EMPIRICAL VALIDATION OF THE CONVENTIONAL MODEL FOR LENGTH OF DAY VARIATIONS DUE TO ZONAL TIDES

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ABSTRACT. The deformations of the Earth caused by the zonal part of the tidal potential induce fluctuations in the rotational speed of the Earth with periods from about 5 days to 18.6 years. Measures for the deviation of the Earth rotation from a uniform motion are the parameter dUT1 (defined as the difference between Universal Time UT1 and atomic time UTC) and its time derivative the length of day (LOD). LOD variations were derived from 23 years of VLBI observations, using the VLBI software package OCCAM61E. We also calculated a second LOD series of two years with the Bernese GPS Software 5.0 for the years 2005 and 2006, analysing data of 113 stations of the global IGS network. The atmospheric influence on LOD was computed from atmospheric angular momentum functions provided by the NCEP and subtracted from the original series. Different sets of tidal terms were estimated in a least squares adjustment introducing the remaining LOD variations as pseudo-observations. The resulting amplitudes were compared with the values of the model recommended in the IERS Conventions 2003 (Defraigne and Smits, 1999).

1. ZONAL TIDAL POTENTIAL

The response of the Earth to the zonal tidal potential results in a periodic change of the principal moments of inertia. According to the conservation of angular momentum this leads to a periodic change of the rotation rate and consequently to variations in dUT1 or LOD, respectively. The tidal potential V_G in a point A can be expanded into spherical harmonics, using Legendre polynomials.

$$V_G(A) = GM \sum_{n=2}^{\infty} \frac{R^n}{r^{n+1}} P_n cos(\psi) \tag{1}$$

with G = gravitational constant, M = mass of the celestial body, r = geocentric distance of the celestial body and R = Earth radius. When the geocentric zenith angle ψ is expressed by geocentric spherical coordinates of the station (θ, λ) and of the tracing point of the celestial body (p, Λ) the tidal potential (limited to second order) can be split up into three families of spherical harmonics (e.g. Melchior, 1978).

$$V_{G,20} = GM \frac{R^2}{r^3} P_{20}(cos\theta) P_{20}(cosp) \tag{2}$$

$$V_{G,21} = \frac{1}{3} GM \frac{R^2}{r^3} P_{21}(cos\theta) P_{21}(cosp) cos(\Lambda - \lambda) \tag{3}$$

$$V_{G,22} = \frac{1}{12} GM \frac{R^2}{r^3} P_{22}(cos\theta) P_{22}(cosp) cos2(\Lambda - \lambda) \tag{4}$$

The equations (3) and (4) are denoted sectorial and tesseral functions and describe the short period tides with semi-diurnal and diurnal variations. Function (2) is called zonal function. It depends on the slowly varying latitude of the tracing point of the celestial body only and therefore describes the medium and long period tides. Major periods are fourteen and twenty eight days for the moon and six months for the sun. Accordingly these are the periods of the largest terms of the LOD variations induced by the zonal tidal deformations.

2. LOD TIME SERIES FROM VLBI AND GPS

We processed 23 years of VLBI observations using the VLBI software package OCCAM Version 6.1 (least squares approach, Gauss-Markov model). The so-called VLBI Intensive sessions, as well as all geodetic 24-hour experiments, except those, which are not suitable for the estimation of Earth orientation parameters due to their limited extension in one or more components, were included in the solution (time span: 1984-2006). Coordinates of stations and sources were fixed to the ITRF2005 resp. ICRF Ext.2 reference frames. Hence the estimates are Earth orientation, troposphere and clock parameters. Nutation offsets, polar motion and dUT1 were set up once per session, as regards the 24-hour experiments. In the analysis of Intensive sessions, nutation and polar motion have to be fixed to a priori values and solely one dUT1 value is estimated per session. LOD was not directly estimated as dUT1-rate, but the generated dUT1 time series was converted to LOD after processing. The a priori models applied in VLBI analysis are listed in Table 1.

For the generation of the GPS-based LOD series observation data of 113 stations, which belong to the IGS05 reference frame sites, was processed by means of Bernese GPS Software Version 5.0. The selected observation period was 2005-2006. Hence we derived a two-years LOD time series. Due to the enormous computing time expenditure of GPS data processing, the generation of a longer time series was not feasible so far. LOD was estimated in 6-hour intervals. The geodetic datum was defined, imposing a no-net-rotation condition on the IGS05 station coordinates. The satellite positions were fixed to the final orbits provided by CODE (Center for Orbit Determination in Europe). All calculations were performed using absolute antenna phase center corrections. The applied a priori models correspond to those introduced in VLBI processing (Table 1).

