RELATIVITY IN FUNDAMENTAL ASTRONOMY: SOLVED AND UNSOLVED PROBLEMS

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ABSTRACT. Nowadays it is no longer necessary to justify the importance of consistent relativistic modelling in the field of fundamental astronomy. Although in the last 20 years the theoretical foundations of relativistic modelling have been elaborated with a lot of care, there are a number of issues, mostly of practical character, that still require both theoretical discussions and practical implementations. These 'gray' areas of the modelling include modelling of rotational motion of celestial bodies, correct inclusion of multipole structure of the bodies in the translational equations of motion, interplay between numerical accuracy and analytical "order of magnitude" of various relativistic terms, relativistic scaling of astronomical quantities and units of measurements. An overview of the relativistic issues in the field of fundamental astronomy is given here from this critical point of view.

1. INTRODUCTION

The field of applied relativity has emerged about 40 years ago, when the growing accuracy of observations and the new observational techniques (like, radar ranging) have made it necessary to take relativistic effects into account on a routine basis. Since that time, applied relativity has evolved into one of the basic ingredients of fundamental astronomy, the discipline that includes celestial mechanics, astrometry, time scales and time dissemination etc. On the one hand, that development required significant theoretical efforts. Triggered also by the needs of applications at an engineering level, special theoretical techniques have been developed to construct the so-called local reference system (like GCRS) and to derived the equations of translational and rotational motion of a system of N bodies having arbitrary composition and shape. On the other hand, astronomers and engineers had to re-think and to re-formulate their problems in a language compatible with general relativity. The need to change the way of thinking from Newtonian "common sense" to relativistic is probably the source of many of the difficulties that non-experts have with relativity. In the same time the relativity itself is quite simple and elegant at least in the post-Newtonian approximation.

It seems to be natural and advantageous to review the status of applied relativity from time to time, and clearly formulate problems which we can consider as well understood and "solved" and also those problems which we have to classify open ones. Previous attempts to formulate unsolved problems in the field of celestial mechanics (including relativistic celestial mechanics) were undertaken by Brumberg & Kovalevsky (1986) and Seidelmann (1986). This short review is by no means intended to serve as a full update of those publications, but represents just a step in that direction.

2. THE IAU 2000 FRAMEWORK

The IAU 2000 framework for relativistic modelling (Soffel et al., 2003) represents a self-consistent theoretical scheme enabling one to model any kind of astronomical observations in the post-Newtonian approximation of general relativity. The framework has three main theoretical ingredients:

1. The theory of local reference systems (e.g. Geocentric Celestial Reference System, GCRS).
2. The post-Newtonian theory of multipole expansions of gravitational field.
3. Careful investigation of the orders of magnitude of various effects that has allowed to make the post-Newtonian reduction formulas for time scales as simple as possible.

The local reference systems have two fundamental properties:

- **A.** The gravitational field of external bodies (e.g. for GCRS all solar system bodies expect the Earth) is represented only in the form of a relativistic tidal potential which is at least of second order in the local spatial coordinates and coincides with the usual Newtonian tidal potential in the Newtonian limit.
- **B.** The internal gravitational field of the subsystem (e.g. the Earth for the GCRS) coincides with the gravitational field of a corresponding isolated source provided that the tidal influence of the external matter is neglected.

These two properties guarantee that the coordinate description of the local physical processes in the vicinity of the considered body (e.g. in the vicinity of the Earth in the case of GCRS) is as close as possible to the physical character of those processes. This means, for example, that if some relativistic effect is present in the coordinates (e.g., of a satellite of that body) the effect cannot be eliminated by selecting some other ("more suitable") coordinates and therefore has physical character.

It should be noted that although only one local reference system – GCRS – is defined by the IAU 2000 framework explicitly, the framework foresees GCRS-like local reference systems for each solar system body for which the local physics (e.g. the structure of the gravitational field and the theory of rotational motion) should be precisely formulated. For example, modelling of LLR data requires a local Celenocentric Celestial Reference System. Recent projects aimed at precise modelling of the rotational motions of Mercury and Mars will have to use the corresponding reference system for Mercury and Mars, respectively. All these local systems are defined by the same formulas as those given in the IAU 2000 framework for the GCRS, but with index E interpreted as referring to the corresponding body.

Moreover, a local reference system defined by the same IAU 2000 formulas, but constructed for a massless observer (with index E referring to a fictitious "body" of mass zero), is suitable to describe physical phenomena in the vicinity of that observer and, in particular, to define measurable quantities (observables) produced by that observer. The relation between this point of view and several standard ways to describe observables in general relativity is described by Klioner (2004).

