_{km}$, $B_{km}$ are to be determined and $n_l$ are some integer numbers.




The Earth

For the Earth the component, orthogonal to the plane of the fixed ecliptic J2000.0 is determined:

$$\sigma^{Z} = \{19''.1988821 - 0''.00017685T + \ldots + 10^{-6}\dot{\lambda}_3[-34''.285\cos\lambda_3 + 149''.227\sin\lambda_3 +$$

$$+T(-7''.539\cos\lambda_3 - 5''.682\sin\lambda_3) + T^2(0''.261\cos\lambda_3 - 0''.291\sin\lambda_3) + \ldots] + \ldots\} / \text{tjy}$$

From the paper of Brumberg and Bretagnon (2002) one can use the developments representing the geodetic rotation of the Earth in Euler angles for constructing the same component:

$$\sigma^{Z}_{BB} = \{19''.19883018 - 0''.00026965T + 10^{-6}\dot{\lambda}_3[(-34''.28\cos\lambda_3 + 149''.22\sin\lambda_3) +$$

$$+T(-7''.54\cos\lambda_3 - 5''.69\sin\lambda_3) + T^2(0''.30\cos\lambda_3 - 0''.29\sin\lambda_3)]\} / \text{tjy}.$$

$\lambda_3 = 1.75347029148 + 6283.0758511455T$ is the mean longitude of the Earth.

All symbols $\dot{\lambda}_j$ mean the time derivatives of the mean longitudes .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From the paper of Brumberg and Bretagnon (2002) one can use the developments representing the geodetic rotation of the Earth in Euler angles for constructing the same component:

$$\sigma^Z_{BB} = \{19.''19883018 - 0.''00026965\mathrm{T} + 10^{-6}\dot{\lambda}_3[(-34.''28\cos\lambda_3 + 149.''22\sin\lambda_3) +$$

$$+\mathrm{T}(-7.''54\cos\lambda_3 - 5.''69\sin\lambda_3) + \mathrm{T}^2(0.''30\cos\lambda_3 - 0.''29\sin\lambda_3)]\} / \mathrm{tjy}.$$

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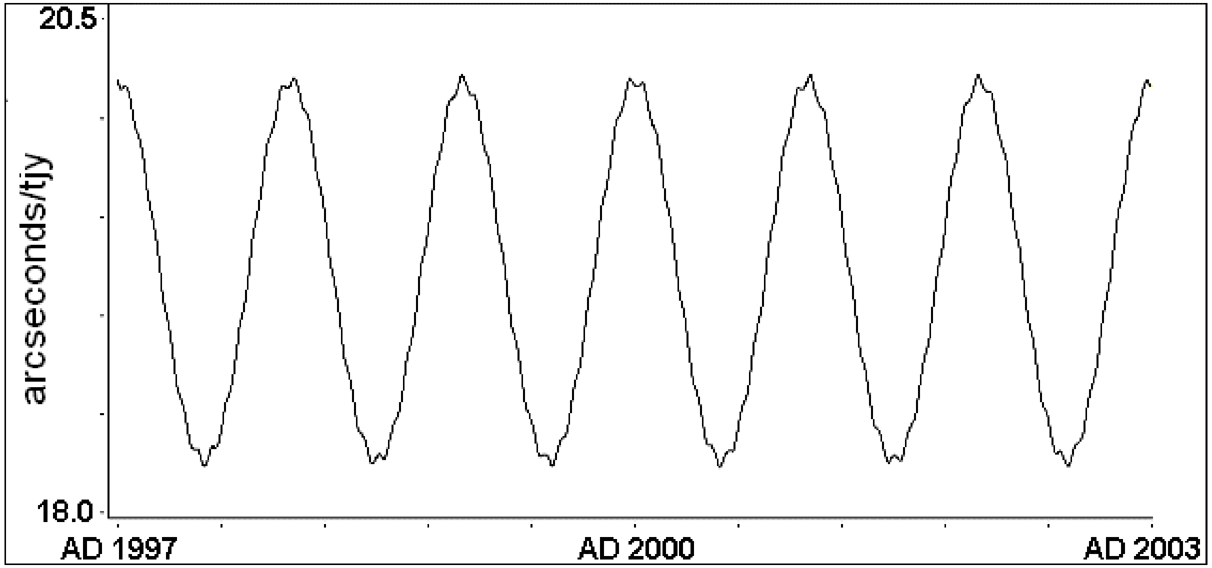
All symbols $\dot{\lambda}_j$ mean the time derivatives of the mean longitudes .

