# VLBI OBSERVATIONS OF NUTATION, ITS GEOPHYSICAL EXCITATIONS AND DETERMINATION OF SOME EARTH MODEL PARAMETERS

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ABSTRACT. Very Long-baseline Interferometry (VLBI) has been used to observe nutation for more than 25 years, with ever increasing accuracy. The amplitudes and phases of individual nutation terms are sensitive to some parameters of the geophysical Earth model. Dominant are the resonant period $P$ and quality factor $Q$ of retrograde Free Core Nutation (FCN) that depend on the flattening of the outer fluid core. The period (in celestial frame) is approximately equal to 430 days. In principle, all nutation terms are affected by these resonance effects, but the most affected is the annual retrograde term that is closest to the resonance. Nutation is dominantly driven by external torques, and it contains, as a multiplicative factor, the dynamical ellipticity of the Earth. Relatively small (near-diurnal in terrestrial frame) geophysical excitations are amplified by the resonance, so they also have non-negligible influence on nutation. The aim of the present study is to take these effects into consideration and determine the period and quality factor of the Earth at the FCN frequency. An attempt to derive from them also the dynamical flattening of the whole Earth and its fluid core is made, taking into account other effects, such as electro-magnetic coupling between the core and the mantle.

## 1. INTRODUCTION - RESONANCES IN EARTH ORIENTATION

Due to the existence of a flattened fluid and rigid inner core, there are strong resonances in near-diurnal (in terrestrial frame) part of the spectrum, leading to a significant modification of nutation amplitudes, and also to a non-negligible influence of geophysical excitations in nutation. The resonances are characterized by Mathews-Herring-Buffet (Mathews et al. 2002, paper further referred to as MHB) transfer function, the strongest one being the retrograde Free Core Nutation (FCN):

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_\circ \left[ 1 + (1+\sigma) \left( Q_\circ + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right]. \quad (1)$$

This transfer function, which is expressed in a complex form, gives a ratio between the non-rigid and rigid Earth model nutation amplitude for a given terrestrial frequency $\sigma$ (expressed in cycles per sidereal day). $e_R = (C - A)/A = 0.0032845075$ is the ellipticity of the rigid Earth used to compute the 'rigid' solution. The other parameters are in general complex constants, among which $s_j$ are the resonant frequencies (of Chandler wobble, retrograde FCN, prograde FCN and Inner Core Wobble, respectively). Recently, we demonstrated (Vondrák and Ron 2007) that the atmospheric and oceanic effects can excite free core nutation, and that their contributions of nutation are of the order of 100$\mu$as.

## 2. ESTIMATION OF PARAMETERS $N_\circ$ AND $s_2$

We use the basic equation (1), together with the observed values of some nutation amplitudes, to estimate some of the parameters on its right-hand side. Parameter $N_\circ$ is a common multiplier in Eq (1) that is closely connected with the dynamical ellipticity of the whole Earth, parameter $s_2$ is the complex resonant frequency due to the flattening of outer fluid core. We estimate these parameters from the observed amplitudes and phases of seven nutation terms, with periods 6798, 3399, 365.26, 182.62, 121.75 and 13.66 days. To this end, we use the VLBI-based celestial pole offsets (IVS 2005) with respect to the IAU 2000 nutation (Mathews et al. 2002) and 2006 precession (Capitaine et al. 2003) models. The

IAU2000 model of nutation contains an empirical correction in prograde annual nutation ('MHB Sun-synchronous correction'), supposedly due to atmospheric excitation, which we removed from the observed amplitude at this frequency. We kept all other parameters in MHB transfer function constant, and made the solution in successive approximations. The following series of VLBI celestial pole offsets were used:

- Geoscience Australia (AUS) solution in the interval 1984.0–2007.6;
- Goddard Space Flight Center (GSFC) solution in the interval 1979.6–2007.2;
- Paris Observatory (OPA) solution in the interval 1984.0–2007.1;
- U.S. Naval Observatory (USNO) solution in the interval 1979.6–2007.2;
- International VLBI Service (IVS) combined solution in the interval 1979.6–2006.9.

