## ESTIMATION OF COEFFICIENTS OF DIFFERENTIAL EQUATIONS MODELING THE POLAR MOTION

Y. AKIMENKO, E. SPIRIDONOV, E. TSURKIS

Schmidt's Institute for Physics of the Earth RAS, Moscow, Russia spiridonov@ifz.ru

ABSTRACT. We dealt with Liouville equations which describes the polar motion and presents a system of first-order equations with coefficients depending on T (period of free nutation) and Q (mantle quality factor). The actual problem is to evaluate these coefficients (i.e. T and Q) by:1) excitation functions series used for constructing right hand-side of Liouville equations and 2) polar coordinates series which one can interpret as a solution of the equations. Validity of this task which is typical inverse problem is shown. Practically we solved a number of "direct" ones under different meanings of T and Q. The preferred values of parameters to be estimated are: T=425-440 days and Q=20-60. Disagreement between our model based on Liouville equations and data series is conditioned by physical reasons, as far as mathematical problem is valid. Possible causes of such disagreement are discussed. In this work, we attempted to find optimal values T and Q as parameters fitted by the numerical integration of the pole motion. Also, the stability of values (T and Q) depending on time, length of data sets and different variants of excitation functions (right-hands sides of equations) was investigated.

## 1. MODELS

Five classes of models were examined. They differ one from another by excitation functions and length of data sets. The calculations of each model were carrying out in two variants: without and with variation of initial conditions (Models 1\_1-5\_1 and Models 1\_2-5\_2 accordingly). The free nutation period T and the quality factor Q at the resonance frequency were varied from 400 to 500 days (a step 2.4 h) and from 1 to 500 (a step of 0.1) accordingly. For each model we calculate the dispersion values  $D_{opt}$  for the chosen optimum model parameters (T and Q) and the correlation coefficient r between the optimum model and the observed pole motion series (in percent). The value  $D_{opt}$  was determined by the formula:  $D_{opt} = 1 - D_{res}/D_{pm}$ , where  $D_{res}$  is the dispersion of the residual series defined above and  $D_{pm}$  is the dispersion of the observed pole motion series.

Model 1. The right-hand sides of equations of motion (1) formed by excitation functions based on sum of OAM derived from R.Gross data sets an AAM (NCEP/NCAR, 2005).

Model 2 = Model 1, but the data here averaged by 10-days sinusoidal window (step 10 days).  $D_{opt}$  of this models accounts for more than 60-70%. The optimum quality factor here equal about  $30 \pm 10$  and free nutation period T=435-439 days.

Model 3. As distinct from Models 1 and 2 the length of OAM and AAM excitation functions series in this model is twice as many (1962-2002). The excitation functions are the same. (Step = 10 days.) This model describes the least part of the pole motion sets dispersion. After initial condition selection Model  $3_2$  give  $D_{opt} = 55 - 58\%$ . Without this selection the model describes only 25-30% of dispersion. This model describes about 50% pole motion dispersion if  $Q = 40 \pm 20$  (for Y-component) and Q = 20 - 130 (for X-component) in the range of period about T = 432 - 438 days. As a whole the Model describes the observed pole motion unsatisfactorily. In order to look into this fact we analyzed the changes of T and Q values in time. For that we calculate the values of parameters over the sequential short data sets chosen from sets of Model 3. We select seven 10-years sequential intervals with 5-years overlapping from 1962 till 2002. Lowest values of Q and D% correspond to period of time from 1962 till 1977. In time interval from 1972 till 1982 the Q value (for y-component) tends to infinity. Most likely it is a consequence of the best phase agreement of excitation functions and observed pole motion data sets. Following years

(1977-2002) the Q's values lies as a rule within the limits from 20 to 70. Optimum values of T lies in the range from 420 to 450 days.

Model 4. In contrast to previous models we use in this model the OAM mass terms were calculated by us from TOPEX/POSIEDON satellite altimetry data. Then we added these mass terms data sets with Gross's OAM motion terms and AAM excitation functions. The value of Dopt% obtained in the Model 4.2 for x- and y-components greater by 10% than one for Models 1.2 and 2.2. At the same time, we have here the greater uncertainty of Q values, calculated in Model 4.2, in comparison with previous models. Apparently it is a consequence of the short length of the TOPEX/POSEIDON project data sets. Thus, the use of the TOPEX data for calculation of OAM leads to adequate results in process of the pole motion modeling. However, short lengths of sets don't allow evaluate the quality factor value.

Model 5 constructed by excitation functions of Model 1 (OAM+AAM) plus water storage hydrology excitation functions(HAM, Jianli Chen). The Dopt% values of Model 5\_1 obtain only 50% after initial conditions variation. It indicates to still insufficient quality of water storage excitation function data sets. (T=434-438 days, Q=30-40).

## 2. REFERENCES

Spiridonov, Eugene and Tsurkis, Elias. Modeling of the Earth's Pole Motion from Data on the Atmospheric and Oceanic Angular Momenta over 1980-2002, Izvestiya, Physics of the Solid Earth, 2006, Vol. 42, No 2, pp 149-155.