# The Earth (fragment)

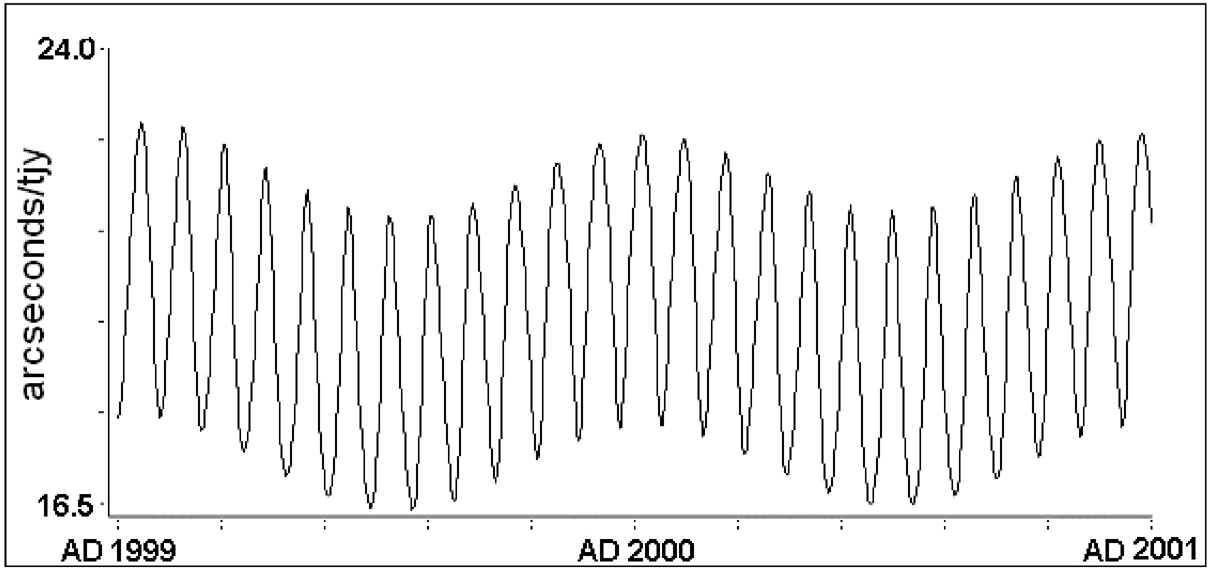



The difference in the coefficients of the secular term is explained by the different theories of the orbital motions (VSOP87 and DE404/LE404): $\Delta\sigma^{Z} = T\,0.''00008611/\text{tjy}$.

This term is nearly equal to the difference between the secular terms in $\sigma^{Z}$ and in $\sigma^{Z}_{BB}$ .

Taking into account also completely different methods of the determination of the coefficients of $\sigma^{Z}$ and $\sigma^{Z}_{BB}$, one can state that they are in a good accordance. Thus the method of the determination of the geodetic rotation velocity vector is very accurate and effective. This method is applied to the other bodies of the solar system.

# The Moon (fragment)




The geodetic rotation of the Moon is determined not only by the Sun but also by the Earth.

$$\sigma^z = \{19''.4950402 \; + \; 0''.0000062\,\mathrm{T} + \ldots + 10^{-6}\dot{D}\left(30''.212\cos D - 0''.001\sin D + \ldots\right) + \ldots$$

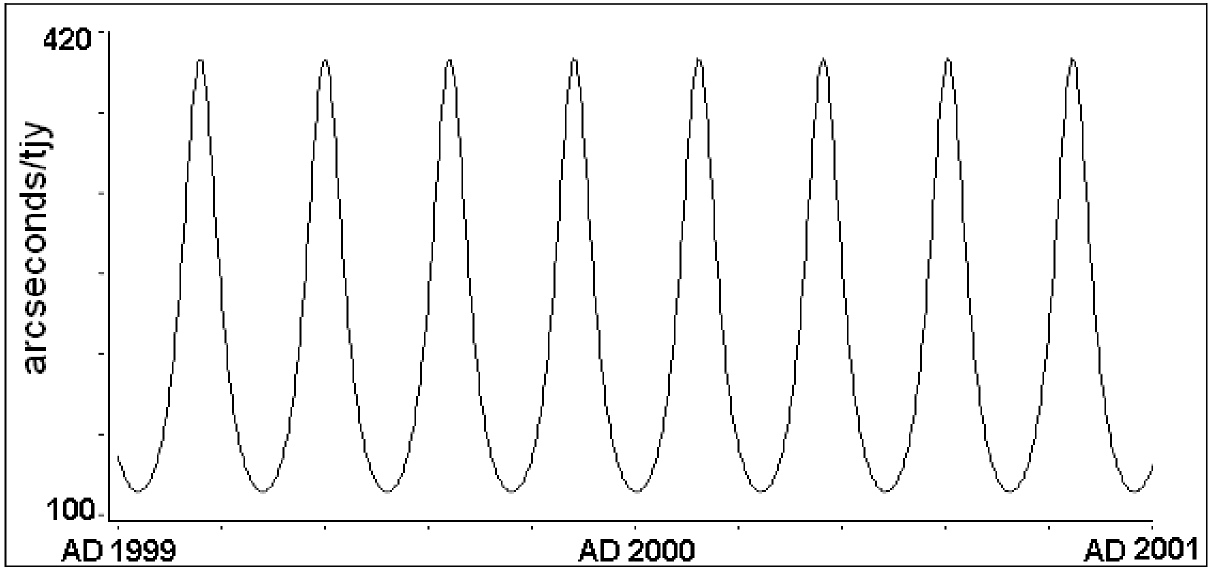
$$+10^{-6}\dot{\lambda}_3\left[-34''.280\cos\lambda_3 + 149''.204\sin\lambda_3 + \mathrm{T}\left(-7''.559\cos\lambda_3 - 5''.683\sin\lambda_3\right) + \right.$$

$$\left. +\mathrm{T}^2\left(0''.261\cos\lambda_3 - 0''.292\sin\lambda_3\right) + \ldots\right] + \ldots\} / \mathrm{tjy}$$

$D = 5.19846640063 + 77713.7714481804\ \mathrm{T}$, $\dot{D}$ is the time derivative of the argument ,

$D = \lambda_{10} - \lambda_3 + 180^\circ$, $\lambda_{10}$ is the mean geocentric longitude of the Moon.

