Estimation of Earth Interior Parameters From a Bayesian Inversion of VLBI Nutation Data

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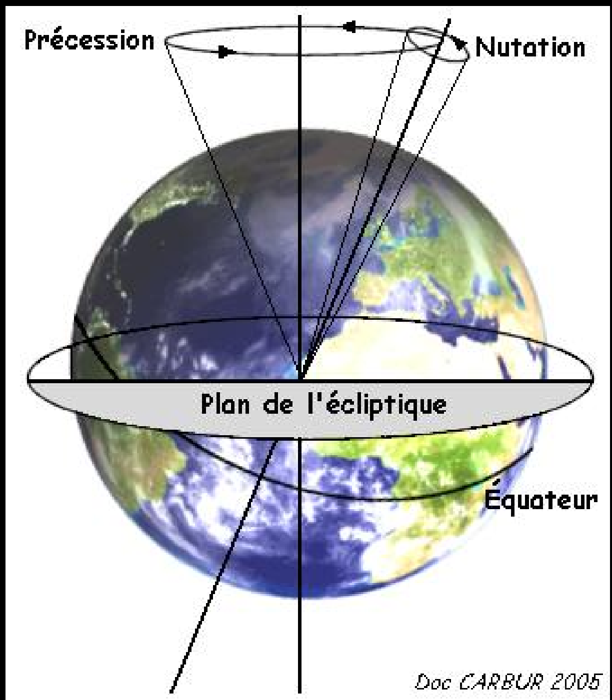
Nutation and the Earth interior



The Moon, the Sun and the planets exert a gravitational torque on the equatorial bulge

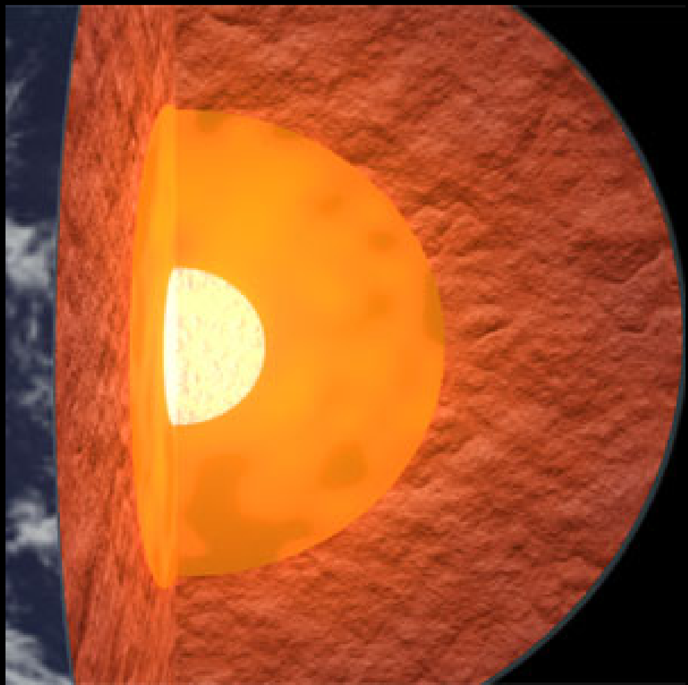
Response of the Earth: precession/nutation

The torque is known very accurately from celestial mechanics

The rotational response of the Earth depends on its internal structure

Nutation and the Earth interior

Earth interior model:



3 ellipsoidal layers: mantle, liquid outer core, and solid inner core

- spheroidally stratified (e.g. constant density on spheroidal surfaces)
- characterized by its moments of inertia: A, C, A^f, C^f,... and thus its dynamical ellipticity: $e \equiv (C - A)/A$, e^f,...