Let us also mention that the IAU 2000 framework by no means restricts the freedom to use any other reference systems for the analysis and modelling of various phenomena. In some toy models possessing some special symmetries it may be advantageous to reflect those symmetries directly in the choice of the coordinates. This could help to formulate the problem and its solution in a simpler way. If those other reference systems are defined correctly, one can always find a coordinate transformation between the IAU 2000 reference systems and those other reference systems. The IAU 2000 framework suggests a standard choice of the reference systems. That standard choice can be used by those who do not want to care about the relativistic formulation by themselves. On the other hand, the IAU 2000 framework is also a particular set of reference systems in which all results and parameters obtained by different groups can be compared and combined, even if those groups use different relativistic formulation in their work.

3. THE IAU 2000 FRAMEWORK AND PPN FORMALISM

The IAU 2000 framework has been formulated within Einstein's general relativity. On the other hand, it is clear that modern high-accuracy astronomical observations open one of the most important ways to test the validity of general relativity. Most of the best current estimates of many relativistic effects come from high-accuracy astrometry (Will, 2006). A popular way to quantitatively test general relativity in the post-Newtonian approximation is to estimate from observations numerical parameters in the models formulated in the so-called Parametrized Post-Newtonian (PPN) formalism (e.g., Will, 2003). The PPN formalism is a phenomenological scheme covering a broad class of possible theories of gravity in the weak-field slow-motion (post-Newtonian) approximation. Many metric theories of gravity were investigated by the authors of the PPN formalism and a generic form of the post-Newtonian metric tensor of a system of N bodies was derived. That PPN metric tensor is a generalization of the BCRS metric tensor given in the IAU 2000 framework and contains a number of numerical parameters. At least two such numerical parameters are well known in the astronomical community: β and γ. These parameters have been often determined from observations.

Two attempts to generalize the general-relativistic theory of local reference systems onto the PPN formalism were undertaken until now: Klioner & Soffel (2000) and Kopeikin & Vlasov (2004). Although the two investigations are based on similar ideas and partially agree with each other, some important details were treated differently. Future investigations should clarify which approach is more adequate for practical modelling of observations.

4. WELL-UNDERSTOOD PROBLEMS

Before proceeding to unsolved problems, it seems to be appropriate to give a list of problem which can be considered as solved ones.

- *Post-Newtonian* relativistic reference systems. This includes the theory of both global and local reference systems in the framework of general relativity, relativistic time scales, time synchronization and dissemination.
- *Post-Newtonian* equations of motion for test particles and massive bodies having only masses and no further structure of the gravitational field (the so-called Einstein-Infeld-Hoffmann (EIH) equations).
- Multipole structure of the *post-Newtonian* gravitational field. The Blachet-Damour multipole moments are used also in the IAU 2000 framework and represent a physically adequate and convenient way to deal with gravitational fields of arbitrary structure in general relativity.
- *Post-Newtonian* equations of motion of bodies with multipole structure.
- *Post-Newtonian* equations of rotational motion.
- *Post-Newtonian* theory of light propagation.
- Some properties of the *post-post-Newtonian* effects, but by no means so detailed understanding as for the post-Newtonian approximation.

Although all these topics are very well investigated from the theoretical point of view, it does not mean that no difficulties in practical use of these results can occur. A number of known difficulties are discussed in Section 6 below.

5. UNSOLVED AND POORLY KNOWN ISSUES

Let us now give a list of problems are still unsolved.

- Embedding of the post-Newtonian BCRS in the cosmological background. This question could be important for the interpretation of high-accuracy observations (Gaia, VLBI, etc.). Although some efforts in this direction have been started, the problem is far from being solved.
- Post-post-Newtonian relativistic reference systems (especially, the post-post-Newtonian definition of local reference systems like the GCRS)
- Multipole structure of the post-post-Newtonian gravitational field. The Blanchet-Damour moments are defined only in the post-Newtonian approximation. No similar results in the post-post-Newtonian approximation are known.
- The post-post-Newtonian equations of motion for N-body system. The post-post-Newtonian equations of motion (and even higher-order ones) are only known for a system of 2 bodies.

On the other hand, it is clear that the last 3 topic are currently not very interesting from the practical point of view, the only application being binary and double pulsars. The situation can, however, change very quickly if such observational techniques as laser ranging between spacecrafts become operational. Such projects like Lisa and Astron may need the post-post-Newtonian equations of motion to predict the motion of drag-free spacecraft with required accuracy.