Parameter	Model
Nutation	IAU2000A
Ocean loading	FES2004
sub-daily ERP variations	IERS2003

Table 1: A priori models applied in VLBI/GPS processing

3. ESTIMATION OF TIDAL TERMS

Before the LOD time series can be analysed to extract the tidal signals of interest, it is necessary to eliminate other geophysical signals from the observed LOD variations. The largest impact, to be considered, is atmospheric excitation. The LOD variations induced by the atmosphere are calculated from atmospheric angular momentum functions provided by the NCEP (U.S. National Centers for Environmental Prediction; Salstein and Rosen, 1997). Short period variations in LOD with diurnal and semi-diurnal periods are induced by ocean tides. These fluctuations were already considered within the VLBI/GPS processing by applying the conventional model. The series were additionally "cleaned" by subtracting linear trends and very low frequency variations. From the remaining signal the main zonal tidal terms were estimated at predefined periods. The periods are in principle waves of the tidal potential, which are very well known from the orbital motion of the moon and sun (and planets). The amplitudes of the tidal terms were determined in a least squares adjustment, introducing the LOD variations as pseudo-observations. As observation equations we employed a spectral representation of the tidal variations $\delta LOD(t)$ as follows

$$\delta LOD(t) = \sum_{i=1}^{n} [A_i cos\xi_i(t) + B_i sin\xi_i(t)] \quad with \quad \xi_i(t) = \sum_{j=1}^{5} N_{ij} F_j(t) \tag{5}$$

A_i and B_i denote the cosine and sine amplitudes belonging to the tidal wave i, while n specifies the number of tides considered. The angle argument $\xi_i(t)$ is built as a linear combination of the five fundamental arguments F_j. Each period is specified by a sequence of integer multipliers N_{ij}, where the subscript i labels the tide (Simon et al., 1994). The so-called out-of-phase terms, which are represented by the sine terms here, were in fact neglected in this study, because their expected magnitude is close to zero, according to the IERS conventional model.

In terms of VLBI-derived LOD we solved for 10 major terms, 20 sideband terms and one test term, with periods from 6.85-365.26 days. For the test term we chose the period 16.63 days, where no tidal signal is expected. The magnitude of the test term should give an estimate of the noise level of the observed variations. In case of GPS-based LOD, where the time series covers just two years, only the amplitudes of periods below 35 days were estimated, i.e. 8 major terms, 17 sideband terms and the same test term were determined. Since it is impossible to separate the sideband terms from the major terms with only two years of data, we imposed additional constraints on the estimated amplitudes in order to account for the contribution of the sidebands. To specify, the sideband contribution was considered in the adjustment as a fraction of the main terms, assuming that the relative size of the excitation A is proportional to the relative size of the amplitude V of the waves in the tidal potential: $A_i' = \frac{V_i'}{V_i} A_i$. Since the sideband terms were introduced as additional conditions for the main terms, they were not estimated directly within the adjustment, but calculated a posteriori from the condition equations. Hence no formal errors can be given for the sideband terms, determined in this way. Figure 1 shows the amplitude spectra of the LOD variations after the signal processing steps.

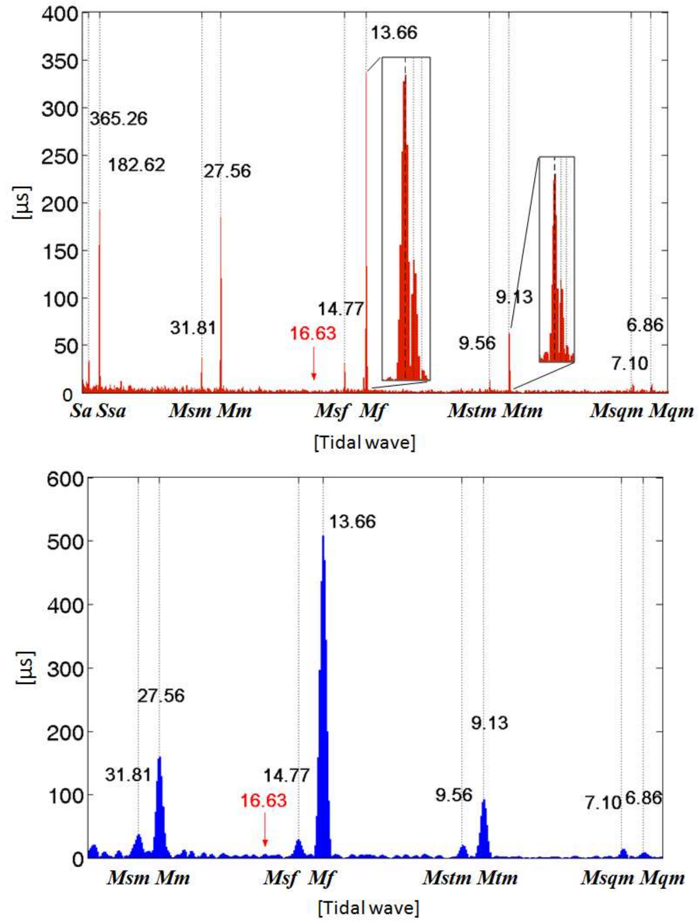

Figure 1: Amplitude spectra of δLOD VLBI (top) and δLOD GPS (bottom)