- anelastic: the Earth is deformable with a delayed response to the forcing $\longrightarrow$ complex compliances describing the deformability of each layer

Nutation and the Earth interior

- Interactions between the layers:
  - inertial coupling (pressure on the elliptical boundaries)
  - gravitational coupling
  - other couplings (friction, topographic, electromagnetic,...) not modeled and described by general coupling constants
- External geophysical fluids: ocean, atmosphere

Geophysical parameters of this Earth model:

Dynamical ellipticities, compliances, coupling constants between the 3 layers

Nutation and the Earth interior

Nutation measurement:

VLBI technique: very accurate data

Data used: GSFC “gsf2007b.eops”

Main interest of nutation to study Earth interior:

- Gravitational torque known very accurately from celestial mechanics
- Nutation measured very precisely with VLBI technique
- Earth response to the torque depends on internal structure

$\longrightarrow$ VLBI data allow to constrain parameters of the Earth interior model

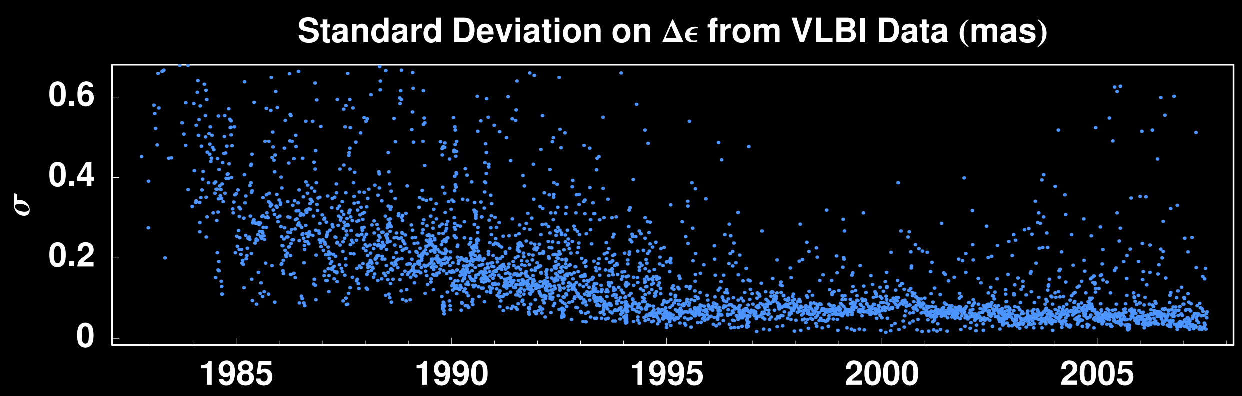
Parameters Estimation

I. Direct estimation from the VLBI *time series*

Main advantages:

- All the available data are used, no extraction of amplitudes at given frequencies (as in MHB)
  $\longrightarrow$ No loss of information

- Takes into account the time variable error on VLBI data



Time domain nutation model:

$$\begin{aligned}\hat{\eta}(t) &= \sum_{\sigma} \tilde{\eta}(\sigma) e^{i \arg(\sigma, t)} \\ &+ \left(P \sin(\epsilon_0) + i \frac{d\Delta\epsilon}{dt} \right) (t - t_0) \\ &+ c_{\psi} \sin(\epsilon_0) + i c_{\epsilon}\end{aligned}$$

Periodic terms

Linear rates

Constant offsets

Parameters Estimation

II. Inversion method

Setting:

$$\underbrace{d_i}_{\text{data}} = \underbrace{m(x_i, \theta)}_{\text{model}} + \underbrace{e_i}_{\text{measurement error}} + \underbrace{\epsilon_i}_{\text{modeling error}}$$

