Prospects for Improving the Masses of (1) Ceres and (4) Vesta Prior to Dawn's Arrival at These Dwarf Planets

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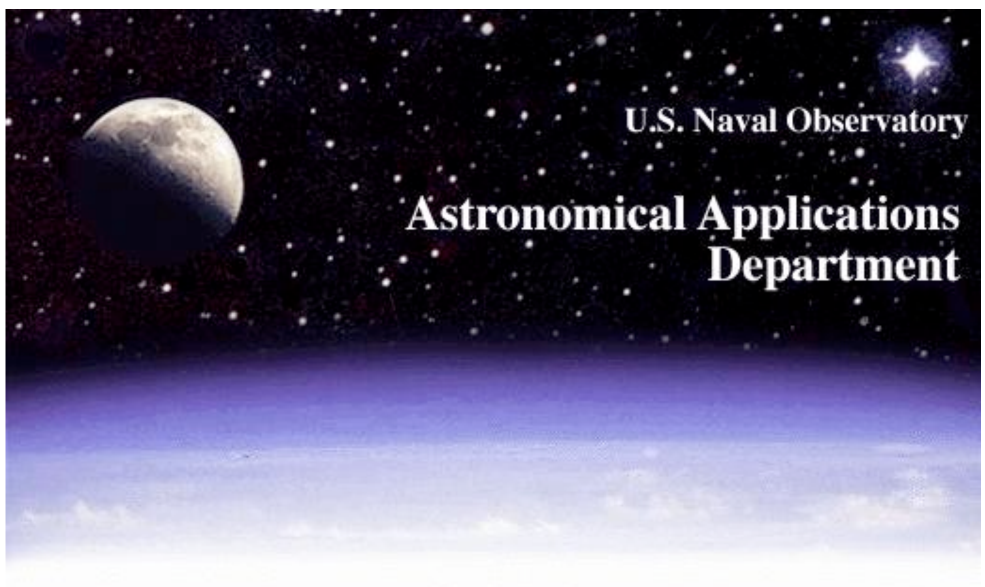


In its purest form, minor planet mass determination can be reduced to a scattering problem.

