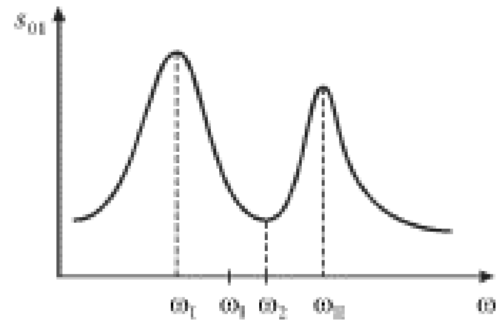
Harmonic Models of Tide-Generating Potential of Terrestrial Planets



by

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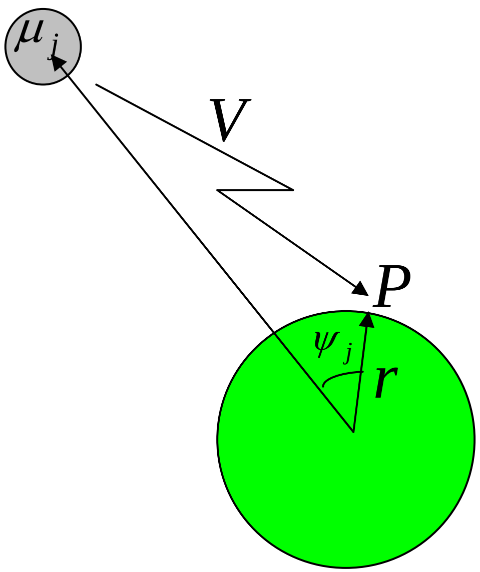
Planet Tide-Generating Potential (TGP)

The classical representation of the TGP generated by the Sun, planetary moons and other planets at point P on the planet's surface at epoch *t* is

$$V(t) = \sum_j \mu_j \sum_{n=2}^{\infty} \frac{r^n}{r_j^{n+1}(t)} P_n(\cos\psi_j(t))$$



where

V - TGP value at P;

r - planetocentric distance to P;

μ_j - gravitational parameter of j^{th} body;

r_j - planetocentric distance to j^{th} body;

ψ_j - angle between P and j^{th} body;

P_n – Legendre polynomial of degree n.

Harmonic Development of the Planet TGP

$$V(t) \equiv \sum_{n=2}^{\infty}\sum_{m=0}^{n} V_{nm}(t) = \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{r}{R_{pl}}\right)^{n} \overline{P}_{nm}(\sin\varphi')$$
$$\times[C_{nm}(t)\cos m\theta(t) + S_{nm}(t)\sin m\theta(t)]$$

where

To be expanded to Poisson series

$$C_{nm}(t) = \frac{1}{2n+1}\sum_{j}\frac{\mu_j}{R_{pl}}\left(\frac{R_{pl}}{r_j(t)}\right)^{n+1} \overline{P}_{nm}\left(\sin\delta_j(t)\right)\cos m\alpha_j(t),$$
$$S_{nm}(t) = \frac{1}{2n+1}\sum_{j}\frac{\mu_j}{R_{pl}}\left(\frac{R_{pl}}{r_j(t)}\right)^{n+1} \overline{P}_{nm}\left(\sin\delta_j(t)\right)\sin m\alpha_j(t),$$

R_{pl} is the planet mean equatorial radius;

$\theta(t)$ is local mean sidereal time at $P(r,\varphi',\lambda)$;

$\theta(t) = \lambda + W(t)$ ($W(t)$ is the planet prime meridian angle);

$\alpha_j(t)$, $\delta_j(t)$ are the right ascension and declination of the j^{th} attracting body.

Expansion of the TGP Coefficients

Step 1: Calculation of numerical values for $C_{nm}(t),\ \dot{S}_{nm}(t)$

For Mercury & Venus: values are calculated over the time interval 1000-3000 with a sampling step of 1 day.
Source of Sun/planets ephemerides: DE-406 (Standish 1998).

For Mars: values are calculated over the time interval 1900-2100 with a sampling step of 0.01 days (because of the fast motion of Phobos and Deimos).
Source of Sun/planets ephemerides: DE-406 (Standish 1998).
Source of Phobos/Deimos ephemerides: MARTSAT theory (Kudryavtsev et al. 1997).

Step 2: The sampled values are processed by a spectral analysis method (Kudryavtsev 2004). Unlike the classical analysis Fourier, the expansion is here done to Poisson series.

Method of Expansion to Poisson Series

The method allows to expand long-term series of sampled values (e.g., DE/LE-406 covers 6,000 years). As a result, close frequencies are better separated.

Let function $f(t)$ be sampled over the interval [-*T*, *T*].

We look for representation of $f(t)$ in the form

$$f(t) \approx A_0 + A_1 t + \ldots + A_p t^p + \sum_{k=1}^{N} \left\{ \left[A_{k0}^c + A_{k1}^c t + \ldots + A_{kp}^c t^p \right] \cos\omega_k(t) + \left[A_{k0}^s + A_{k1}^s t + \ldots + A_{kp}^s t^p \right] \sin\omega_k(t) \right\}$$

where $\omega_k(t)$ are some pre-defined polynomilal arguments

$$\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \ldots + \nu_{kq} t^q.$$

(In our study $\omega_k(t)$ are various combinations of multipliers of Delaunay arguments, planets and moons mean longitudes.)