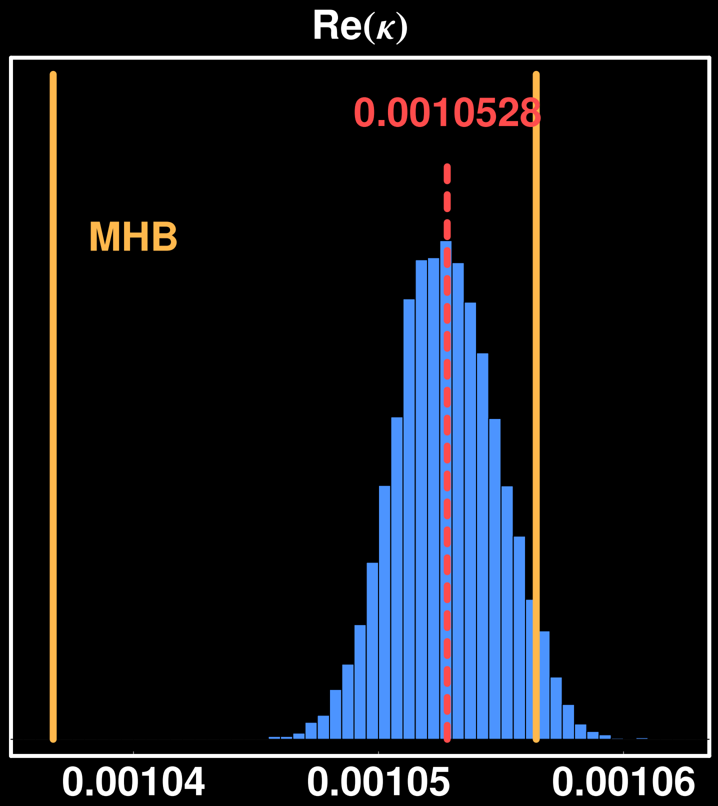
Modeling error takes into account the imperfections in the nutation model: simple Earth interior model, ocean tides, atmosphere, free FCN not modeled,...

We choose to use the **Bayesian inversion method** because:

- No linearization of the model
- Easy to include modeling uncertainties

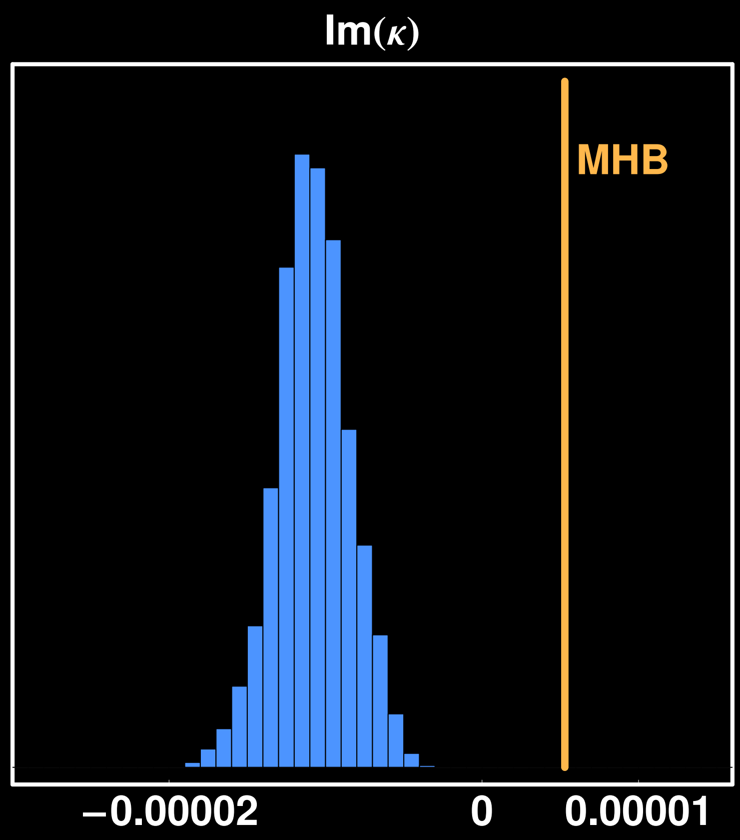
Parameters Estimation

Bayesian inversion: The parameters are random variables and the result of the method is the estimation of their probability density function (pdf).