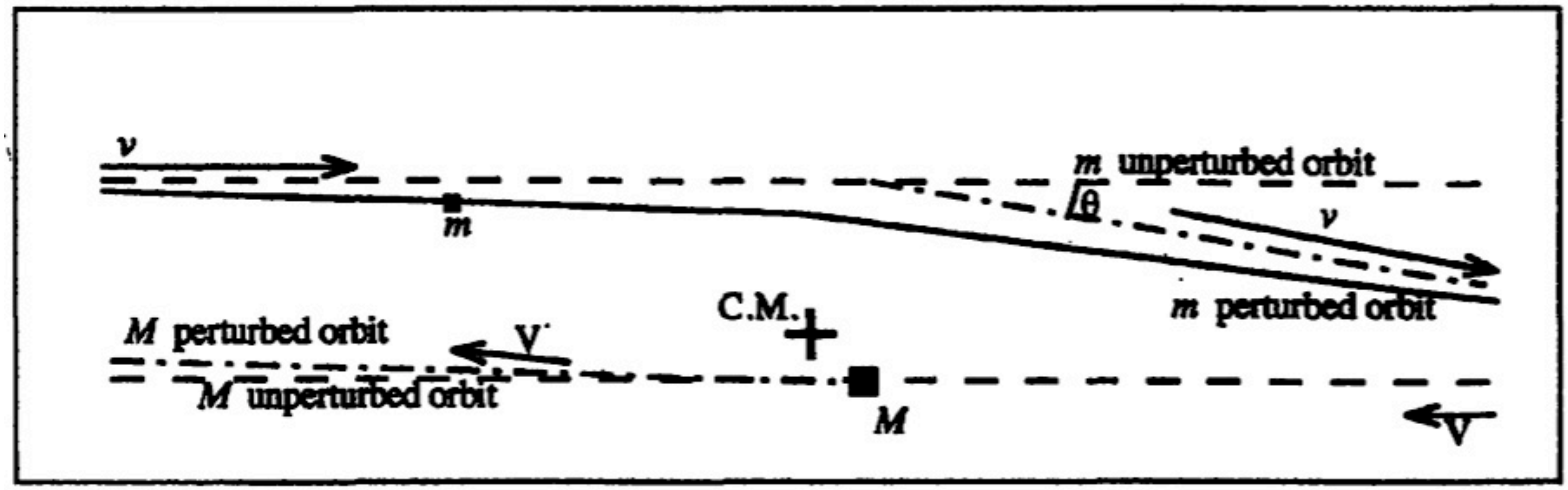
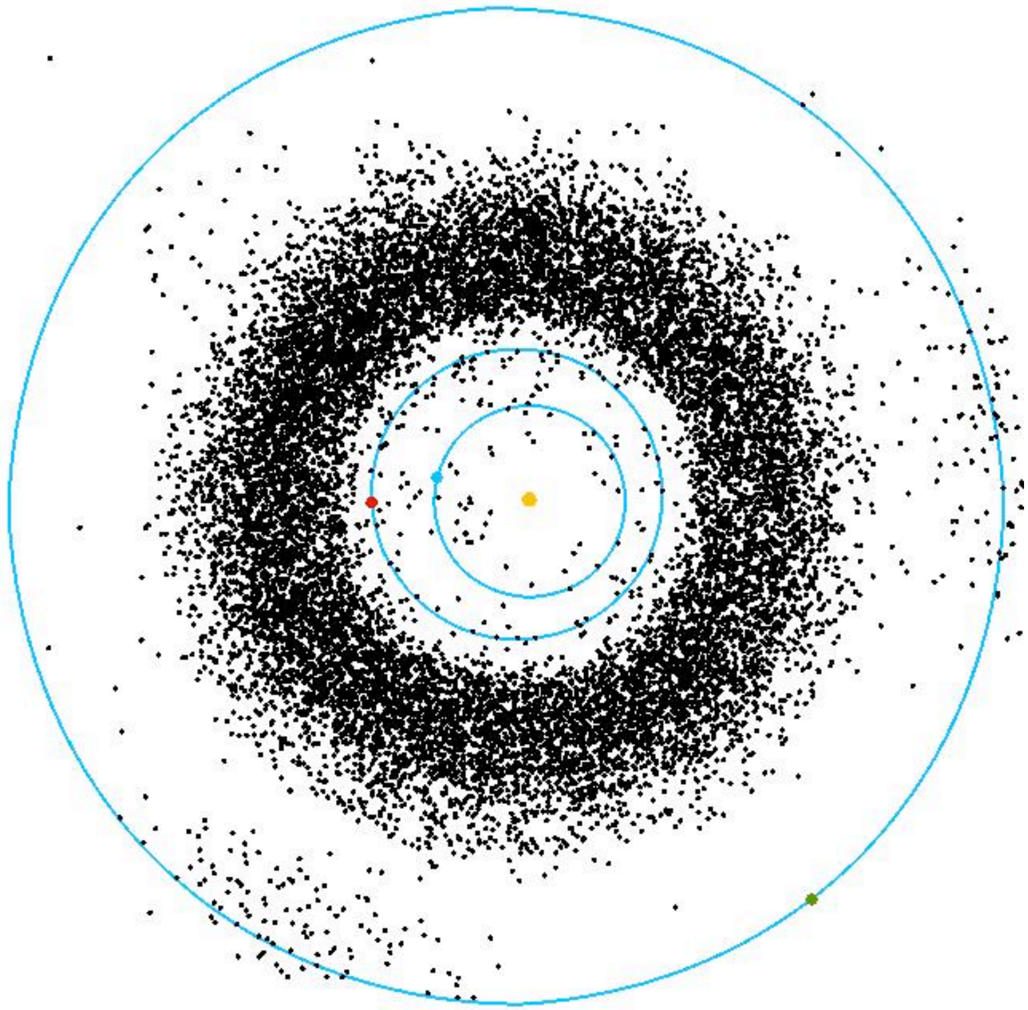


FIG. 1. The scattering of a small asteroid (m) by a more massive asteroid (M) in the center of mass frame of reference.

$$\tan 1/2\ \theta = G\,(M + m) / v^2\, b$$

The problems in determining θ are:



1. It is usually small
2. The perturbing body is immersed within the main belt itself. Thus, there are many small perturbations that serve to add noise to the large perturbation.

The idea is to look for encounters that are strong enough that a minor planet's mass can be determined in a short amount of time before the multiple small perturbations of other minor planets on the test body become a limiting factor.

