Relativity in Fundamental Astronomy: solved and unsolved problems

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Relativistic effects in fundamental astronomy

Several general-relativistic effects are confirmed with the following precisions:

- VLBI $\pm 0.0003$
- HIPPARCOS $\pm 0.003$
- Viking radar ranging $\pm 0.002$
- Cassini radar ranging $\pm 0.000023$
- Planetary radar ranging $\pm 0.001$
- Lunar laser ranging I $\pm 0.00045$
- Lunar laser ranging II $\pm 0.005$

No way to model all these kind of data without relativity

More precise data to come:
Gaia, BepiColombo, LLR with APOLLO, etc.
Why general relativity?

• Newtonian models cannot describe high-accuracy observations:
  • many relativistic effects are many orders of magnitude larger than the observational accuracy

• The simplest theory which successfully describes all available observational data:

APPLIED RELATIVITY

a multidisciplinary research field
The IAU 2000 framework

- Three standard astronomical reference systems were defined
  - BCRS (Barycentric Celestial Reference System)
  - GCRS (Geocentric Celestial Reference System)
  - Local reference system of an observer

- All these reference systems are defined by the form of the corresponding metric tensors.

Technical details:
- Brumberg, Kopeikin, 1988-1992
- Damour, Soffel, Xu, 1991-1994
- Klioner, Voinov, 1993
- Klioner, Soffel, 2000
- Soffel, Klioner, Petit et al., 2003
General relativity for space astrometry

- Relativistic reference systems
  - Relativistic equations of motion
  - Equations of signal propagation
  - Definition of observables
  - Relativistic models of observables
    - Astronomical reference frames
    - Observational data

Coordinate-dependent parameters
Parametrized post-Newtonian (PPN) formalism

• K. Nordtvedt, C. Will (1970-)

  • covers a class of possible metric theories of gravity in the weak-field slow-motion (post-Newtonian) approximation:

    many metric theories of gravity were investigated and a generic form of the post-Newtonian metric tensor of a system of N bodies was derived.

  • the metric tensor contains 10 numerical ad hoc parameters.

  • two most important parameters are $\gamma$ and $\beta$ ($\gamma = \beta = 1$ in GRT)

  • all predictions of the theories can be expressed using these parameters

• Klioner, Soffel, 2000; Kopeikin, Vlasov, 2004: GCRS with PPN parameters
What is well known theoretically

- post-Newtonian relativistic reference systems
- post-Newtonian equations of motion for test particles and massive bodies having only masses
- multipole structure of the post-Newtonian gravitational field
- post-Newtonian equations of motion of bodies with multipole structure
- post-Newtonian equations of rotational motion
- post-Newtonian theory of light propagation

- some features of the post-post-Newtonian effects
  (by no means so detailed understanding as for the post-Newtonian approximation)
What is poorly known theoretically

• embedding of the post-Newtonian BCRS in the cosmological background

...could be important for the interpretation of high-accuracy observations (Gaia, VLBI, etc.)
Some efforts have been started by a number of people

• post-post-Newtonian relativistic reference systems (especially, GCRS)
• multipole structure of the post-post-Newtonian gravitational field
• detailed post-post-Newtonian equations of motion and their solution and analysis

...currently not interesting from the practical point of view...
What is thought to be clear theoretically, but represents a lot of difficulties in practice

- **post-Newtonian equations of motion of bodies with multipole structure**
  
  very complicated and implicit algorithm for post-Newtonian approximation; never applied for numerical calculations

  possible source of “fly-by anomaly”? unexpected additional increase of the velocity of several spacecrafts after close approach with the Earth

- **post-Newtonian equations of rotational motion**
  
  complicated, but clear, explicit algorithm

  first implemented by Klioner, Soffel, Le Poncin-Lafitte (2007)
What is thought to be clear theoretically, but represents a lot of difficulties in practice

Also much “simpler” issues…

- meaning of relativistic coordinate time scales

  TCG is NOT “time at geocenter”!
  TCB is NOT “time at barycenter”!
  TT is NOT “time on the rotating geoid”!