- Estimation of the mean and standard deviation of the pdf
- We compare our estimates with those of MHB (3 σ intervals)

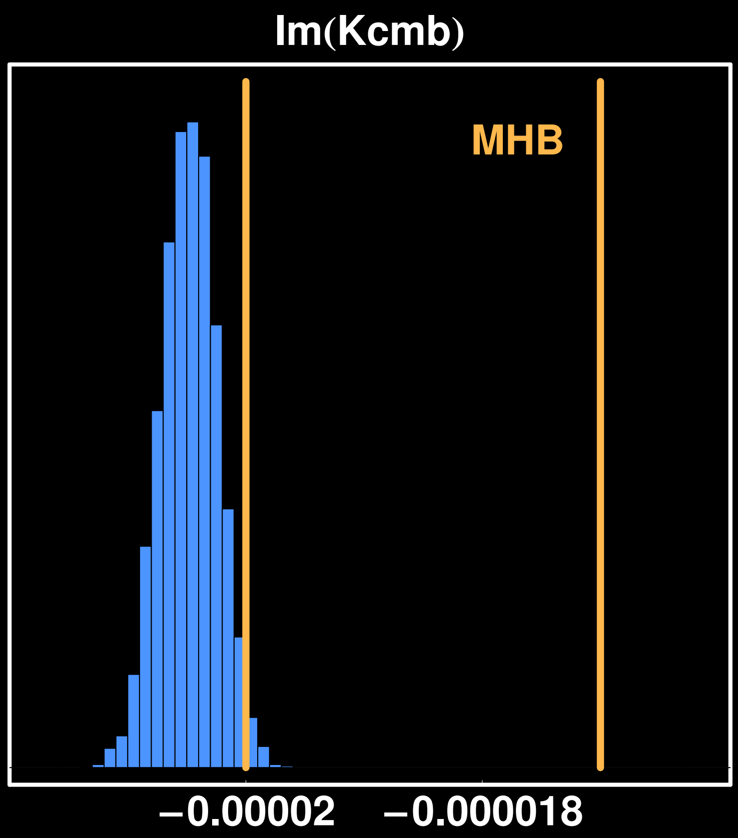
Results: Compliance of the whole Earth



Imaginary part

- Imaginary component of the compliance: due to the delay of the deformational response
- In MHB this parameter is not estimated but computed theoritically
- Difference from MHB due to the 0.7 scaling factor introduced by MHB in the ocean tides current contribution