The parameters $N_\circ$ (real) and $s_2$ (complex) were determined in weighted least-squares estimation, and from the latter one the FCN period $P$ and quality factor $Q$ computed:

$$P = 0.99727/\left[\mathrm{Re}(s_2)+1\right], \quad Q = -\mathrm{Re}(s_2)/2\mathrm{Im}(s_2) \tag{2}$$

The results are displayed in Tab. 1; first five rows show our estimation from different VLBI solutions, with no geophysical excitation applied. The last row displays the MHB values, for comparison. The table

Table 1: Results of estimating the parameters from VLBI observations

| Solution | $P$ | $Q$ | $N_\circ$ |
|---|---|---|---|
| AUS | $-429.98 \pm 22$ | $19985 \pm 837$ | $1.00001100 \pm 357$ |
| GSFC | $-430.11 \pm 18$ | $19544 \pm 633$ | $1.00001046 \pm 297$ |
| OPA | $-430.27 \pm 16$ | $19839 \pm 593$ | $1.00000826 \pm 260$ |
| USNO | $-430.07 \pm 18$ | $19348 \pm 646$ | $1.00000818 \pm 295$ |
| IVS | $-430.22 \pm 16$ | $20741 \pm 645$ | $1.00000921 \pm 256$ |
| MHB | $-430.21$ | 19998 | 1.00001224 |

shows a very good agreement of all estimated parameters; they are mutually consistent within the limits of their formal uncertainties, and they are also very close to MHB values.

## 3. GEOPHYSICAL EXCITATIONS, ESTIMATION OF SOME EARTH PARAMETERS

Next we calculate the influence of geophysical effects, namely the atmospheric and oceanic excitations, on nutation. We consider that the existing equatorial excitation series $\chi$ (in complex form) are all given in terrestrial reference frame, so they must be first transformed into $\chi'$ in celestial (non-rotating) frame, by using a simple formula $\chi' = -\chi \mathrm{e}^{\mathrm{i}\phi}$, where $\phi$ is the Greenwich sidereal time. Because we are interested only in long-periodic motions in celestial frame (i.e., precession-nutation) we smoothed out all excitations with periods shorter than 10 days, using the smoothing (Vondrák 1977) with $\varepsilon = 6\ \mathrm{day}^{-6}$. The following geophysical excitations were used, all in 6-hour intervals, to estimate the effects in nutation:

- Atmospheric Angular Momentum (AAM) functions (pressure and wind terms): NCEP/NCAR reanalysis in 1983.0–2007.0 (Salstein 2005); ERA40 in 1979.0–2001.0 (Dobslaw and Thomas 2007);
- Oceanic Angular Momentum (OAM) functions (matter and motion terms): ECCO model in 1993.0–2006.2 (Gross et al. 2005); Ponte model in 1993.0–2000.5 (Ponte and Ali 2002); OMCT model in 1979.0–2001.0 (Dobslaw and Thomas 2007).

These series were further used in the following combinations:

- NCEP AAM with inverted barometer (IB) correction (1983.0–2007.0),
- NCEP AAM without IB correction (1983.0–2007.0),
- NCEP AAM(IB) + ECCO OAM (1993.0–2006.2),
- NCEP AAM (IB) + Ponte OAM (1993.0–2000.5), and
- ERA40 AAM + OMCT OAM (1983.0–2001.0)

to integrate numerically the broad-band Liouville equations after Brzezinski (1994), here expressed in complex form:

$$\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f = -\sigma_C\left\{\sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \mathrm{i}\left[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w\right]\right\}. \tag{3}$$

$P = \mathrm{d}X + \mathrm{i}\mathrm{d}Y$ is geophysically excited motion of Earth's spin axis in celestial frame, $\sigma'_C = 6.32000 + 0.00237\mathrm{i}$, $\sigma'_f = -0.0146011 + 0.0001533\mathrm{i}$ rad/day are the Chandler and FCN frequencies in celestial frame, and $\sigma_C = \sigma'_C - \Omega$ is the Chandler frequency in terrestrial frame ($\Omega = 6.30038$ rad/day is the angular speed of Earth's rotation). $\chi'_p, \chi'_w$ are excitations (matter and motion term) in celestial frame, and $a_p = 9.2 \times 10^{-2}$, $a_w = 5.5 \times 10^{-4}$ are dimensionless numerical constants. Numerical integration of Eq (3), using four-order Runge-Kutta method and 6-hour step, yields the geophysical contribution in time domain; an example of such integration is depicted in Fig. 1, where both pressure and wind term are considered. The comparison with VLBI observations (IVS solution) shows a very good agreement.