6. ISSUES REPRESENTING PRACTICAL DIFFICULTIES

The problems listed in Section 4 above can be considered as solved from the theoretical point of view. However some of them still represent a lot of difficulties for non-experts. Some other problems being well understood theoretically still wait for practical implementation in numerical calculations. Let us give some examples:

- Although the general form of the *post-Newtonian* equations of motion of bodies with multipole structure is well known, these equations have never been applied explicitly in their full complexity in a numerical code. Important applications here are modelling of the figure-figure interaction in the Earth-Moon system for LLR, the influence of the structure of the Earth's gravity on the motion of spacecrafts during the fly-by maneuvers, etc. An improvement of practical models is necessary here.

- Numerical calculations with the *post-Newtonian* equations of rotational motion are rather tricky. Although the equations themselves have been formulated about 15 years ago, the first numerical results have appeared only recently (Klioner, Soffel, & Le Poncin-Lafitte, 2007).

- Relativistic time scales as a part of the IAU 2000 framework are traditionally difficult to understand for "Newtonian-thinking people". These time scales are (1) TCB and TCG as coordinate times of BCRS and GCRS, respectively, as well as (2) TDB and TT as scaled versions of them (Soffel et al., 2003; IAU, 2006). Although the concept of a coordinate time is crystal clear for people trained in relativity, coordinate time scales may sometimes be very confusing for people using "Newtonian common sense". In the literature one can sometimes meet wrong statements about astronomical time scales. For example, the following statements are **wrong**: (a) TCB is the time in the barycenter of the solar system, (b) TCG is the time at the geocenter, (c) TT is the time on the rotating geoid, (d) an ideal clock put in these three locations would keep TCB, TCG and TT, respectively. A discussion of these and other issues concerning time scales can be found in (Brumberg, Kopeikin, 1990; Klioner, 2008).

- One more **wrong** statement about time scales is that for TDB no location could be found where an ideal clock would keep it. and that this implies some non-SI "TDB seconds". This statement is probably one of the main reasons to introduce "TDB units" in various documents describing astronomical reduction algorithms. Arguments why the scaling from TCB to TDB does not imply any change of units have been put forward by Klioner (2008). Additional discussions and educational efforts are necessary here to achieve a consensus.

Another group of difficulties is related to the fact that the mathematical techniques commonly in use in relativity and in fundamental astronomy are sometimes very different. One example of different mathematical languages in these two fields is the expression for the torque in the rotational equations of motion. The relativistic torque cannot be written in terms of Legendre polynomials and their derivatives as it is the case with the Newtonian torque. Special mathematical machinery of symmetric trace-free (STF) tensors should be used for the relativistic torque. The corresponding mathematical expression takes totally different form compared to the Legendre polynomials and this makes them difficult to understand for the astronomical community (see Klioner, Soffel & Le Poncin-Lafitte (2008) for further details).

Another example is related to analytical and numerical orders of magnitude. In typical calculations in relativity the terms are taken into account or dropped based on their analytical order of smallness with respect to c^{-1} (or in some cases with respect to the Newtonian gravitational constant G). For example, the post-Newtonian approximation consists in taking into account all terms in the equations of motion of the order of c^{-2} and neglecting all higher-order terms. On the other hand, for practical calculations we should be more interested in numerical magnitudes of various terms rather than in their analytical orders of smallness.

As an example of this controversy let us consider the post-post-Newtonian expression for the Shapiro delay (gravitational time retardation) in the gravitational field of one spherically symmetric body with mass M in the framework of an extended version of the PPN formalism. A light ray (a photon) is propagating from position $\mathbf{x}_0$ where it is situated at moment t_0 to another position $\mathbf{x}_1$. The goal is to find moment t_1 at which the light ray reaches $\mathbf{x}_1$. Denoting $m = \frac{GM}{c^2}$, $R = |\mathbf{x_1} - \mathbf{x}_0|$, $x_0 = |\mathbf{x}_0|$, and

$x_1 = |\mathbf{x}_1|$ one has (Klioner, Zschocke, 2007)

$$
\begin{aligned}
c\,(t_1 - t_0) \quad = \quad & R \, + \, (1+\gamma)\, m \, \log \frac{x + x_0 + R}{x + x_0 - R} \\
& + \frac{1}{8}\, \epsilon \, \frac{m^2}{R} \left(\frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\
& + \frac{1}{4}\, (8(1+\gamma) - 4\beta + 3\epsilon)\, m^2 \, \frac{R}{|\mathbf{x} \times \mathbf{x}_0|} \, \arctan \frac{x^2 - x_0^2 + R^2}{2|\mathbf{x} \times \mathbf{x}_0|} \\
& - \frac{1}{4}\, (8(1+\gamma) - 4\beta + 3\epsilon)\, m^2 \, \frac{R}{|\mathbf{x} \times \mathbf{x}_0|} \, \arctan \frac{x^2 - x_0^2 - R^2}{2|\mathbf{x} \times \mathbf{x}_0|} \\
& + \frac{1}{2}\, (1+\gamma)^2\, m^2 \, \frac{R}{|\mathbf{x} \times \mathbf{x}_0|^2}\, (x - x_0 - R)\, (x - x_0 + R) + \mathcal{O}(c^{-6}),
\end{aligned} \tag{1}
$$