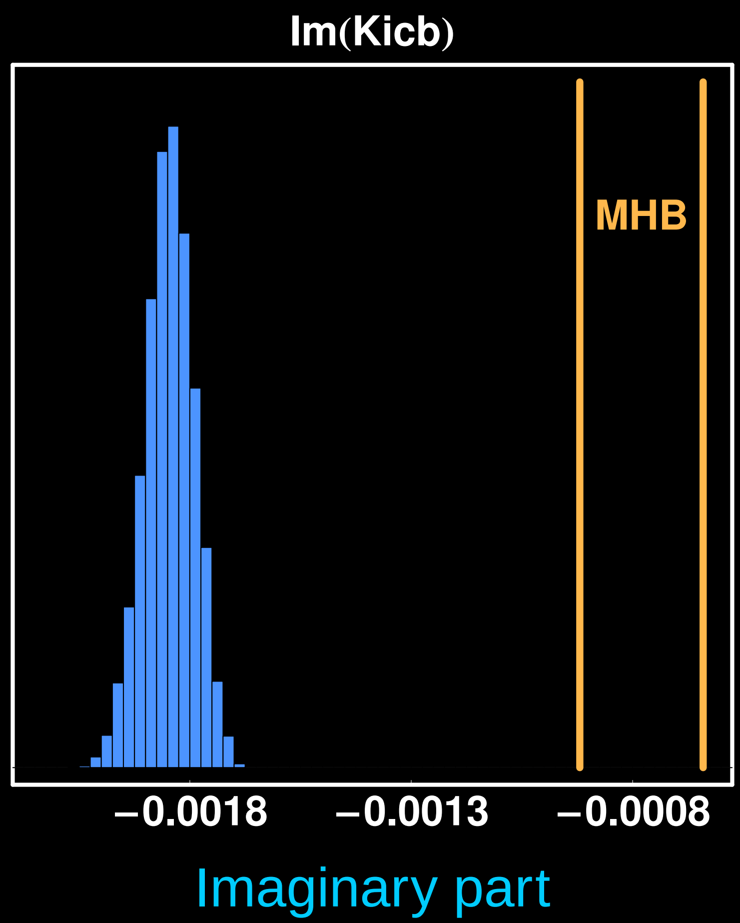
Results: Coupling constant at the CMB



Imaginary part

- Different from MHB but there is still an overlap between the 3σ domains
- Error 2 times smaller
- Resulting Q of the FCN mode: $\simeq 14000 \pm 600$
  MHB obtained: $\simeq 19000 \pm 1400$