- relativistic scaling of astronomical quantities

  scaling does not mean changing of units

Applied Relativity as multidisciplinary research topic

“Cultural” problems:

- Different mathematical languages

Example: STF tensors are used instead of spherical harmonics etc. to described the bodies’ structure in the equations of motion

\[
\frac{d}{dT_{CG}} \left(C^{ab \omega^b} \right) = \sum_{l=1}^{\infty} \frac{1}{l!} \varepsilon_{abc} M_{bL} G_{cL} + \varepsilon_{abc} \Omega_{\text{iner}}^b C^{cd \omega^d}.
\]
STF approach to compute the torque

1. For any STF tensor: \( T_L = \sum_{m=-l}^{l} T_{lm} \hat{Y}_L^{lm} \)

   \( 2l + 1 \) real numbers

   STF basis

2. For the multipole moments of the Earth: \( M_L = \sum_{m=-l}^{l} M_{lm} \hat{Y}_L^{lm} \)

   equivalent to the \( 2l + 1 \) harmonic coefficients

3. For the tidal moments: \( G_L = \sum_{m=-l}^{l} G_{lm} \hat{Y}_L^{lm} \)

   functions of the ephemeris data
STF approach to compute the torque

4. The torque:
\[
\sum_{l=1}^{\infty} \frac{1}{l!} \varepsilon_{abc} M_{bL} G_{cL} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \sum_{m'} \alpha_{lmm'}^a M^{lm} G^{lm'}
\]

\[m' = -m - 1, -m + 1 \quad \text{for} \quad a = 1, 2\]

\[m' = -m \quad \text{for} \quad a = 3\]

This is equivalent to the classical formulation with Legendre polynomials for Newtonian tidal potentials.

For relativistic tidal potential the STF tensor is the most efficient way also from the computational point of view.
Legendre polynomials vs. STF tensors for the Newtonian torque

\[ \mu \text{as} \]

years from J2000
Applied Relativity as multidisciplinary research topic

Further “cultural” problems:

• only numerical magnitude is interesting for practical work
• in relativity analytical orders of magnitude are used

The situation is similar to “analytical expansions” (e.g., in powers of eccentricities) in classical celestial mechanics…

\[ c^{-2}, c^{-4}, e^4, \ldots \]
Post-post-Newtonian light propagation?

Full post-post-Newtonian expression for the Shapiro time delay with PPN parameters (Klioner, Zschocke, 2007):

\[
c \tau = R + (1 + \gamma) m \log \frac{x + x_0 + R}{x + x_0 - R} \\
+ \frac{1}{8} \alpha \epsilon \frac{m^2}{R} \left( \frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\
+ \frac{1}{4} \alpha (8(1 + \gamma) - 4\beta + 3\epsilon) m^2 \frac{R}{|x \times x_0|} \arctan \frac{x^2 - x_0^2 + R^2}{2|x \times x_0|} \\
- \frac{1}{4} \alpha (8(1 + \gamma) - 4\beta + 3\epsilon) m^2 \frac{R}{|x \times x_0|} \arctan \frac{x^2 - x_0^2 - R^2}{2|x \times x_0|} \\
+ \frac{1}{2} (1 + \gamma)^2 m^2 \frac{R}{|x \times x_0|^2} (x - x_0 - R) (x - x_0 + R).
\]

\[m = \frac{GM}{c^2}\]

The higher-order terms give up to 10 meters. Are all these terms relevant?
Post-post-Newtonian light propagation?

NO!

The only numerically relevant term can be written as

\[ c \, \tau = R + (1 + \gamma) \, m \, \log \frac{x + x_0 + R + (1 + \gamma) \, m}{x + x_0 - R + (1 + \gamma) \, m} \]

This has already been derived by Moyer (2003) in a different way.