The most significant way in which the change in the orbit presents itself is as the cumulative change in longitude arising from the change in the perturbed bodies semimajor axis.

$$n = \frac{l_1 - l_0}{t_1 - t_0}$$

The uncertainty in the semimajor axis is:

$$\sigma_a = \frac{2}{3}\frac{a\sigma_n}{n}$$

And the uncertainty in the mass of the perturbing asteroid is:

$$\sigma_M = M\frac{\sigma_{\Delta a}}{\Delta a}$$

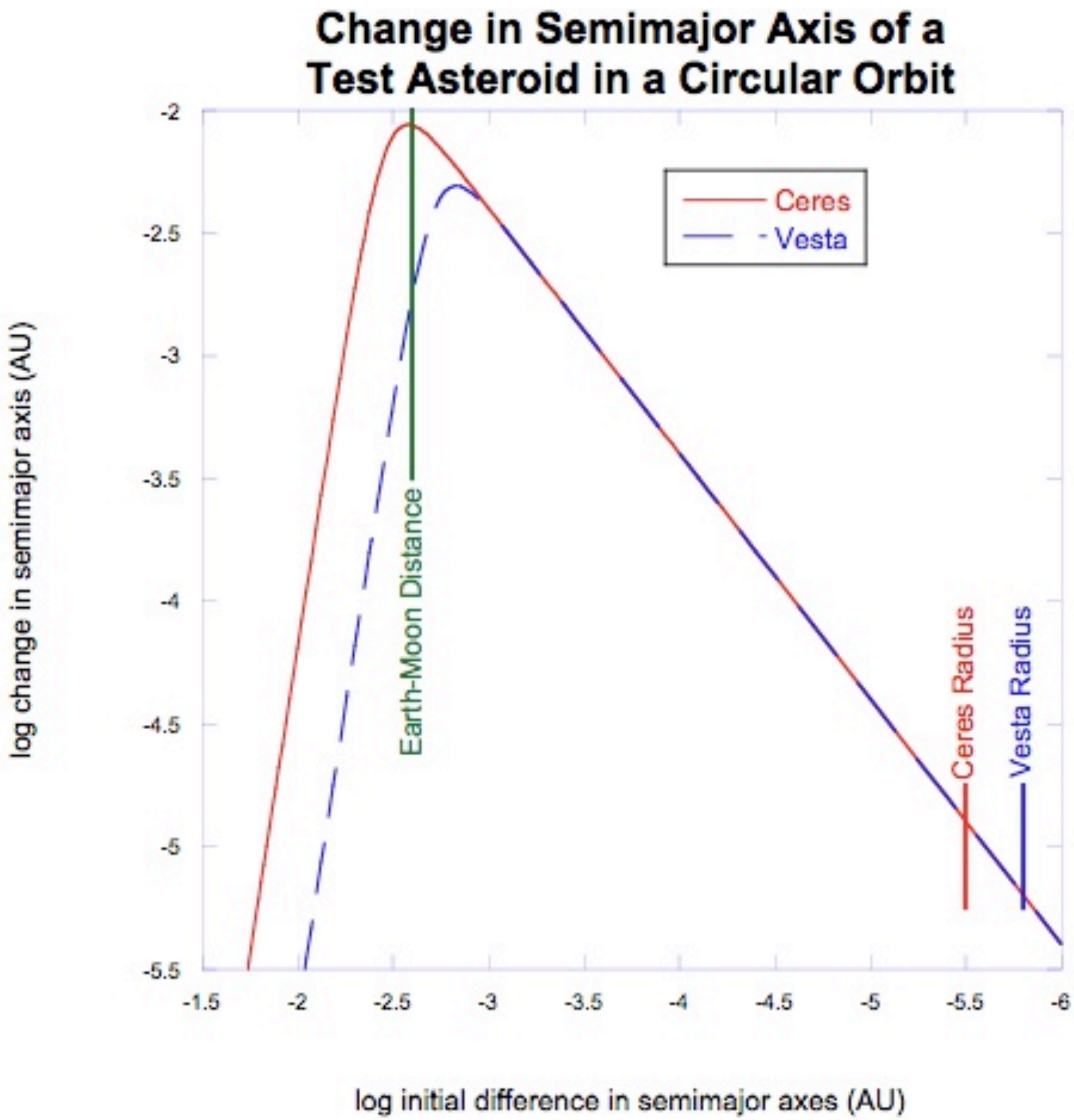
Using the best current optical astrometry, the minimum Δa required to estimate the mass of Ceres to 5% in one year is:

9.6×10^{-6} AU

Two tests of the method:

1. Agrees with the Konopliv *et al*. uncertainty in the masses for Ceres and Vesta from observations of Mars.
2. Agrees with Virtanen *et al*. estimate of the uncertainty in *a* from GAIA.