Results: Coupling constant at the ICB

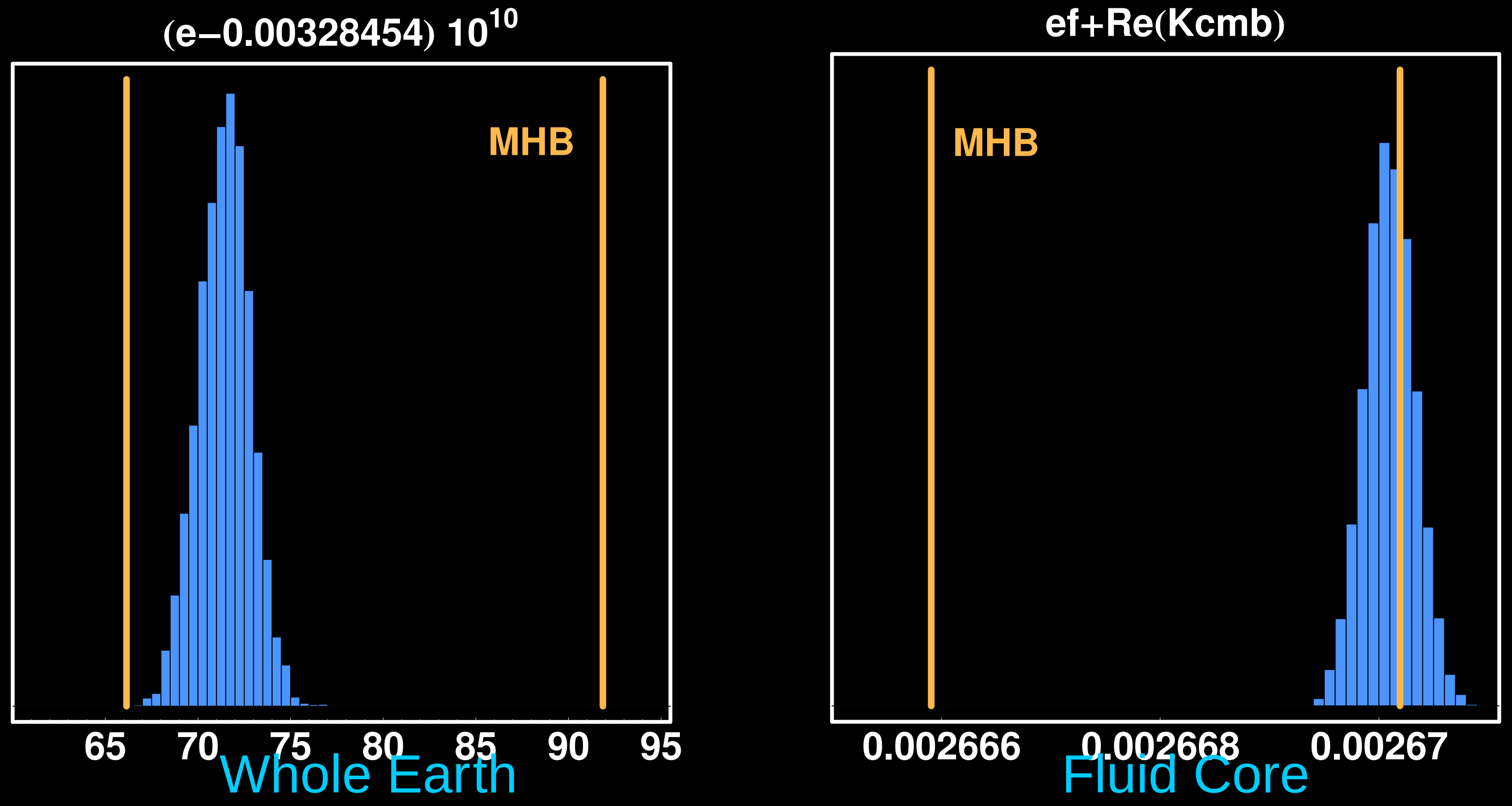


- The mean value is more than 2 times larger than MHB (but still same order of magnitude)
- Similar estimated error as MHB. The real part has even a larger error
  $\longrightarrow$ more realistic ?
- Resulting Q of the FICN mode: $\simeq 270 \pm 30$.
  MHB's value is: $\simeq 640 \pm 100$

Conclusions

- The Earth interior parameters can be estimated *directly* from the time domain VLBI data. No need of the extraction of amplitudes at some given frequencies (as done in MHB)
- Estimated error on the parameters always smaller than in MHB except for the parameters at the ICB, which seems reasonable
- Differences between this computation and MHB due to:
  - inversion strategy
  - 7 years of additional data
  - empirical parameters not justified (e.g. 0.7 factor)
- K^{ICB} is the most different parameter between our estimation and MHB. Its interpretation in terms of physical coupling mechanisms must be done with care

Results: Dynamical ellipticities

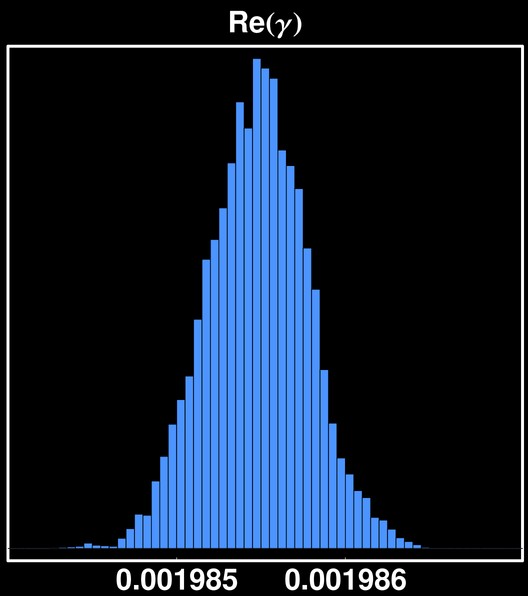


- Estimations in agreement with MHB
  In particular, the FCN frequency is in agreement with their one
- Errors 3 times smaller

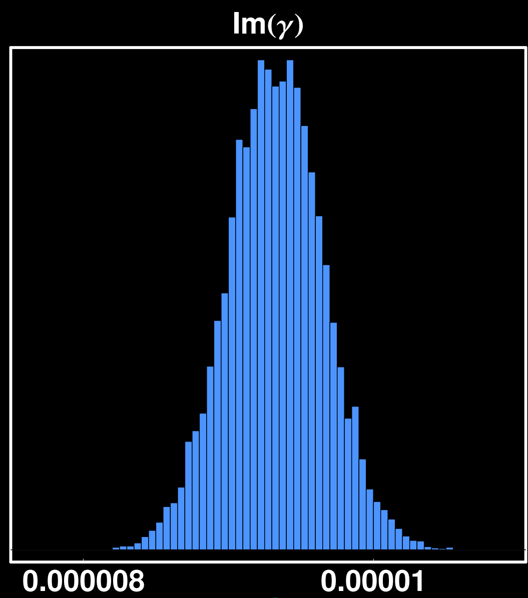
Results: Compliances

- In this paper: complex anelastic compliances estimated from VLBI data
- In MHB: real elastic compliances estimated from VLBI data + anelastic correction computed independently