where β, γ and ϵ are three numerical parameters of the extended PPN formalism. All these three parameters are equal to unity in general relativity and may have different numerical values in other theories of gravity. The post-post-Newtonian terms can attain about 10 meters for experiments in the solar system and should be taken into account for high-accuracy data. We see that this expression is quite complicated compared to the usual post-Newtonian one given by the first line of (1). However, if one estimates the numerical magnitudes of the post-post-Newtonian terms in (1) it turns out that only the last term is numerically relevant. The last term can be written together with the main post-Newtonian term in a compact way

$$
c\,(t_1 - t_0) = R \, + \, (1+\gamma)\, m \, \log \frac{x + x_0 + R + (1+\gamma)\, m}{x + x_0 - R + (1+\gamma)\, m}\,. \tag{2}
$$

All other post-post-Newtonian terms together can be estimated as

$$
c\,\delta\, t_1 \le \frac{m^2}{d} \, \left(\frac{3}{4} + \frac{15}{4}\, \pi \right)\,. \tag{3}
$$

where d is the impact parameter of the light ray with respect to the gravitating body. Since $d \ge L$, L being the radius of the body, one concludes that for any solar system experiments Eq. (3) gives at most 4 cm for a Sun-grazing ray. These terms can therefore be neglected for all present and planned experiments. Eq. (2) coincides with Eq. (8-54) of Moyer (2000) who derived this equations in a different and inconsistent way.

7. CONCLUSION

Applied relativity is a multidisciplinary research field. Progress here requires dedicated efforts both from the side of theoretical work and from the side of practical implementation of relativistic concepts and ideas into every-day astronomical practice. It is clearly a challenge to combine knowledge in theoretical general relativity and in practical observational techniques and modelling of complex astronomical phenomena (e.g. Earth rotation). For this reasons, one often fuzzily divides scientists into "experts in relativity" and "people doing practical calculations". For the former kind of people it is quite difficult to understand what the second kind of people really need for their work. Vice verse, for "practical people" it is not always easy to understand what the "theorists" suggest. Clearly, educational efforts on both sides are indispensable for further progress in the field of applied relativity.

8. REFERENCES

Brumberg, V.A., Kopejkin, S.A. 1990, "Relativistic time scales in the solar system", Cel.Mech.dyn.Astron., 48, 23

Brumberg, V.A., Kovalevsky, J., 1986, "Unsolved Problems of Celestial Mechanics" Cel.Mech. 39, 133

IAU, 2006, "Re-definition of Barycentric Dynamical Time, TDB", Resolution adopted by the XXVIth General Assembly

Klioner, S.A., 2004, "Physically adequate proper reference system of a test observer and relativistic description of the GAIA attitude", Phys.Rev. D 69, 124001

Klioner, S.A., 2008, "Relativistic scaling of astronomical quantities and the system of astronomical units", A&A , forthcoming

Klioner, S.A., Soffel, M., 2000, "Relativistic celestial mechanics with PPN parameters", Phys.Rev.D 62, 024019

Klioner, S.A., Soffel, M., Le Poncin-Lafitte, Ch., 2008, "Towards the relativistic theory of precession and nutation", this volume

Klioner, S.A., Zschocke, S., 2008, "Parametrized post-post-Newtonian analytical solution for light propagation", available from Gaia Livelink archive as GAIA-CA-TN-LO-SK-002-1, to be published in Classical and Quantum Gravity

Kopeikin, S., Vlasov, I., 2004, "Parametrized post-Newtonian theory of reference frames, multipolar expansions and equations of motion in the N-body problem", Physics Reports, 400, 209

Moyer, T.D., 2000, "Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation", Jet Propulsion Laboratory, California Institute of Technology

Seidelmann, P.K., 1986, "Unsolved Problems of Celestial Mechanics – The Solar System" Cel.Mech. 39, 141

Soffel, M. et al., 2003, "The IAU 2000 resolutions for astrometry, celestial mechanics and metrology in the relativistic framework: explanatory supplement", AJ , 126, 2687–2706

Will, C.M., 1993, Theory and experiment in gravitational physics (Cambridge: Cambridge University Press)

Will, C.M., 2006, "The Confrontation between General Relativity and Experiment", Living Review Relativity 9, 3, http://www.livingreviews.org/lrr