The main tidal terms are marked with the corresponding periods in days and the names for the tidal waves in Darwin's notation. The upper spectrum (VLBI-derived LOD variations) reveals that it is possible to separate the adjacent sidebands from the main terms with a time series of over 20 years. Most of the very close periods differ by only one cycle in 18.6 years (period of the lunar ascending node), which implicates that an observation period of at least this length is necessary to separate these terms. For example two distinct peaks at the fortnightly band are magnified, demonstrating the separability of the *Mf'* tidal wave (13.63 d) from the main tidal wave *Mf* (13.66 d). The lower picture, which is the spectrum of the GPS-based LOD variations, shows on the contrary, that these terms cannot be separated using a time span of two years. Examining again the *Mf* tidal band, exhibits that the two terms accumulate to one single peak with increased amplitude in the spectral analysis. Thus the results of the frequency analysis support the previously introduced approach of estimating the contribution of the sideband terms with additional conditions in the adjustment, in case they are not resolvable. This applies for all sideband terms estimated from GPS-derived LOD variations and also for the sideband of the annual tidal wave *Sa* (365.26 d) determined from VLBI-derived LOD variations. The final amplitudes determined in the least squares adjustment are compared to the amplitudes of the IERS Conventions model in Figure 2.

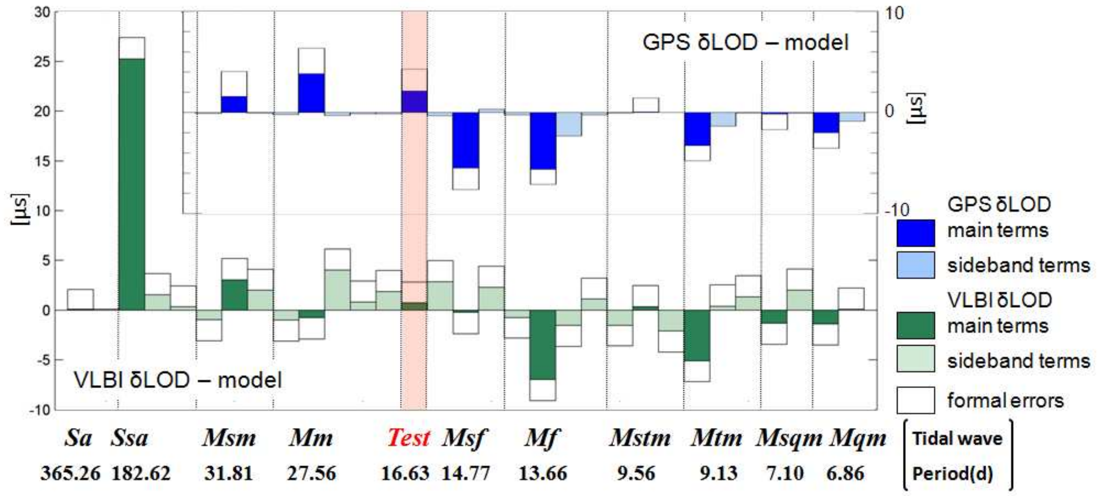

Figure 2: Comparison of tidal amplitudes: observed - IERS model

4. DISCUSSION

The magnitude of the major part of the estimated terms agrees with the amplitudes specified in the IERS model. The values differ less than 5 μsec and the differences are well within the range of three times the formal errors of the adjustment. The agreement is slightly worse for the *Msf* and *Mf* tidal waves in terms of GPS-derived LOD variations and for the *Mf* and the *Mtm* tidal waves determined from VLBI-based LOD variations. For the semi-annual *Ssa* wave, which was estimated from VLBI δLOD only, we found a remarkable discrepancy of 25 μsec between the observed and the theoretical amplitude. In general the two solutions (VLBI/GPS), which are obtained independently, confirm each other and show a very similar pattern regarding the sign of the differences; this is particularly true for the Mf and Mtm tidal waves. As for the difference in *Msf* we can state that periods in the fortnightly band are always critical in GPS analysis, because they are possibly affected by aliasing due to imperfect reduction of the influence of short period ocean tides and processing characteristics. The reasons for the differences in the other terms mentioned are suspected to be either insufficient modelling of the atmospheric influence or inexact consideration of medium and long period ocean tides in the theoretical model, or even a combination of both.

5. REFERENCES

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