# Mercury (fragment)




Since Mercury is the nearest planet to the Sun then it is clear that its geodetic rotation has to be the most significant in the solar system.

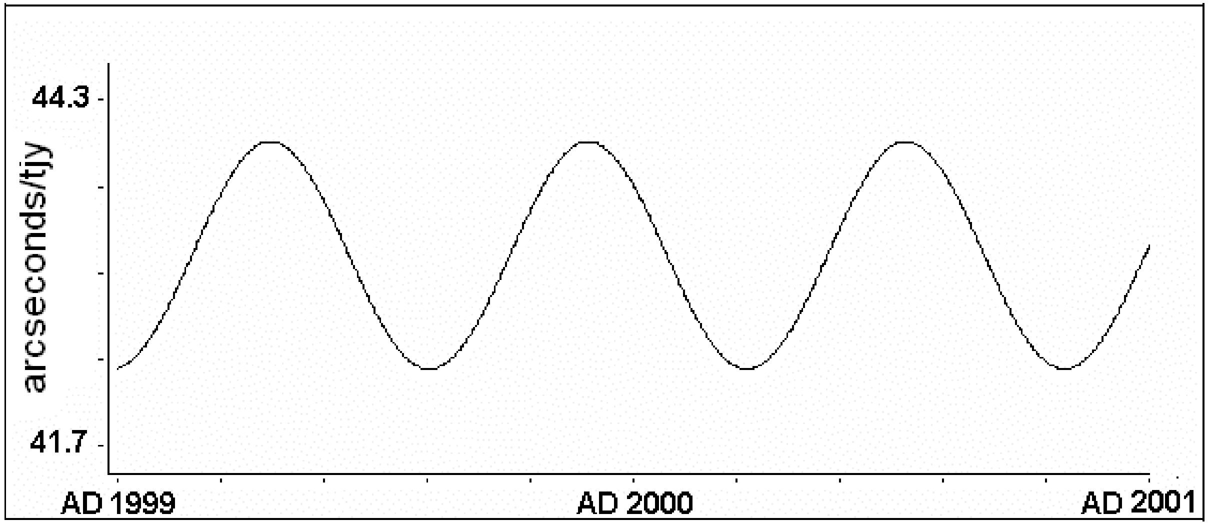
$$\sigma^Z = \{214''.905 + 0''.012T + \ldots$$

$$+10^{-6}\dot{\lambda}_1\,[1086''.273\cos\lambda_1 + 4882''.196\sin\lambda_1 + T(-134''.507\cos\lambda_1 + 35''.242\sin\lambda_1) +$$

$$+T^2(-0''.439\cos\lambda_1 - 1''.687\sin\lambda_1) + \ldots] + \ldots\}\,/\,\mathrm{tjy}.$$

$\lambda_1 = 4.40260867435 + 26087.903145742T$ is the mean longitude of Mercury.

# Venus (fragment)



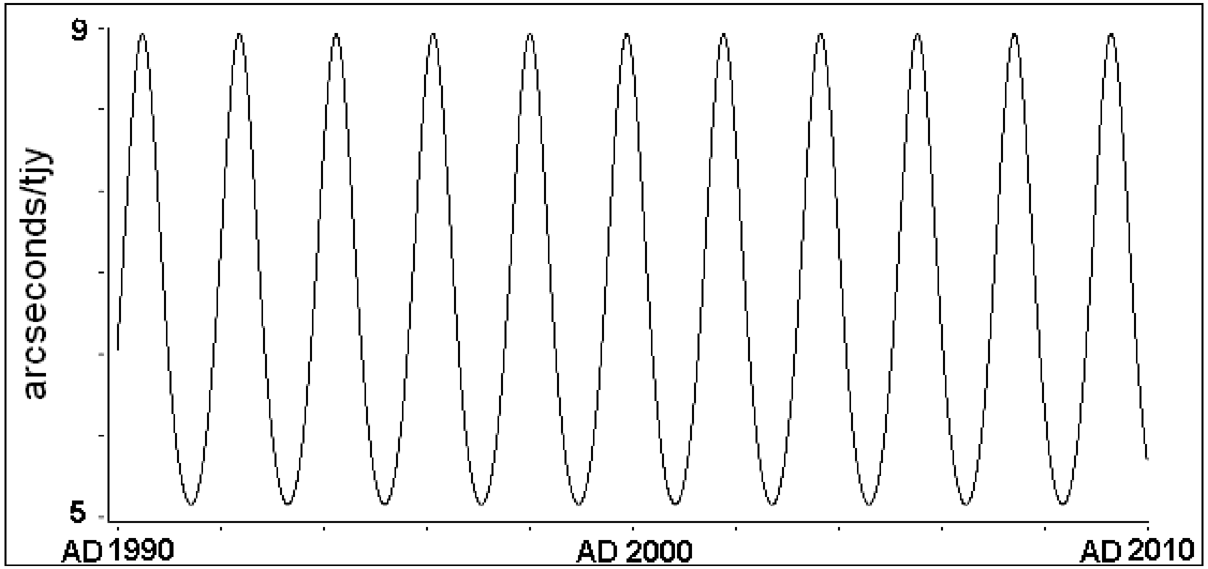

$$\sigma^{Z} = \{43''.12350 - 0''.0005712\,T + \ldots$$

$$+10^{-6}\dot{\lambda}_2\left[-56''.907\cos\lambda_2 + 64''.182\sin\lambda_2 + T\left(3''.958\cos\lambda_2 - 4''.574\sin\lambda_2\right) + \right.$$

$$\left. +T^2\left(0''.062\cos\lambda_2 + 0''.242\sin\lambda_2\right) + \ldots\right] + \ldots\} / \text{tjy}.$$