The method is completely general and may be used with any massive minor planet given an estimated mass.


Change in Semimajor Axis of a
Test Asteroid in a Circular Orbit
Ceres
Vesta
Earth-Moon Distance
Ceres Radius
Vesta Radius
log change in semimajor axis (AU)
log initial difference in semimajor axes (AU)
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6

Encounters with Vesta Prior to October 2011

Perturbed Asteroid	Date	a (AU)	v (m/s)	δa (AU)
89391 2001 VU_{108}	2005.87	2.344	55	-3.8×10^{-6}
2003 GH_7	2006.31	2.392	–85	4.4×10^{-7}
2001 XG_{49}	2007.18	2.330	164	-4.3×10^{-8}
1999 CF_{34}	2008.76	2.405	–114	6.4×10^{-8}
2004 RO_{69}	2011.54	2.369	–38	1.2×10^{-4}

Major Encounters with Ceres Prior to February 2015

Perturbed Asteroid	Date	a (AU)	v (m/s)	δa (AU)
2004 BW_{137}	2005.69	2.865	6	4.7×10^{-4}
2000 GT_{118}	2006.34	2.729	18	2.2×10^{-5}
4325 Guest	2008.58	2.749	18	1.3×10^{-4}
52179 1998 FV_{130}	2012.52	2.866	10	7.2×10^{-5}
2000 EM_{61}	2013.14	2.802	–1	6.2×10^{-2}
104241 2000 EB_{134}	2014.08	2.734	20	-3.2×10^{-5}

These perturbed bodies are quite dim. Their V magnitudes at mean opposition are:

2004 BW_{137}	20.6
4325 Guest	15.7
2000 EM_{61}	18.7

Thus,

- They can only be observed for a few weeks near opposition.
- Only Guest's positions may be reduced using the very accurate catalogs such as the UCAC.
- Using less accurate catalogs the individual positions may be an order of magnitude less accurate.

2004 BW_{137}

- The encounter with Ceres occurred in 2005. Thus, its usefulness is dependent on pre-existing observations.
- The *AstDys* web site gives an uncertainty in a of 5 x 10^{-7} AU from 34 observations made between 2000 and 2005. Thus, the potential exists to determine the mass of Ceres to under 1%.

2000 EM_{61}

The potential change in its semimajor axis is extremely high.

However, it is also in a regime where the change in a is sensitive to initial conditions.

> A increase of $1ms^{-1}$ in the encounter velocity will decrease the change in a to 1/32. its calculated value.

