CONCISE ALGORITHMS FOR PRECESSION-NUTATION

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ABSTRACT. The precession-nutation models based on the IAU 2000A nutation series involve several thousand amplitude coefficients, many under 1 μas in size, and the sines and cosines of about 1350 angles. For the many applications that do not require the utmost accuracy this represents an unnecessary or even excessive computational overhead. The IAU 2000B model offers one alternative, an order of magnitude smaller than IAU 2000A and delivering classical nutation components of 1 mas accuracy in the current era. In a recent paper (Capitaine & Wallace 2007), the main results of which are provided here, we looked at other options, based on series for the CIP coordinates and the CIO locator and with the GCRS to CIRS rotation matrix as the end product. Truncation of the series provides most of the savings, but certain other measures can be taken also. Three example formulations are presented that achieve 1 mas, 16 mas and 0.4 arcsec accuracy throughout 1995-2050 with computational costs 1, 2 and 3 orders of magnitude less than the full models. A few examples of possible applications are presented.

1. INTRODUCTION

The IAU 2000A model for precession-nutation includes terms at about 1350 frequencies, and coefficients as small as 0.1 μas. Although for many applications the size of the model is not an important consideration, this is not always the case. Use of the full models is natural when the application demands it and/or computing resources are ample. But often a lower accuracy will suffice, and then the full models are an unnecessary overhead. Sometimes computing resources are so limited that the full models are unaffordable: unless some interpolation or look-up scheme is devised, a simplified model must be used. The IAU recognized this need, adopting in addition to the full-accuracy model an equinox based lightweight alternative of 1 mas accuracy, namely 2000B (McCarthy & Luzum 2003). The recent paper by Capitaine & Wallace (2007, CW07) addresses the more general problem of how to construct a concise and efficient CIO based model that achieves any given level of accuracy, and the present paper presents some of the results of that work.

2. METHODS

The transformation from celestial (GCRS) to terrestrial (ITRS) coordinates can be written out as:

$$\mathbf{v}_{\mathrm{ITRS}} \quad = \quad \mathbf{R}_{\mathrm{PM}} \cdot \mathbf{R}_3(\theta) \cdot \mathbf{R}_{\mathrm{NPB}} \cdot \mathbf{v}_{\mathrm{GCRS}} \qquad (1)$$

where the vectors $\mathbf{v}_{\mathrm{GCRS}}$ and $\mathbf{v}_{\mathrm{ITRS}}$ are the same direction with respect to the two reference systems, the matrix $\mathbf{R}_{\mathrm{NPB}}$ represents the combined effects of frame bias and precession-nutation and defines the directions of the celestial intermediate pole and origin (CIP and CIO), $\mathbf{R}_3(\theta)$ is Earth rotation angle and the matrix $\mathbf{R}_{\mathrm{PM}}$ is polar motion. The objective is to devise formulations for $\mathbf{R}_{\mathrm{NPB}}$ that achieve different compromises between accuracy and computing costs over a specified time interval.

There are several ways of forming $\mathbf{R}_{\mathrm{NPB}}$ (see Capitaine & Wallace 2006), and abbreviated forms of any of these could be developed. But the method based on direct series for (i) the CIP coordinates X, Y and (ii) the quantity $s+XY/2$ that locates the CIO is particularly attractive. Separate treatment of bias, precession and nutation is avoided, each of the three series makes the same contribution to final accuracy (e.g. no $\sin\epsilon$ factors to consider), and other aspects can be optimized individually. Furthermore, simple truncation of the series is likely to deliver a nearly optimal result, without resorting to least-squares fitting or harmonic analysis. The method includes a number of opportunities for trading off accuracy against computing costs and these have been developed in CW07. In summary:

- Truncating the X, Y series is where the biggest savings lie. Each term in X or Y consists of a sine and cosine component at a given frequency. The "purist" approach is to regard each term as a vector and to truncate based on modulus – so that coefficients are either dropped or retained in pairs. But because almost all terms have phases such that either the sine or cosine coefficient dominates, truncating by individual coefficient, i.e. usually retaining only one of the pair, avoids wasteful and ineffectual tiny values in the final series. The relationship between number of retained coefficients and the 1995-2050 CIP accuracy is shown in Figure 1.

