THREE-YEAR SOLUTION OF EOP BY COMBINATION OF RESULTS OF DIFFERENT SPACE TECHNIQUES

V. ŠTEFKA[1], I. PEŠEK[2], J. VONDRÁK[1]
[1] Astronomical Institute, Academy of Sciences of the Czech Republic
Boční II, 14131 Prague 4, Czech Republic
e-mail: stefka@ig.cas.cz, vondrak@ig.cas.cz
[2] Dept. of Advanced Geodesy, Faculty of Civil Engineering, CTU in Prague
Thákurova 7, CZ-16629 Prague
e-mail: pesek@fsv.cvut.cz

ABSTRACT. The method of non-regular combination of results of different techniques, namely GPS, VLBI, SLR and Doris, to obtain unique values for both the Earth orientation parameters (EOP) and the station coordinates needs the EOP at the adjacent epochs to be suitably constrained to each other. Modified smoothing algorithm was used as this constraint. Weighting controls smoothness of the combined EOP, so that it can also be used to filter out undesirable frequencies from the solution. To do it, a transfer function was empirically estimated from two combinations of EOP and used to compute three-year solution. The result is compared with IERS c04 series and presented here.

1. INTRODUCTION

Orientation of the Earth's body in space is described by five angles, called Earth orientation parameters, EOP, which tie the Earth-fixed coordinate system ITRF to the celestial reference frame. The EOP are two coordinates of the intermediate pole with respect to the ITRF, x_p, y_p, a time correction $UT1-UTC$, which characterizes irregularity of the Earth's proper rotation and, finally, two components of the celestial pole offset, dX, dY, which denote the observed corrections to the adopted precession-nutation model (they are not considered in the present paper because they are measured only by one technique, VLBI). International reference frame ITRF is realized by geocentric rectangular coordinates of reference points of a set of stations (observatories) equipped by one or more high precision observation techniques.

The space geodesy techniques used to produce the EOP and station coordinates are Global Position System (GPS), Very Long-Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), and recently also Doris, all of them working with a high internal accuracy. The individual techniques, though, are referred to different standards and constants, and use different mathematical models, so that their results suffer form mutual systematic differences and biases.

For deriving a function from scattered data, a smoothing (Vondrák, 1977) is widely used. The method is designed to find the most probable function values as a compromise between the least squares fit and the demanded function's smoothness. We implemented this smoothing to the non-rigorous combination (Štefka and Pešek, 2007) as a more sophisticated approach to tie EOP at the adjacent epochs. Before using the method with real observations, we tested it with simulated data in order to derive a transfer function.

2. NON-RIGOROUS COMBINATION

The basic idea of the method is to combine station position vectors, x_C, in the celestial reference frame (Pešek and Kostelecký 2006). These vectors are functions of both the Earth orientation parameters and the station coordinates x_T,

$$x_C = PN(t)R_3(-GST)R_1(y_p)R_2(x_p)x_T, \tag{1}$$

where $PN(t)$ is precession-nutation matrix and R_i is the matrix of rotation around the i-th axis. Input data for the combination consist of M sets of EOP $(x_p, y_p, UT1-UTC, dX, dY)_m$ and corresponding sets of station coordinates $x_m, m = 1, ...M$, as derived by analysis centers for individual techniques.

To make the combination more stable, parameters $p = p_1, ...p_7$ of a individual seven-parametric transformation formula are derived for each technique, instead of corrections to the station coordinates themselves.

The partial derivative of the formula (1) with respect to U, which stands for any unknown parameter (EOP and p), yields observation equations of the form

$$\sum_{m=1}^{m=M} \frac{\delta x_C}{\delta U_m} dU_m = x_{C|obs} - x_{C|0} + v, \tag{2}$$

where the "observed" vectors $x_{C|obs}$ are calculated from the respective input solution, $x_{C|0}$ are functions of adopted a priori values of the unknowns.