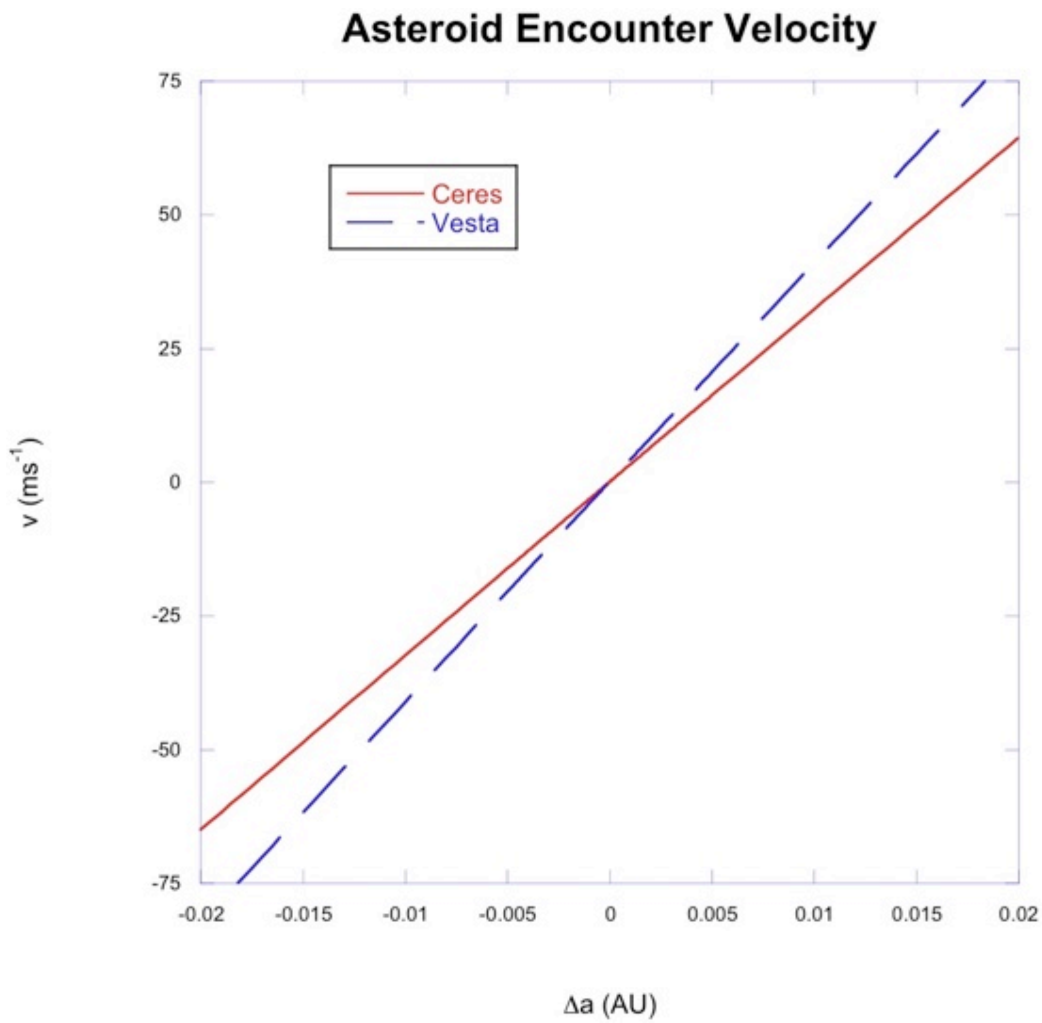
The End

- It would take an error of 3×10^{-4} AU in a of 2000 EM61 to cause an error of $1 ms^{-1}$
- The current uncertainty in a is 5×10^{-6} AU (from the *AstDys* web site)
- The calculations were made using osculating elements
- The expected change in osculating elements from planetary perturbations should be well under 10^{-3} AU.
- Thus, 2000 EM_{61} should provide an excellent mass for Ceres.

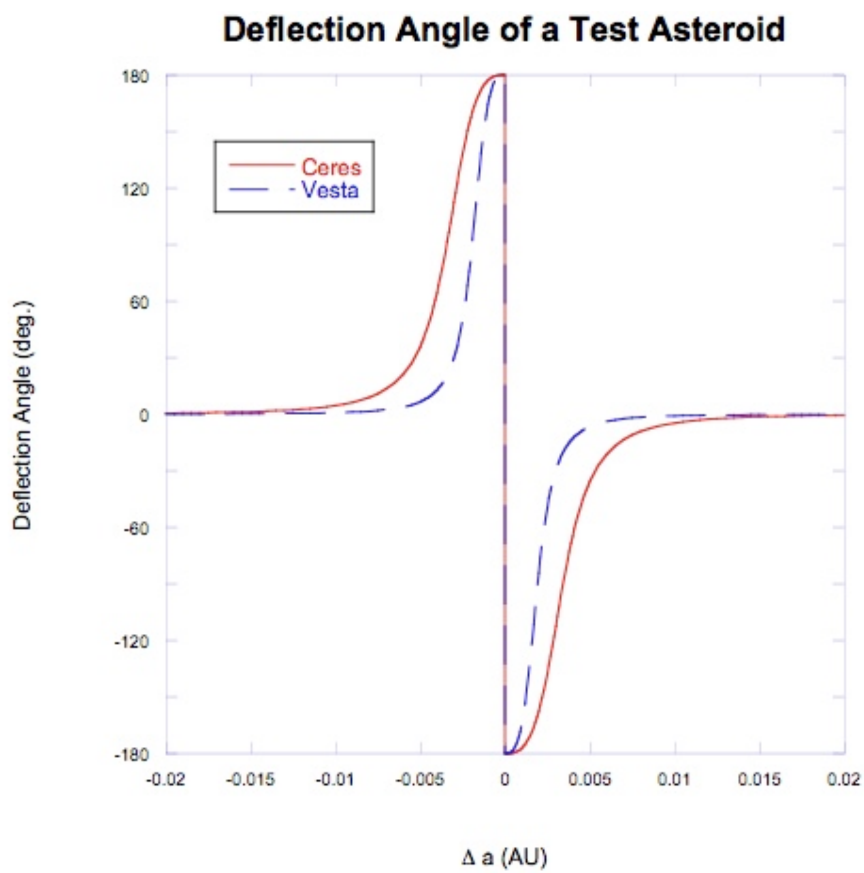
- There are no encounters prior to Dawn's arrival at Vesta useful for improving the determination of its mass.
- There are several encounters that may allow improvement in determining Ceres' mass.