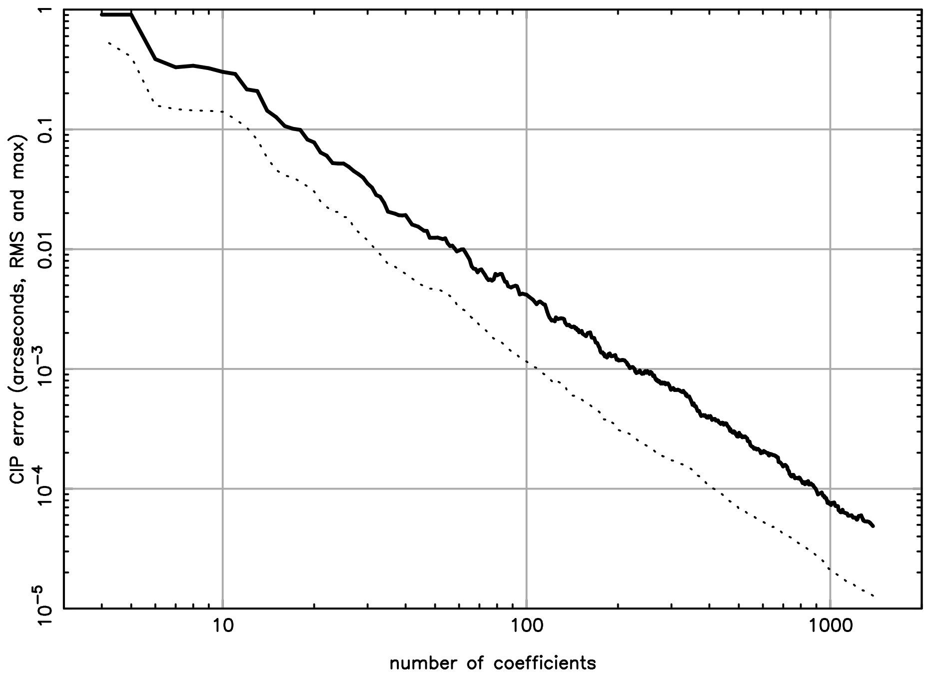

Figure 1: The variation of CIP accuracy with differing cut-offs applied to the individual coefficients of the X, Y series. The horizontal axis is the number of retained coefficients, each of which is either a sine or cosine term at a particular frequency and power of (t) and contributes to either X or Y. The vertical axis is the error in the position of the CIP, compared with that predicted by the full series. The heavy line shows the maximum error during the interval 1995-2050; the dotted line is the RMS error in the same interval.

- The $s + XY/2$ series is much shorter than those for X and Y but there are still opportunities for worthwhile savings. Only a handful of terms is needed to achieve 1 mas, and for the most concise models s can be neglected altogether. Note that the X and Y used to remove the $XY/2$ term do not have to be very accurate, so that whatever approximate values have already been calculated will be more than adequate.

- There are obvious opportunities for approximating the $\mathbf{R}_{\mathrm{NPB}}$ matrix elements, exploiting the facts that the CIP z-coordinate is nearly unity and the angle s is small. During 1995-2050, accuracies of a few μas can be achieved without resorting to trigonometric functions or square roots, and even a matrix that contains only the values 0, 1, X and Y achieves 0.1 arcsecond accuracy.

- The series for X, Y and $s + XY/2$ are functions of the fundamental arguments, a set of 14 angles. They comprise the five Delaunay variables l, l', F, D and Ω, eight planetary longitudes, and the general precession. Each is a polynomial in time: the expressions for the Delaunay variables use five coefficients (i.e up to t^4) , all the others just two. Potential savings, from omitting unused arguments and truncating the series for the Delaunay variables, are always modest, but worthwhile for the more approximate $\mathbf{R}_{\mathrm{NPB}}$ formulations.

- The full X, Y series contain terms with periods from 3.5 days to almost 100 millennia. In a restricted time span, for example 1995-2050, the terms of longer period produce nearly fixed offsets in X and Y. Using 1000 years as the cut-off eliminates 33 terms and gives offsets of $-634.2\,\mu$as in X and $+1421.45\,\mu$as in Y. These can be combined with the CIP bias and used for all the concise formulations.

3. EXAMPLE CONCISE FORMULATIONS

Table 1 summarizes the performance of three models obtained with the techniques just described, using the SOFA implementation of the full IAU 2000A model as the reference. Figures for IAU 2000B are also included, for comparison.

model	*coeffs*	*freqs*	*RMS*	*worst*	*speed*
reference	4006	1309	-	-	1
IAU 2000B	354	77	0.28	0.99	7.6
CPN_b	229	90	0.28	0.99	15.3
CPN_c	45	18	5.4	16.2	138
CPN_d	6	2	160	380	890
		mas	mas		

Table 1: Three concise models (designated CPN_b, CPN_c and CPN_d) compared.