$\lambda_2 = 3.17614652884 + 10213.2855462110\,T$ is the mean longitude of Venus.

# Mars (fragment)



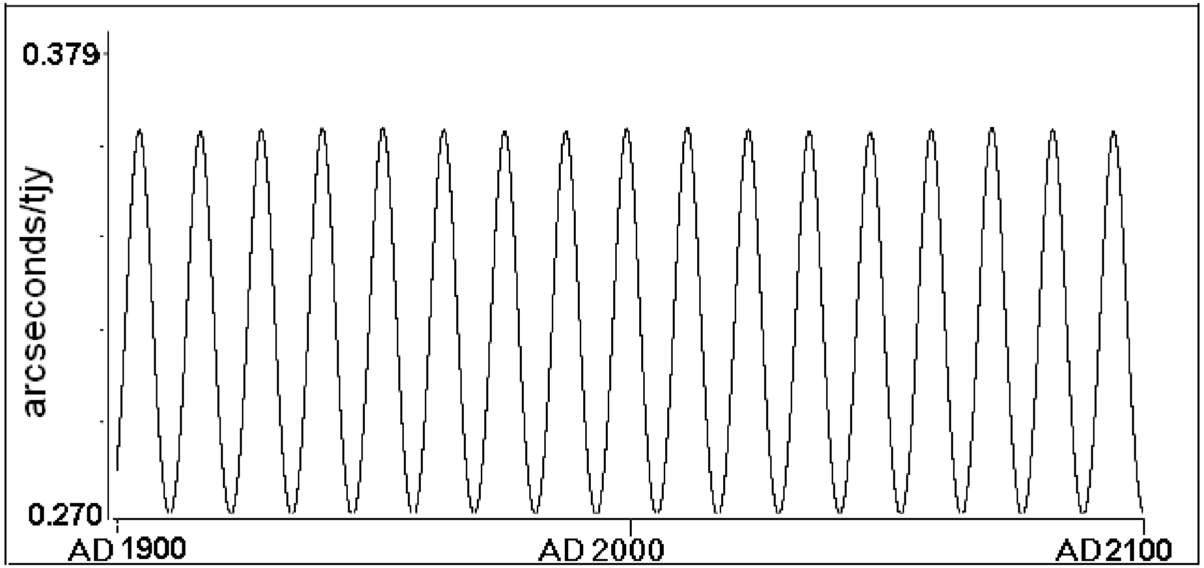

$$\sigma^{Z} = \{6''.755879 - 0''.0001435\mathrm{T} + \ldots$$

$$+10^{-6}\dot{\lambda}_4 \left[516''.062\cos\lambda_4 - 229''.326\sin\lambda_4 + \mathrm{T}\left(22''.803\cos\lambda_4 + 37''.729\sin\lambda_4\right) + \right.$$

$$\left. +\mathrm{T}^2\left(-0''.784\cos\lambda_4 + 1''.349\sin\lambda_4\right) + \ldots\right] + \ldots\} / \mathrm{tjy}.$$

$\lambda_4 = 6.20347594486 + 3340.6124266998\,\mathrm{T}$ is the mean longitude of Mars.