In particular, observations of 2004 BW_{137}, 4325 Guest, and 2000 EM_{61} may make it possible to determine Ceres' mass to an uncertainty of less than 1%.

- Current State and Practical Limits of Asteroid Mass Determinations
- Filtering to Find Candidate Perturbed Bodies
    - Coplanar Circular Orbits
    - Elliptical Non-coplanar Orbits
- Filtering Results for Ceres and Vesta



The pre-encounter velocity of a test body with respect to Ceres and Vesta as a function of Δa, assuming both bodies are on circular, coplanar orbits.



The deflection of a test body with respect to Ceres and Vesta as a function of Δa, assuming both bodies are on circular, coplanar orbits.

The change in the semimajor axis of the perturbed body is:

$$\delta a = \frac{2\mathbf{v} \cdot \Delta \mathbf{v} a_t^2}{\mu_\odot}$$

After a bit of math the change in the semimajor axis of the perturbed body is found to be:

$$\delta a \approx \left(\Delta a - \frac{3 \Delta a^2}{4 a_m} \right) [1 - \cos\theta]$$

Parameters for determining the masses of Ceres and Vesta to 5% in a year assuming coplanar circular orbits.

Quantity	Ceres (AU)	Vesta (AU)
Maximum Distance	0.0125	0.0060
Minimum Distance	5×10^{-6}	5×10^{-6}
Radius	3.2×10^{-6}	1.7×10^{-6}
Maximum δa	0.0044	0.0025
Distance of Maximum δa	0.0026	0.0015

The non-coplanar elliptical case is much more difficult than the coplanar elliptical case.

Thus, rather than searching for an analytic solution, a number of metrics to limit the phase space are devised.

Since the velocity of the encounter is no longer dictated by the distance, an estimate of the maximum encounter velocity is needed.

Take 1/2 the maximum velocity of an encounter where the perturbed body just skims the surface of the massive body

$$v_{\max} = 2^{\frac{1}{4}} \sqrt{\frac{\mu_M}{r}}$$

v_{max}(Ceres) = 430 m s^{-1}

v_{max}(Vesta) = 350 m s^{-1}

Inclination

1. Any motion out of the plane of the massive body increases the distance as the two bodies move away from the nodes.

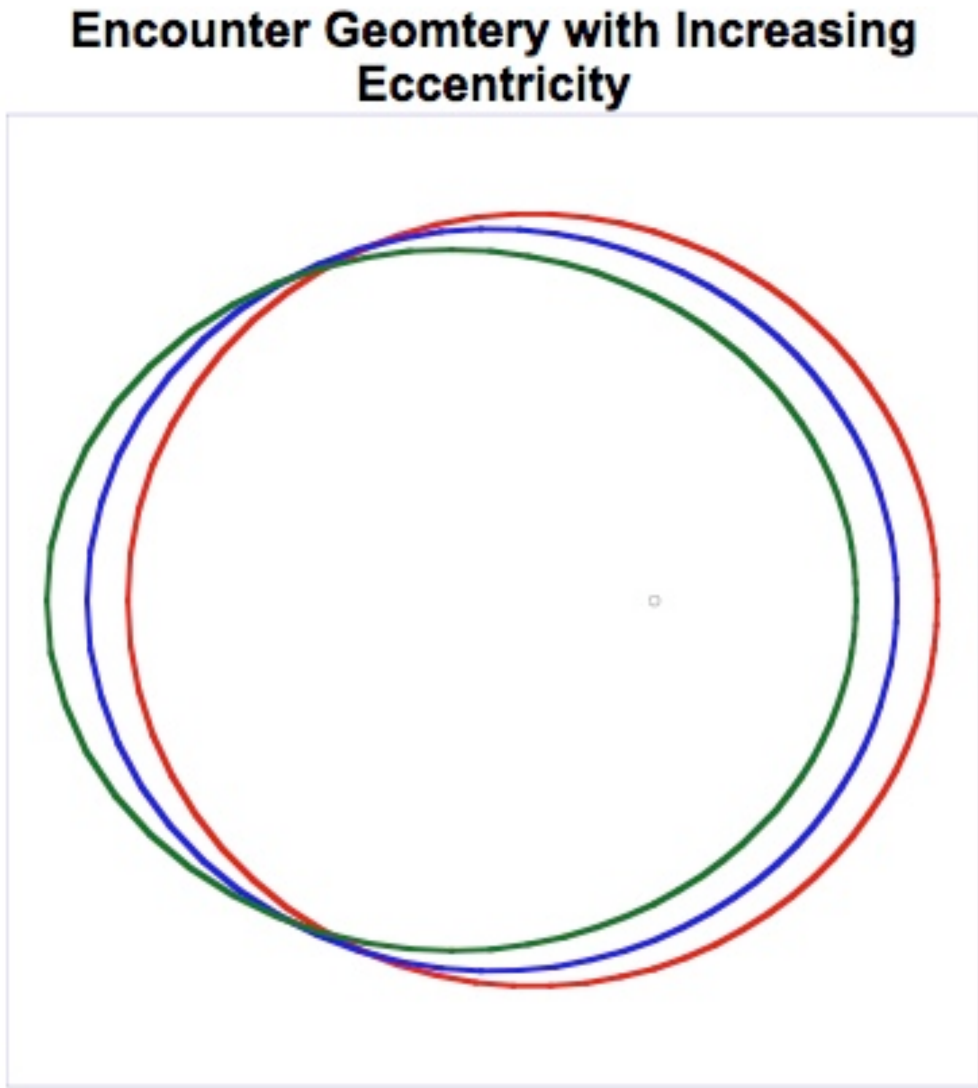
   $i_{\rm max}$(Ceres) ≈ 15.′5 $i_{\rm max}$(Vesta) ≈ 8.′7