Compliance for the fluid core



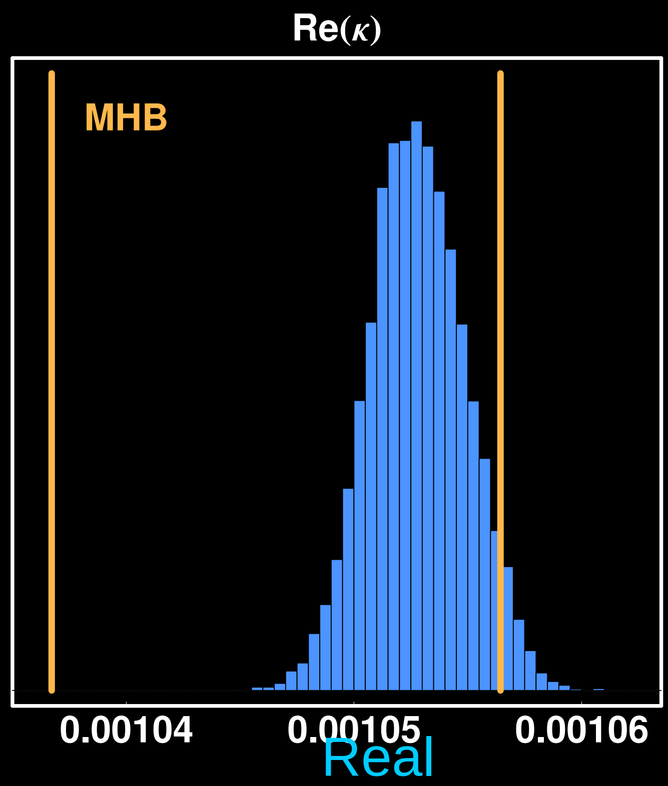
Real

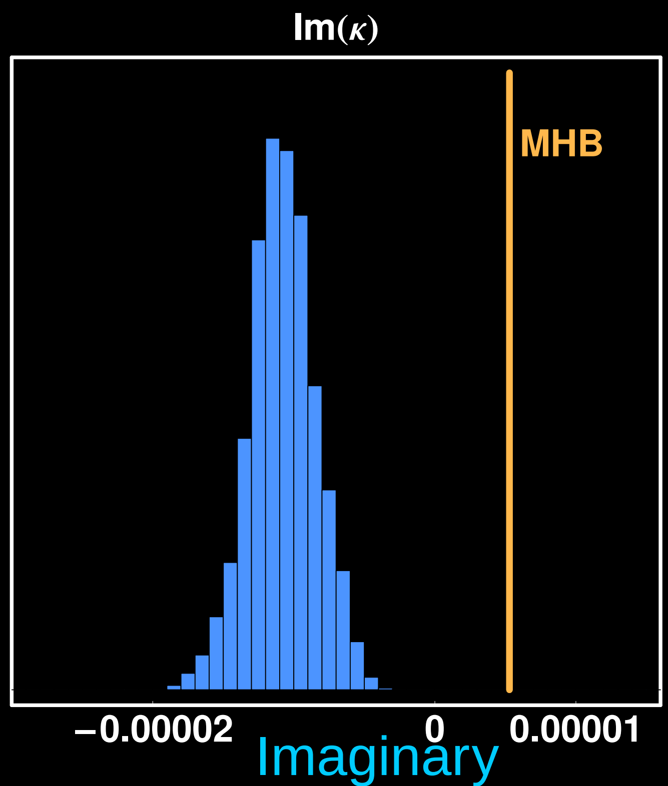


Imaginary

NB: Can not be compared with MHB because unknown anelastic correction

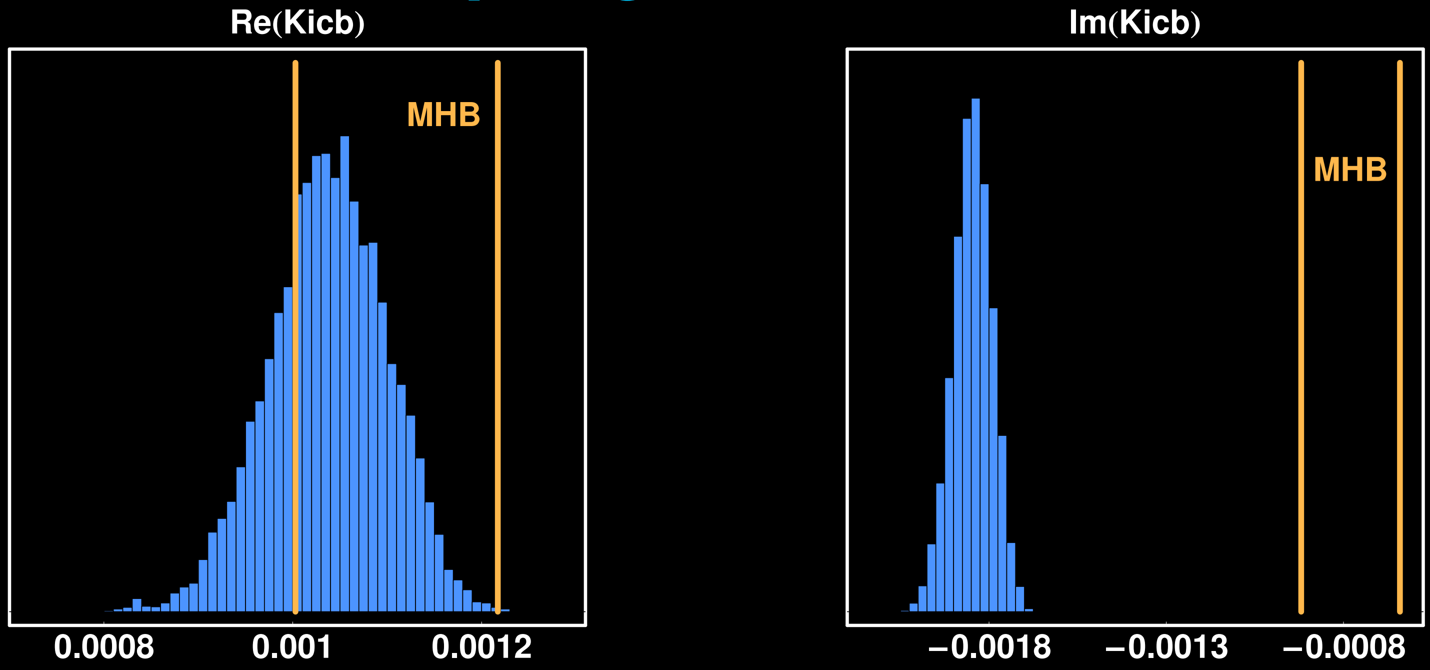
Compliance for the whole Earth





- Real: in agreement with MHB, smaller error
- Imaginary: Difference from MHB due to the 0.7 scaling factor introduced by MHB in the ocean tides current contribution

Results: Coupling constant at the ICB



Both component have larger error (more realistic ?)

- Real: in agreement with MHB $\longrightarrow$ FICN frequency in agreement
- Imaginary:
  - More than 2 $\times$ larger than MHB, but still same order of magnitude
  - Q of the FICN mode: $\simeq 270 \pm 30$, while MHB's value is $\simeq 640 \pm 100$