All other terms can be estimated as

\[ c \, \delta \, \tau \leq \frac{m^2}{d} \left( \frac{3}{4} + \frac{15}{4} \pi \right) \]

This gives maximally 4 cm for Sun-grazing ray, and much less in typical cases…

Similar situation with light deflection, with post-post-Newtonian equations of motion, etc.
Applied Relativity as multidisciplinary research topic

“Social” or educational problems:

- People doing “practical work” have limited knowledge of relativity and often cannot understand the details of the suggested relativistic models.
- People working in relativity have limited experience with real data and often cannot judge if what they suggest is at all relevant.

A. Basic education in relativity must be a part of astronomical education
B. Discussions are necessary

V.M.Lipunov
IAU Commission 52
“Relativity in Fundamental Astronomy”

• Created by the IAU in 2006

• Present Organizing Committee:

Sergei A. Klioner, President
Gérard Petit, Vice President
Victor A. Brumberg
Nicole Capitaine
Agnès Fienga
Toshio Fukushima
Bernard Guinot
Cheng Huang
François Mignard
Ken Seidelmann
Michael Soffel
Patrick Wallace
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• is thought to provide a stimulate the exchange on relativistic issues

• Current projects:
  • Compile a list of unsolved problems
  • Frequently asked questions
  • Relativistic glossary for astronomers
  • “Task teams”, an ad hoc discussion group for very well posed issues:
    • One task team is currently activated: “TDB units”

Question: “What are the units of TDB?”
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• Topics to be discussed in the future:

1. Ecliptic or “ecliptic image” in the GCRS
   to be started soon with the input of the Earth rotation project in Dresden

2. The system of astronomical units
   difficult subject that should be discussed by a broader community
Do we need astronomical units?

- The reason to introduce astronomical units was that angular measurements were many orders of magnitude more accurate than distance measurements.

- BUT
  - The situation has changed crucially since that time!
  - Solar mass is time-dependent just below current accuracy of ephemerides
    \[ \frac{\dot{M}_{\text{Sun}}}{M_{\text{Sun}}} \sim 10^{-13} \text{ yr}^{-1} \]
  - Very confusing situation with astronomical units in relativistic framework

- Why not to define AU conventionally as fixed number of meters?
- Do you see any good reasons for astronomical units in their current form?
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- All interested members of the IAU are kindly invited to join:
  
  http://astro.geo.tu-dresden.de/RIFA
Backup
Content

1. No way to live without relativity
2. IAU 2000 Framework as a standard tool to model
3. PPN formalism as a standard tool to test
4. Is everything OK? Not quite: main problems
5. Example I: numerical magnitude vs. analytical order of magnitude
6. Example II: SFT tensors
7. Example III: scaling and units, astronomical units
Data analysis models compatible with IAU 2000

- Ephemeris construction (JPL, IMCCE, IAA)
- VLBI
- LLR
- SLR
- Hipparcos, Gaia
- time transfer algorithms
- pulsar timing
- GPS, standard model, according to Ashby (2003)
- Galileo (as a copy of GPS)

Some minor problems may still exist…
Observations against General Relativity?

• Are there some experimental evidences against General Relativity?

“Candidates”:

“Pioneer anomaly”: unexpected additional constant acceleration of two Pioneer spacecrafts directed towards the Sun $8.5 \times 10^{-9} \, \text{m/s}^2$

Criticism: “dirty” models of the spacecrafts (heat radiation from RTG, etc.)

“Fly-by anomaly”: unexpected additional increase of the velocity of several spacecrafts

Criticism: the used model has not been checked from the relativistic point of view

Could the story of the Le Verrier’s discovery of the perihelion advance repeat? A lot of care should be taken in dealing with this delicate question…
Linear drifts between time scales

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<th>Pair</th>
<th>Drift per year (seconds)</th>
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Coordinate-dependent parameters
Astronomical observation

physically preferred
global inertial
coordinates

observables are
directly related to
the inertial
coordinates
Astronomical observation

no physically preferred coordinates

observables have to be computed as coordinate independent quantities