# Jupiter (fragment)



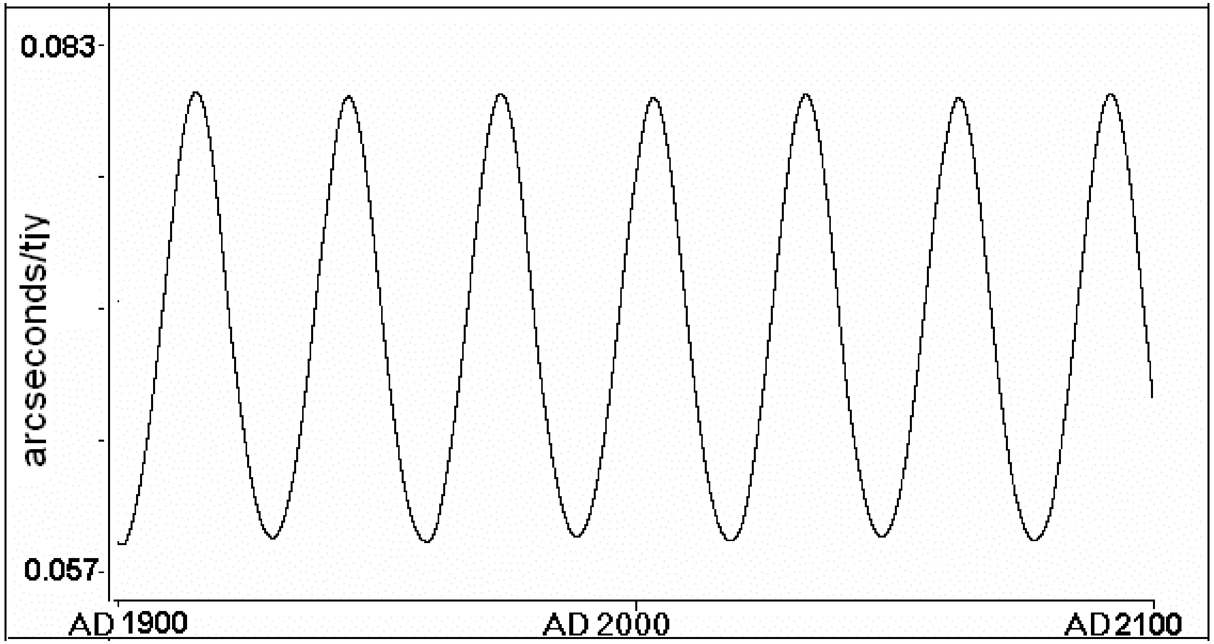

$$\sigma^{Z} = \{0''.311586 \;-\; 0''.000058\,\mathrm{T} + \ldots$$

$$+10^{-6}\,\dot{\lambda}_5\left[82''.830\cos\lambda_5 + 21''.289\sin\lambda_5 + \mathrm{T}\left(1''.955\cos\lambda_5 + 3''.997\sin\lambda_5\right) + \right.$$

$$\left. +\mathrm{T}^2\left(0''.923\cos\lambda_5 - 1''.756\sin\lambda_5\right) + \ldots\right] + \ldots\} / \mathrm{tjy}.$$

$\lambda_5 = 0.59954632934 + 529.6909650946\mathrm{T}$ is the mean longitude of Jupiter.

# Saturn (fragment)



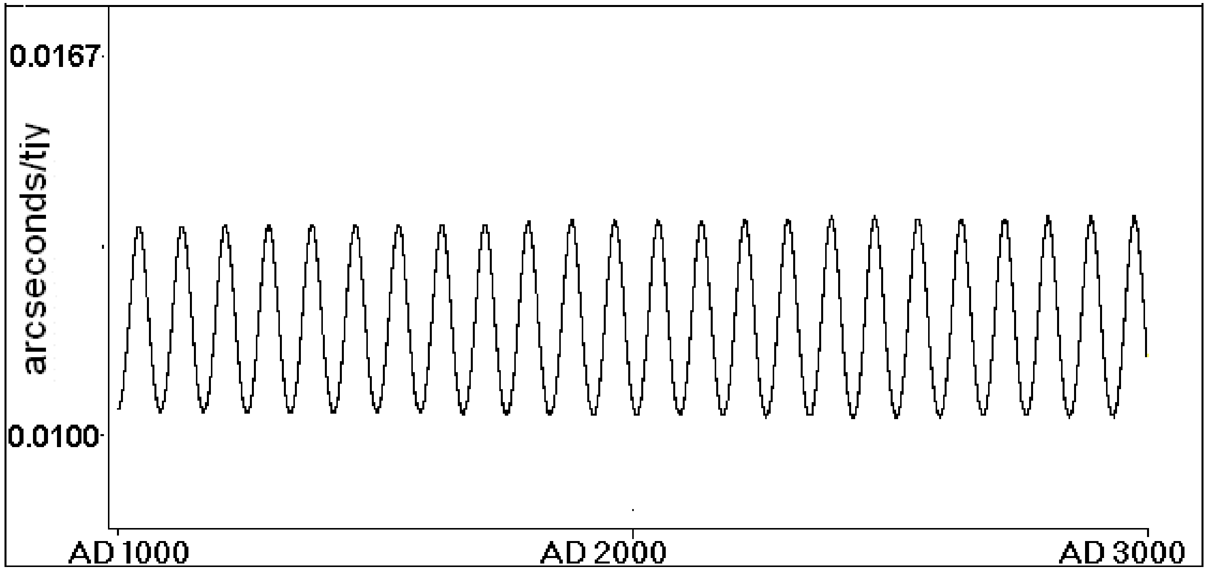

$$\sigma^{Z} = \{0''.068569 - 0''.000224\,T + \ldots +$$

$$+10^{-6}\,\dot{\lambda}_6\,[-2''.710\cos\lambda_6 + 53''.014\sin\lambda_6 + T\,(-5''.126\cos\lambda_6 - 3''.476\sin\lambda_6) +$$

$$+T^2\,(-2''.139\cos\lambda_6 + 5''.548\sin\lambda_6) + \ldots] + \ldots\} \,/\, tjy.$$

$\lambda_6 = 0.87401658845 + 213.2990954380T$ is the mean longitude of Saturn.