To remove singularity of the system (2), a no net-rotation constraint, minimizing mutual shifts and preserving the system as a whole, has to be introduced,

$$\sum p^T p = min, \tag{3}$$

which stabilizes calculating the station coordinates. On the other hand, Earth orientation parameters are calculated for each individual epoch independently of the others. As a consequence, errors in the input data, including station coordinates, are transferred to the EOP and increase their scatter substantially. The effect can be reduced by including constrains, in the form of additional observation equations (pseudo-observations), that tie the values of the respective EOP at adjacent epochs, here denoted generally as E. In the original method they were used

$$dE_i - dE_{i-1} = E_{i-1} - E_i + v. \tag{4}$$

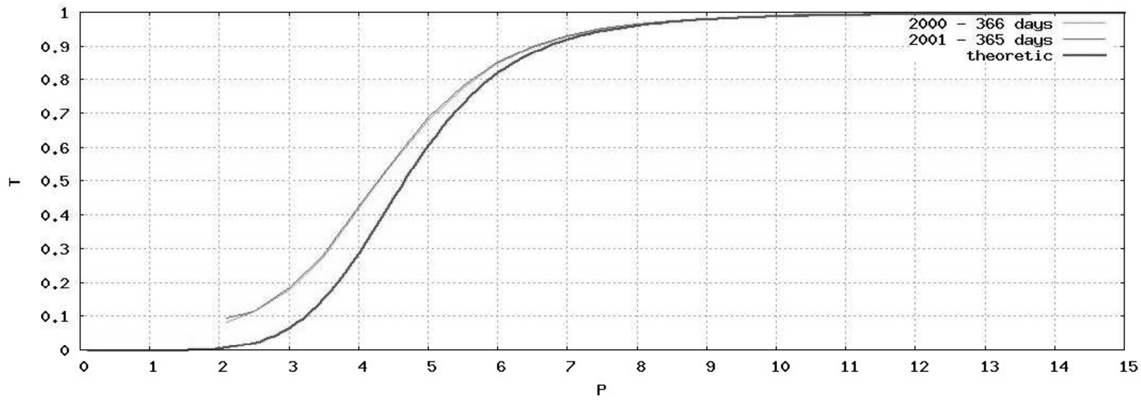

Figure 1: The comparison of the computed transfer functions with the theoretic one. The choice of the numerical value of ϵ was calculated by the formula (7), where $P_{0.99} = 10$ days and $p = 1$

3. AN IMPROVEMENT IN THE LATTER METHOD

The improvement was done by implementation of a method of smoothing (Vondrák 1977). It consists of replacing constraints (4) by the third derivatives of third-order Lagrange polynomial $L_i(x)$ running through the four adjacent points i, $i+1$, $i+2$, $i+3$, i.e.

$$L_i^{'''} = \sum_{k=0}^{3} \left(6 \prod_{j=0, j\neq k}^{3} \frac{1}{(x_{i+k} - x_{i+j})} \right) E_{i+k}. \tag{5}$$

By assigning a weight w to these constrains (5), we can control a smoothness of each series of unknowns x_p, y_p and $UT1 - UTC$, respectively. The following rule applies: the bigger the weight w, the smoother is the solution. That means that we can use three different values of w (w_1, w_2, w_3), for the three EOP´s, but here we use simply $w = w_1 = w_2 = w_3$.

4. CHOICE OF WEIGHT OF SMOOTHING

To study a relation between the weight and smoothness, we supposed that the observation equations (2) can be expressed as a sum of several periodic functions. Each term can then be smoothed separately and the resulting smoothed function expressed as the sum of individual smoothed terms. Then, by changing values on the RHS of the observation equations in order to simulate a signal with a known period P and amplitude A, we can compute a transfer function T (i.e., the ratio between the amplitude of the smoothed curve and the observed amplitude of a periodic function with frequency f).

Two solutions of transfer functions were computed from independent data sets, each covering a year period, 2000 and 2001, respectively. Both solutions are displayed in Fig. 1 and compared with a modified analytical formula proposed by (Huang&Zhou 1981, 1982) as

$$T = \frac{\epsilon P^6}{61529p + \epsilon P^6}, \tag{6}$$

where p is an average weight of E_i. Alternatively, different formula holds for calculating both coeficients (ϵ and w) if we wish to pass 99% of the amplitude of the periodic process with period $P_{0.99}$ (corresponding to $T = 0.99$):

$$\frac{1}{w(P_{0.99})} = \epsilon(P_{0.99}) = 99p\left(\frac{2\pi}{P_{0.99}}\right)^6. \tag{7}$$