Method of Expansion to Poisson Series (cont.)

1) At the first step we numerically calculate the following scalar products:

$$A_{lk}^{c} = \frac{1}{2T}\int_{-T}^{T} f(t)t^{l}\cos\omega_k(t)\chi(t)dt, \quad A_{lk}^{s} = \frac{1}{2T}\int_{-T}^{T} f(t)t^{l}\sin\omega_k(t)\chi(t)dt,$$

where $\chi(t) = 1 + \cos\frac{\pi}{T}t$ is the weighting function (the Hanning filter).

2) At the second step the coefficients $\left\{A_{kl}^{c}, A_{kl}^{s}\right\}_{0 \le k \le N,\ 0 \le l \le p}$ are orthogonalized.

(Problems: the number of coefficients can reach 10^4-10^5; arguments are polynomials).

Harmonic Development of Mercury TGP

$$C(S)_{nm} = A_0 + A_1 t + A_2 t^2 + \sum_{k=1}^{N} \{ [A^c_{k0} + A^c_{k1} t + A^c_{k2} t^2] \cos \omega_k(t)) + [A^s_{k0} + A^s_{k1} t + A^s_{k2} t^2] \sin \omega_k(t) \}$$

where $\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \nu_{k3} t^3 + \nu_{k4} t^4$

Attracting bodies: Sun, Venus, Earth, Moon, Mars, Jupiter, Saturn

Time interval of the development: **1000-3000**

Minimal amplitude of $C_{nm}(t), S_{nm}(t)$: **10^{-8} m^2/s^2** (max n=3)

The number of terms: **2,345** (in the original format),
1,519 (in modified HW95 format)

Accuracy of gravity tide calculation (max. error): **13 nGal**

Harmonic Development of Venus TGP

$$C(S)_{nm} = A_0 + A_1 t + A_2 t^2 + \sum_{k=1}^{N} \left\{ \left[A_{k0}^c + A_{k1}^c t + A_{k2}^c t^2 \right] \cos \omega_k(t) \right. \\ \left. + \left[A_{k0}^s + A_{k1}^s t + A_{k2}^s t^2 \right] \sin \omega_k(t) \right\}$$

where $\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \nu_{k3} t^3 + \nu_{k4} t^4$

Attracting bodies: Sun, Mercury, Earth, Moon, Mars, Jupiter, Saturn

Time interval of the development: **1000-3000**

Minimal amplitude of $C_{nm}(t), S_{nm}(t)$: **10^{-8} m^2/s^2** (max n=3)

The number of terms: **1,725** (in the original format),
1,120 (in modified HW95 format)

Accuracy of gravity tide calculation (max. error): **20 nGal**

Harmonic Development of Mars TGP

$$C(S)_{nm} = A_0 + A_1 t + A_2 t^2 + \sum_{k=1}^{N} \{ [A_{k0}^c + A_{k1}^c t + A_{k2}^c t^2] \cos \omega_k(t)) + [A_{k0}^s + A_{k1}^s t + A_{k2}^s t^2] \sin \omega_k(t) \}$$

where $\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \nu_{k3} t^3 + \nu_{k4} t^4$

Attracting bodies: Sun, Mercury, Venus, Earth, Moon, Jupiter, Saturn, Phobos, Deimos

Time interval of the development: **1900-2100**

Minimal amplitude of $C_{nm}(t), S_{nm}(t)$: $\mathbf{10^{-6}\ m^2/s^2}$ (max n=7)

The number of terms: **278** (in the original format),
162 (in modified HW95 format)

Accuracy of gravity tide calculation (max. error): **350 nGal**

Examples of the Developments Application

- *Main variations of gravity coefficients due to solid tides*

$$\Delta\overline{C}_{nm}^{ST} - i\Delta\overline{S}_{nm}^{ST} = \frac{k_{nm}}{2n+1}\sum_j \frac{\mu_j}{\mu_{pl}}\left(\frac{R_{pl}}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j} \quad \text{(IERS Conv. 2003)}$$

where k_{nm} are the Love numbers of the planets,

or (Kudryavtsev 2005)

$$\Delta\overline{C}_{nm}^{ST} = \Delta C_{nm}\cos\big(m\times W(t)\big) + \Delta S_{nm}\sin\big(m\times W(t)\big)$$

$$\Delta\overline{S}_{nm}^{ST} = \Delta S_{nm}\cos\big(m\times W(t)\big) - \Delta C_{nm}\sin\big(m\times W(t)\big) \text{, where}$$

$$\Delta C_{nm} \equiv \frac{R_{pl}}{\mu_{pl}}\left(\operatorname{Re}k_{nm}C_{nm} + \operatorname{Im}k_{nm}S_{nm}\right), \quad \Delta S_{nm} \equiv \frac{R_{pl}}{\mu_{pl}}\left(\operatorname{Re}k_{nm}S_{nm} - \operatorname{Im}k_{nm}C_{nm}\right)$$

and C_{nm}, S_{nm} are coefficients of planetary TGP development.

- *Analytical theories of terrestrial planets nutation*