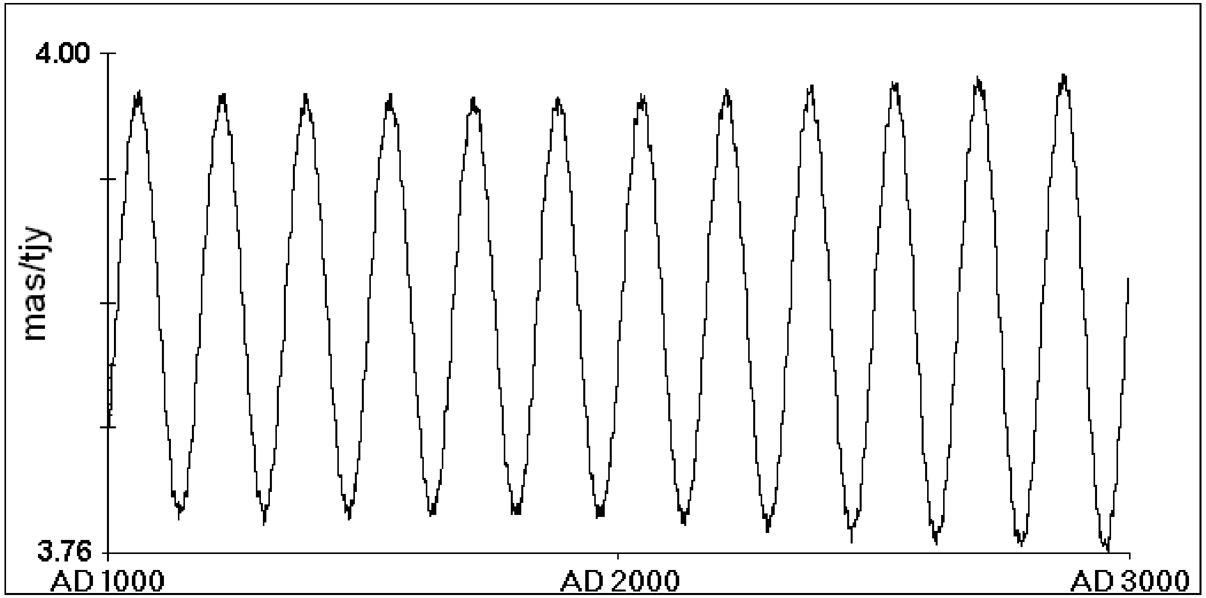
# Uranus




$$\sigma^{Z} = \{0''.011928 + 0''.000089T + \ldots$$
$$+10^{-6}\dot{\lambda}_7\left[-22''.280\cos\lambda_7 + 3''.492\sin\lambda_7 + T\left(-1''.910\cos\lambda_7 - 0''.839\sin\lambda_7\right) + \right.$$
$$\left. +T^2\left(-1''.599\cos\lambda_7 - 2''.146\sin\lambda_7\right) + \ldots\right] + \ldots\} / \text{tjy}.$$

$\lambda_7 = 5.48129370354 + 74.7815985673T$ is the mean longitude of Uranus.

# Neptune



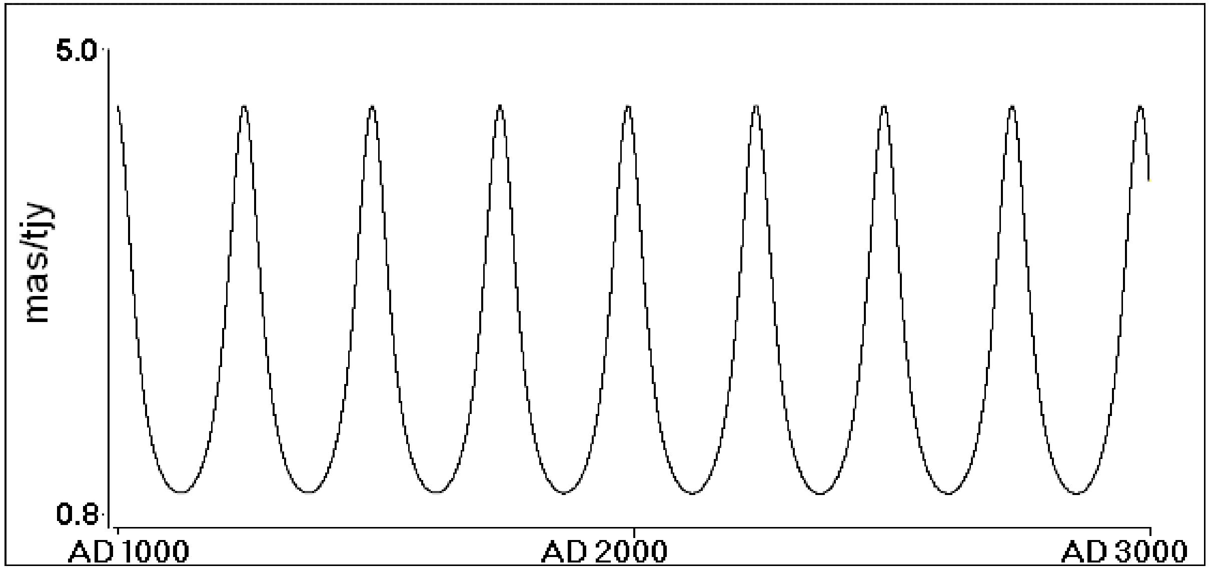

$$\sigma^Z = \{0''.003877 - 0''.000013\,T + \ldots$$

$$+10^{-6}\,\dot{\lambda}_8\,[1''.847\cos\lambda_8 + 1''.773\sin\lambda_8 + T\,(0''.280\cos\lambda_8 + 0''.270\sin\lambda_8) +$$

$$+T^2\,(0''.604\cos\lambda_8 + 1''.167\sin\lambda_8) + \ldots] + \ldots\}\,/\,\text{tjy}.$$

$\lambda_8 = 5.31188611871 + 38.1330356378T$ is the mean longitude of Neptune.