2. Since $\delta a \propto \mathbf{v} \cdot \Delta\mathbf{v}$ any velocity component out of the plane of the massive body contributes nothing to δa.

   $i_{\rm max}$(Ceres) ≈ 0.012 = 41′

   $i_{\rm max}$(Vesta) ≈ 0.009 = 31′

Eccentricity



For two asteroids with the same semimajor axis, but different eccentricities:

$$\cos \nu = -\frac{e_1 + e_2}{e_1 e_2 + 1} \qquad \text{and} \qquad r = a(e_1 e_2 + 1)$$

The components of their velocity vectors at an encounter are:

$$v_x = -\frac{na^2 \sin E}{r} \qquad v_y = -\frac{na^2 \sqrt{1 - e^2} \cos E}{r}$$

Solving for the relative speed, s, of the encounter gives:

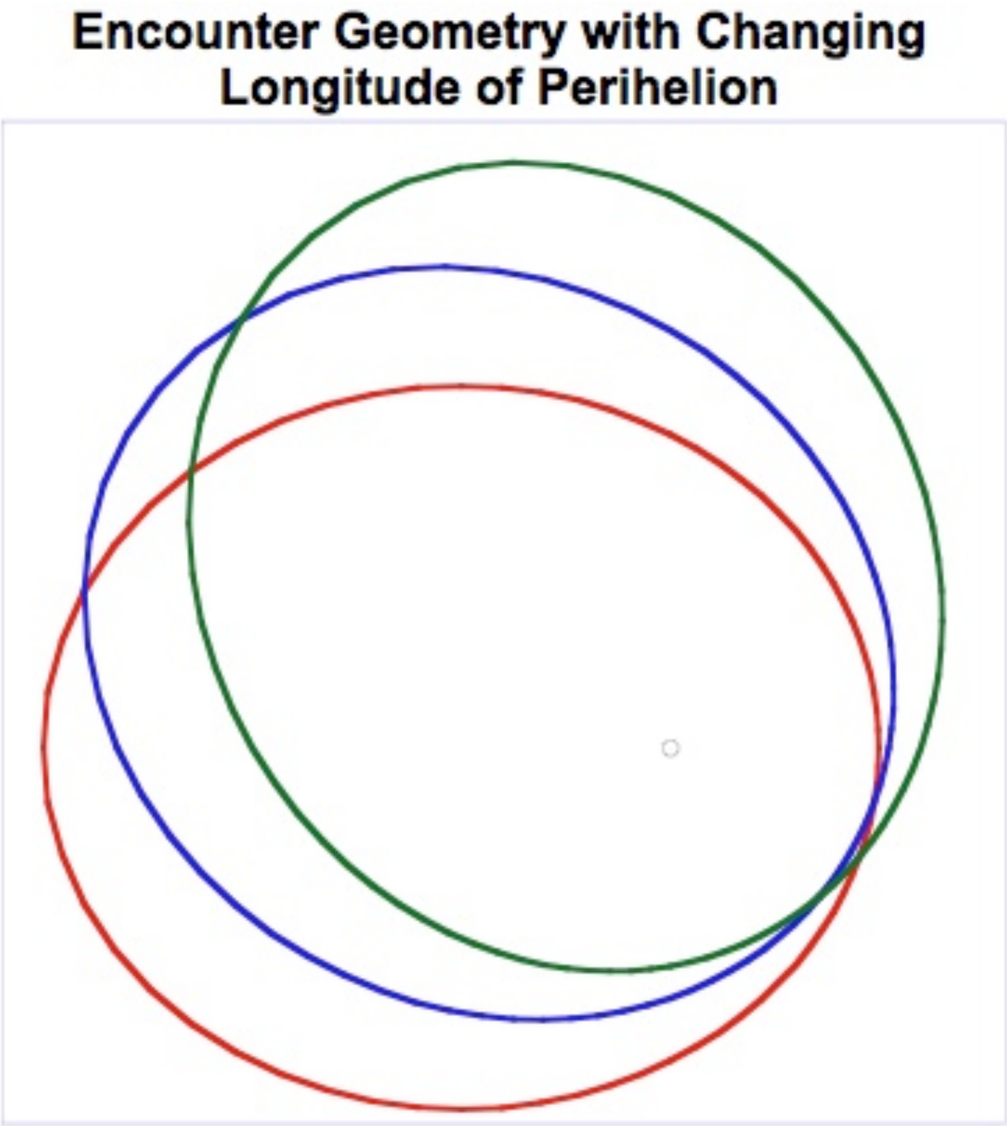
$$s \approx \sqrt{\frac{\mu_\odot}{a}}\Delta e = n\Delta e$$

Using the values for the maximum encounter speed previously calculated gives

Δe_{max}(Ceres) ≈ 0.012

Δe_{max}(Vesta) ≈ 0.009

Longitude of Perihelion



From the equation of a conic section the two ellipses rotated by θ will meet when

$$\cos\nu = \cos(\nu + \theta)$$

This easily solves to

$$\tan\nu = \frac{\cos\theta - 1}{\sin\theta}$$

And with some work you can show

$$\nu = \frac{\theta}{2}$$

Solving for the relative speed, s, of the encounter gives:

$$s \approx v\left(2\psi - \frac{\psi^3}{3} + \frac{3}{4}\psi^3 e\right)$$

Using the values for the maximum encounter speed previously calculated gives