Concise model CPN_b aims to equal the performance of IAU 2000B. The peak errors during 1995-2050 are shown in Figure 2. It was obtained by truncating the X and Y series at $50\,\mu$as and the $s + XY/2$ series at $60\,\mu$as, using slightly simplified expressions for the matrix elements but retaining full-accuracy fundamental-argument expressions. It requires fewer coefficients than IAU 2000B and is twice as fast. Its accuracy is such that the unmodeled free core nutation is itself an important limitation.

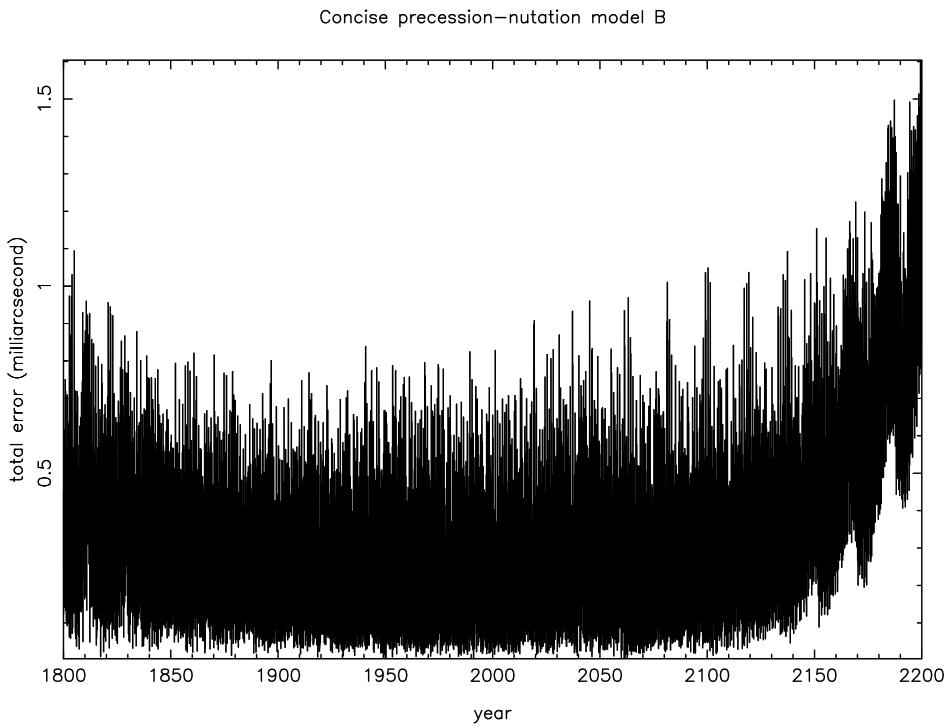

Figure 2: The 400-year performance of the example concise formulation CPB_b. The model achieves better than 1 mas (worst case) during the interval 1995-2050.

Concise model CPN_c achieves 16 mas performance between 1995 and 2050, which is better than the old IAU 1976/1980 model that is still in wide use for low-accuracy applications, and considerably shorter. The X and Y cutoffs are 2.5 mas, leaving only 42 coefficients, plus another three to obtain the CIO locator s. Using 2-coefficient fundamental-argument expressions also offers savings, and a speed well over 100 times faster than the full IAU 2000A model is achieved.

Concise model CPN_d could be a good choice for applications where polar motion will normally be neglected, such as pointing small telescopes. It is almost 1000 times faster than the full IAU 2000A, and despite needing only six coefficients achieves 0.4 arcsec (worst case) during the 1995-2050 test interval.

4. EXAMPLE APPLICATIONS

Only a minority of practical applications require the full accuracies delivered by current models. Examples of applications where somewhat reduced accuracy is acceptable and where improved speed is potentially beneficial include:

- Satellite orbit predictions: see Vallado & Seago (2006).
- Pulsar timing analysis: the recent TEMPO2 analysis software (Edwards et al. 2006) uses IAU 2000B.
- The pointing of telescopes and antennas: accuracy needs are set by the limitations of refraction predictions and the mechanical imperfections of the telescope and mount.
- Occultation predictions.
- The IERS could consider adopting a concise model as an alternative basis for the publication of celestial pole offsets dX, dY. At present, users are put to the expense of computing the full model, only to add corrections to the results. Were the IERS to add to its tabulations dX, dY values with respect to a shorter model (say CPN_b), this would produce an identical final CIP X, Y at a fraction of the computing costs.

5. REFERENCES

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