# Pluto

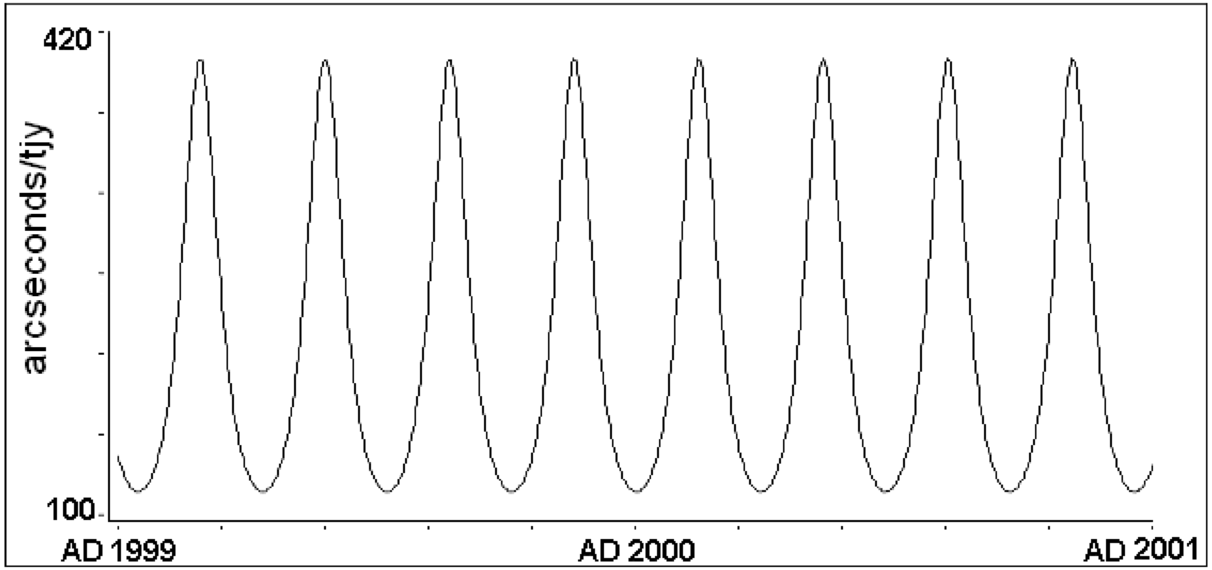



$$\sigma^Z = \{0''.001939 \; + \; 0''.000087\,T + \ldots$$
$$+10^{-6}\,\dot{\lambda}_9\,[57''.447\cos\lambda_9 - 0''.665\sin\lambda_9 + T\,(3''.138\cos\lambda_9 - 24''.301\sin\lambda_9) +$$
$$+T^2\,(38''.590\cos\lambda_9 - 1''.354\sin\lambda_9) + \ldots] + \ldots\}\,/\,\text{tjy}.$$

$\lambda_9 = 0.2480488137 + 25.2270056856T$ is the mean longitude of Pluto, which is determined by the least squares method.

The behavior of the Pluto's component $\sigma^Z$ is similar to that of Mercury.

# Mercury (fragment)




It is explained by the fact that the values of the eccentricities of the orbits of Pluto and Mercury are close to each other.

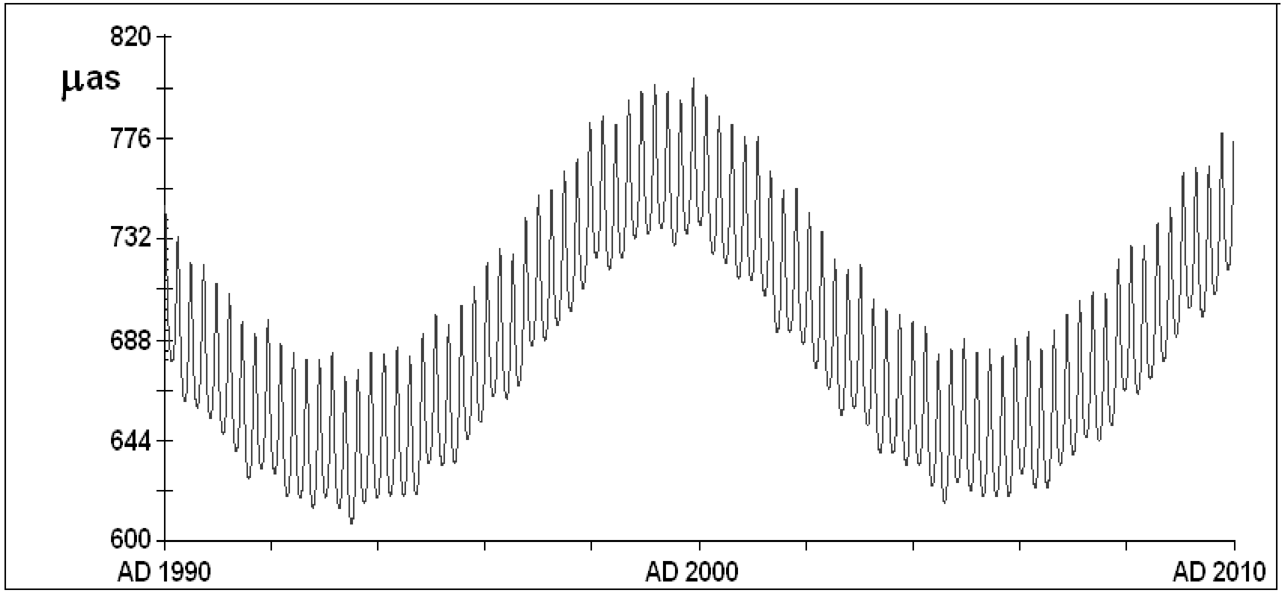
$$\sigma^{Z} = \{214''.905 + 0''.012\mathrm{T} + \ldots$$