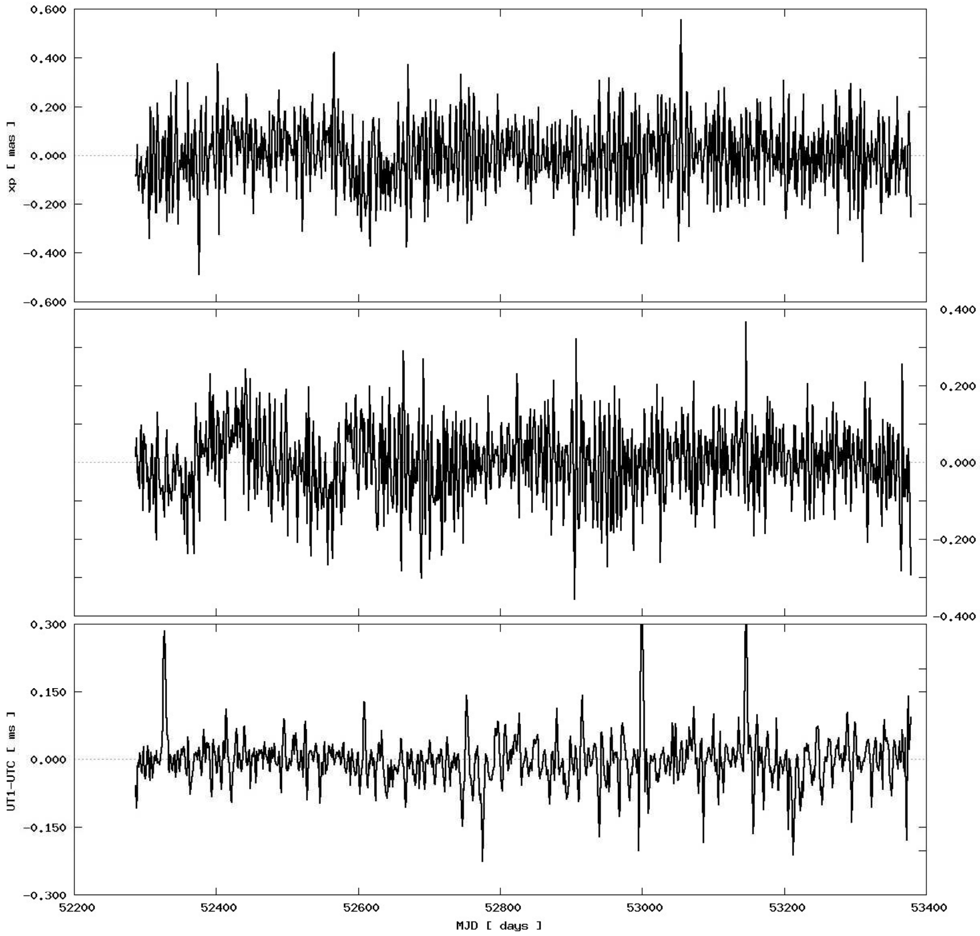

Figure 2: Three-year solution was computed, using $w = 150$, and compared with IERS c04 series. Differences are 0.129 mas, 0.102 mas and 0.0536 mas for the polar motion x_p (top), y_p (center), and the time correction $UT1 - UTC$ (bottom), respectively.

5. DATA AND NUMERICAL SOLUTION

The latter method was tested with the following data: GPS and VLBI data were taken from the IERS Combination Pilot Project database. For SLR, the constrained *ilrsb* solution was used, as published by ILRS analysis centre. Both GPS and SLR are weekly Sinex solutions, from which the EOP and station coordinates were extracted. VLBI data consists of per seance singular normal equation matrices. They were regularized by constraining the station coordinates to the VTRF 2005 frame (Nothnagel, 2005) with the a priori precision of 5 mm. As none of the techniques currently provides the database with the celestial pole offset, only the x_p, y_p, and $UT1-UTC$ are solved for. The techniques enter the combination with the following weights: 1.44 for GPS, 0.8 for SLR, and 1.0 for VLBI. Out of these weights, we used the weight of the constraints for smoothness (5) equal to $w = 150$, assuming there is 99% signal with period greater than 4 days. Three-year solution produced by this method was compared with the IERS c04 series, and the results are displayed in Fig. 2.

6. CONCLUSION

A method for non-rigorous combination of the results of different space geodesy techniques to obtain representative sets of the Earth orientation parameters was modified by implementing the Vondrák´s smoothing. This was done by replacing a simple formula (4) by a more complex one (5), in the level of observation equations. Transfer functions of our method, which was empiricaly computed from two combination of EOP, is in good agreement with the theoretical one. Finally, three-year solution of combination of EOP was calculated, using $w = 150$ to ensure that there is 99% signal with a period greater than 4 days in the solution, and compared with IERS c04 series. The rms differences are 0.129 mas, 0.102 mas and 0.0536 mas for the polar motion x_p, y_p, and the time correction $UT1 - UTC$, respectively.

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