ESTIMATION OF THE TOPOGRAPHIC TORQUE AT THE CORE-MANTLE BOUNDARY ON NUTATION

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ABSTRACT. We study analytically the effect of the existence of CMB (core-mantle boundary) topography on the Earth's nutation. To that aim, we have considered an Earth model with a rigid mantle, a homogeneous and incompressible fluid core, and a slightly non-hydrostatic core-mantle boundary.

1. MOTIVATION AND RESEARCH DESIGN OF THE STUDY

This work is the first in a series of steps carrying out the progress of the European DESCARTES Sub-project entitled: *"Computation of the topographic coupling at the core-mantle boundary and its effect on the nutation"*. The principal milestones of the workplan and their specific objectives are as follows:

[1] Methodology and strategies to obtain the expressions for the topographic coupling:

- To establish the differential equations and boundary conditions describing the problem,

- To study the best analytical method for obtaining the solutions,

- To perform an analytical development of the coefficients describing the dynamic pressure, as function of parameters of the boundary topography,

- To determine the topographic torque.

[2] Comparison with the results from Wu and Wahr (1997) using a numerical technique.

[3] Application of the results to other celestial bodies of the solar system.

2. DIFFERENTIAL EQUATIONS AND PRELIMINARY RESULTS

In order to describe the diurnal wobbles or nutations of the Earth, we consider the Liouville equations for the angular momentum conservation. We have simplified the problem considering a rigid mantle and a homogeneous and incompressible fluid core. With the objective of computing the topographic torque, we have considered a slightly non-hydrostatic core-mantle boundary, from which we have computed the topographic torque. To that aim, we need to establish the equations and the boundary conditions.

- Linearized Navier-Stokes equation:

$$\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\rho_f}\nabla P + \vec{b} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2\vec{\omega}\times\vec{V} - \frac{\partial\vec{\omega}}{\partial t}\times\vec{r} \tag{1}$$

where $\times$ indicates a cross-product, $\vec{V}$ is the velocity of the fluid relative to the reference frame, P is the Eulerian pressure and ρ_f is the fluid density. The body force $\vec{b} = \triangledown\Phi_0 + \nabla\phi_1 + \nabla\phi_e$ (sum of the self unperturbed gravitational attraction $\nabla\Phi_0$, the mass redistribution gravitational attraction $\nabla\phi_1$, and the lunisolar gravitational attraction $\nabla\phi_e$ per unit mass).

- Boundary condition at the core-mantle boundary (CMB): $\vec{n} \cdot \vec{V} = 0$ ($\vec{n}$ is the normal to the surface); it is expressed as a function of the boundary topography; the non-hydrostatic boundary surface is expressed using: $r = r_0 \left[1 + \sum_{n=1} \sum_{m=-n}^{n} \varepsilon_n^m Y_n^m(\vartheta, \lambda) \right]$ (r_0 is the surface mean radius)

- Condition of incompressibility: $\nabla \cdot \vec{V} = 0$.

Wu and Wahr (1997) obtained the solutions of the differential equations (1), after extensive developments, using a numerical technique of integration. Our purpose here is to work out such integration by means of an analytical method in order to do a comparison between the two approaches. To this aim, we have decomposed the velocity as: $\vec{V} = \vec{u} + \vec{v} = \Omega L \vec{q} + \vec{v}$, where L is the maximum radius of the core, $\vec{v}$ is the Poincaré fluid velocity in the case of nutation and $\vec{q}$ is a non-dimensional velocity which equation and conditions can be expressed as:

$$
\begin{cases}
i \sigma_m \vec{q} + 2 \vec{\vec{z}} \times \vec{q} + \nabla \Phi = 0 \\[2mm]
\nabla \cdot \vec{q} = 0 \\[2mm]
\vec{n} \cdot \vec{q} + \Omega^{-1} L^{-1} \vec{n} \cdot \vec{v} = 0
\end{cases}
$$
(2)

where $\Phi = \frac{\phi}{\Omega^2 L^2}$ and $\phi = \frac{p}{\rho_f} + \chi$, Φ being called the non-dimensional dynamic pressure and χ is an unspecified function. The time dependence of the variables is considered as $e^{i\sigma t}$. When introduced in non-dimensional equations as above, the frequency to be used is σ_m instead of σ, where $\sigma = \Omega \sigma_m$. After some algebra of the first equation of (2), one can obtain the following expression for $\vec{q}$ as a function of $\nabla \Phi$:

$$
\vec{q} = \frac{-i \sigma_m}{4 - \sigma_m^2} \left[\nabla \Phi - \frac{2}{i \sigma_m} \vec{\vec{z}} \times \nabla \Phi - \frac{4}{\sigma_m^2} (\vec{\vec{z}} \cdot \nabla \Phi) \vec{\vec{z}} \right] \quad \text{where:} \quad \Phi = \sum_{l=1} a_l^k P_{lk}(\frac{\sigma_m}{2}) Y_l^k(\vartheta, \lambda) \quad (3)
$$

where $\vec{\vec{z}}$ is the unit vector in the z-direction, $P_{lk}(\frac{\sigma_m}{2})$ and $Y_l^k(\vartheta, \lambda)$ are, respectively, the Legendre polynomials and the associated Legendre functions of the first kind.

Using the boundary condition for $\vec{q}$ (third equation of (2)) and the expression of $\vec{q}$ in function of Φ (Eq. (3)), after a lot of developments, one obtains the analytical expressions for the coefficients a_l^k at the first order in the small quantities such as ϵ_n^m. The preliminary results are shown in Table 1, where we have kept only the coefficients significantly higher than $0.1 \epsilon_n^m$.

Coefficients	Term in m_f^+	Term in m_f^-
a_1^0	$0.4\,\epsilon_3^1$	
a_1^1	$0.3\,\epsilon_3^2$	
a_2^0	$-0.7\,\epsilon_2^1$	
a_2^{-1}	$-1.0\,\epsilon_2^0$	
a_2^2		$-0.2\,\epsilon_2^1$
a_3^0	$-3.5\,\epsilon_3^1$	
a_3^1	$0.2\,\epsilon_3^2$	$-0.6\,\epsilon_3^0$
a_3^2	$0.1\,\epsilon_3^3$	$-0.3\,\epsilon_3^1$
a_3^3		$0.04\,\epsilon_3^2$
a_4^0	$-0.3\,\epsilon_2^2$	$0.4\,\epsilon_2^0$
a_4^{-1}	$9.6\,\epsilon_2^0$	

Table 1. Analytical expressions for a_l^k in function of the parameters ϵ_n^m.

3. REFERENCE

Wu, X., Wahr, J.M., 1997, "Effects of non-hydrostatic core-mantle boundary topography and core dynamics on Earth rotation", Geophys. J. Int. 128(1), pp. 18–42.