$$+10^{-6}\dot{\lambda}_1 [1086''.273\cos\lambda_1 + 4882''.196\sin\lambda_1 + \mathrm{T}(-134''.507\cos\lambda_1 + 35''.242\sin\lambda_1) +$$

$$+\mathrm{T}^2(-0''.439\cos\lambda_1 - 1''.687\sin\lambda_1) + \ldots] + \ldots\} / \mathrm{tjy}.$$

$\lambda_1 = 4.40260867435 + 26087.903145742\mathrm{T}$ is the mean longitude of Mercury.

# The Sun (fragment)




The geodetic rotation of the Sun arises when the Sun is orbiting relatively the barycentre of the solar system in the gravitational field of the major planets and the Moon. The vector of the geodetic rotation of the Sun is determined by the orbital motion of the planets.

$$\sigma^z = 0''.000692 \;-\; 0''.298 \cdot 10^{-6} \mathrm{T} + \ldots$$

$$+10^{-6} \dot{\lambda}_1 [6''.226 \cos \lambda_1 + 27''.982 \sin \lambda_1 + \mathrm{T}(-0''.770 \cos \lambda_1 + 0''.206 \sin \lambda_1) + \ldots] + \ldots$$

$$+10^{-6} \dot{\lambda}_5 [55''.823 \cos \lambda_5 + 14''.358 \sin \lambda_5 + \mathrm{T}(1''.407 \cos \lambda_5 + 2''.697 \sin \lambda_5) + \ldots] + \ldots$$

## Table 1. The main secular and periodic terms of the geodetic rotation

| Object | Secular part | Periodic part | Eccentricity of the orbit |
|---|---|---|---|
| Mercury | 214".905 T | $1086''.273\cdot 10^{-6}\sin\lambda_1 - 4882''.196\cdot 10^{-6}\cos\lambda_1$ | 0.206 |
| Venus | 43".124 T | $-56''.907\cdot 10^{-6}\sin\lambda_2 - 64''.182\cdot 10^{-6}\cos\lambda_2$ | 0.007 |
| the Earth | 19".199 T | $-34''.285\cdot 10^{-6}\sin\lambda_3 - 149''.227\cdot 10^{-6}\cos\lambda_3$ | 0.017 |
| the Moon | 19".495 T | $-34''.285\cdot 10^{-6}\sin\lambda_3 - 149''.227\cdot 10^{-6}\cos\lambda_3$ $+30''.212\cdot 10^{-6}\sin D$ | |
| Mars | 6".756 T | $516''.062\cdot 10^{-6}\sin\lambda_4 + 229''.326\cdot 10^{-6}\cos\lambda_4$ | 0.093 |
| Jupiter | 0".312 T | $82''.830\cdot 10^{-6}\sin\lambda_5 - 21''.289\cdot 10^{-6}\cos\lambda_5$ | 0.048 |
| Saturn | 0".069 T | $-2''.710\cdot 10^{-6}\sin\lambda_6 - 53''.014\cdot 10^{-6}\cos\lambda_6$ | 0.056 |
| Uranus | 0".012 T | $-22''.280\cdot 10^{-6}\sin\lambda_7 - 3''.492\cdot 10^{-6}\cos\lambda_7$ | 0.046 |
| Neptune | 0".004 T | $1''.847\cdot 10^{-6}\sin\lambda_8 - 1''.773\cdot 10^{-6}\cos\lambda_8$ | 0.009 |
| Pluto | 0".002 T | $57''.447\cdot 10^{-6}\sin\lambda_9 + 0''.665\cdot 10^{-6}\cos\lambda_9$ | 0.249 |
| the Sun | 0".001 T | $6''.226\cdot 10^{-6}\sin\lambda_1 - 27''.982\cdot 10^{-6}\cos\lambda_1$ $+55''.823\cdot 10^{-6}\sin\lambda_5 - 14''.358\cdot 10^{-6}\cos\lambda_5$ | |

# CONCLUSION

- For the Sun and the superior planets (except Mars) the geodetic rotation is insignificant.
- For the earth group planets and the Moon the geodetic rotation is considerable and has to be taken into account for the construction of the high-precision theories of the rotational motion.
- The geodetic rotation has to be taken into account if the influence of the dynamical figure of a body on its orbital-rotational motion is studied in the post-Newtonian approximation.
- In addition, the lunar laser range processing has to use the relativistic theory of the rotation of the Moon, as well as that of the Earth.

# ACKNOWLEDGMENTS

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