θ_{max}(Ceres) ≈ 0.0060 = 21'

θ_{max}(Vesta) ≈ 0.0045 = 16'

Limits to the Difference in Semimajor Axis, Δa

As with a circular orbit, the relative speed of an encounter depends on the semimajor axis. However, in the case of an elliptical orbit it depends on the current distance from the Sun as well. After some algebra:

$$\Delta a = \left[2\left(\frac{\mu_M}{\mu_\odot}\right)\frac{a^6}{\delta a}\right]^{\frac{1}{5}}$$

Using the values for the minimum change in distance previously calculated gives

$$\Delta a_{\text{max}}(\text{Ceres}) \approx 0.073 \text{ AU}$$

$$\Delta a_{\text{max}}(\text{Vesta}) \approx 0.037 \text{ AU}$$

Approximately 1/2 the values determined for the coplanar circular orbits case.

A Sample of Ceres Masses in the Literature

Mass	Uncertainty	No. Bodies	Reference
5.0	0.2	1	Standish
4.80	0.08	2	Sitarski
4.85	0.06	6	Bowell
4.67	0.09	5	Carpino
4.92	0.07	4	Muinonen
4.71	0.05	7	Rapaport
4.84	0.08	22	Kuznetsov
4.39	0.04	2	Hilton
4.70	0.04	25	Michalak
4.79	0.11	4	Krasinsky

The average σ is 1.7% of the mass determined but the r.m.s. σ of the mass determinations is 9.1%.

Based on the mass determinations since 2000 and careful reading of the papers, I'd estimate the most likely mass for Ceres to be:

$$4.73 \times 10^{-10}\ M_{\odot}$$

With a realistic uncertainty of 3–5%.