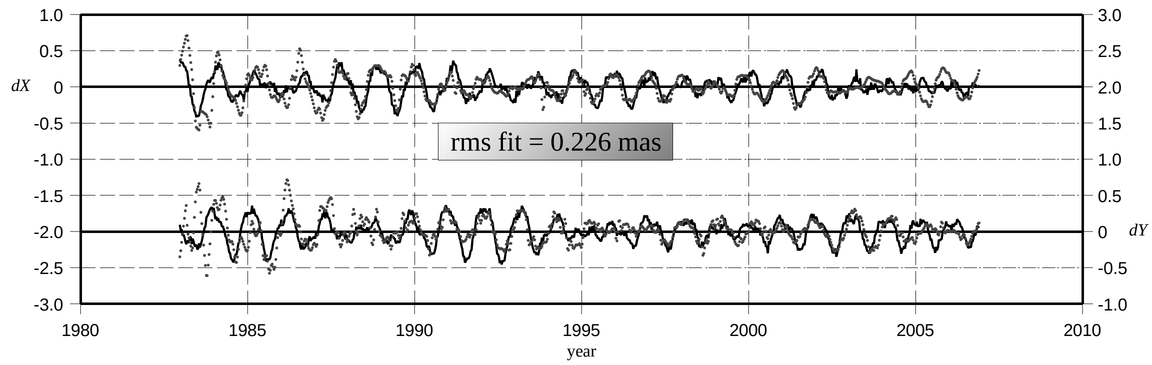


Figure 1: Nutation excited by NCEP AAM(IB), line, and VLBI observations, dots, in mas.

Next we made a spectral analysis of the results, which revealed that all four combinations of geophysical excitation mentioned above yield a similar pattern – there are significant peaks only at FCN, annual and semi-annual periods. Therefore, we made a least-squares estimation of sine/cosine terms in $\mathrm{d}X$, $\mathrm{d}Y$ for these periods, and then calculated the prograde/retrograde complex amplitudes $A^+, A^-$ by using the relation $\mathrm{d}X + \mathrm{id}Y = -\mathrm{i}\sum_k (A_k^+ \mathrm{e}^{\mathrm{i}\omega_k} + A_k^- \mathrm{e}^{-\mathrm{i}\omega_k})$. The results are shown in Tab. 2, in comparison with MHB Sun-synchronous correction (last row). The next step was to remove the 'real' geophysical excitations from the VLBI-observed nutations, instead of the MHB Sun-synchronous correction. To this end, we used the IVS combined solution and the excitations of Tab. 2 with the lowest uncertainties, i.e., NCEP(IB), NCEP and NCEP(IB)+ECCO. Then, the analysis similar to Section 2 was made to derive parameters $P$, $Q$, and $N_\circ$. The results are shown in Tab. 3, where they are compared with VLBI solution from which MHB Sun-synchronous correction was removed (i.e., identical with the row IVS of Tab. 1).

Table 2: Atmospheric and oceanic contributions to nutation and their uncertainties $\sigma$ [$\mu$as]

| | annual | | | | semi-annual | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Excitation | prograde | | retrograde | | prograde | | retrograde | | |
| AAM + OAM | Re | Im | Re | Im | Re | Im | Re | Im | $\sigma$ |
| NCEP(IB) | −3.4 | +109.7 | −69.9 | −18.7 | −44.2 | −55.7 | −1.8 | +0.9 | ±2.7 |
| NCEP | +13.8 | +91.5 | −76.8 | −52.1 | −25.5 | −52.5 | −6.5 | −1.0 | ±4.9 |
| NCEP(IB)+ECCO | −0.6 | +110.7 | −32.2 | −69.1 | −46.5 | −53.9 | −6.2 | −21.8 | ±4.6 |
| NCEP(IB)+Ponte | +80.3 | +107.4 | +86.4 | −135.6 | −26.3 | −62.1 | +9.2 | −9.6 | ±8.5 |
| ERA40+OMCT | −64.9 | +180.3 | −64.3 | +5.6 | −17.9 | −77.3 | +0.3 | +3.4 | ±6.8 |
| MHB Sun-synchr. | −10.4 | +108.2 | – | – | – | – | – | – | – |

Table 3: Results of estimating the parameters from VLBI observations, with geophysical effects removed.

| Solution | $P$ | $Q$ | $N_\circ$ |
|---|---|---|---|
| NCEP(IB) | $-431.00 \pm 23$ | $20530 \pm 911$ | $1.00001437 \pm 368$ |
| NCEP | $-431.18 \pm 21$ | $19892 \pm 781$ | $1.00001506 \pm 335$ |
| NCEP(IB)+ECCO | $-430.96 \pm 19$ | $17584 \pm 636$ | $1.00001708 \pm 1700$ |
| MHB Sun-synchr. | $-430.22 \pm 16$ | $20741 \pm 645$ | $1.00000921 \pm 256$ |

The parameters derived above can be, together with some other parameters, further utilized to estimate the dynamical flattening of the fluid core $e_f$ and of the whole Earth $H_d$. Namely we use the formula, following from MHB expressions (37) and (38):

$$e_f = \tilde{\beta} - \frac{A_m}{A}\left[\mathrm{Re}(s_2) + 1\right] - \mathrm{Re}(K^{CMB}) - \frac{A_s}{A_f}\mathrm{Re}(K^{ICB}), \qquad (4)$$

giving the relation between the flattening and parameters $\tilde{\beta}$ (compliance), $A, A_m, A_s, A_f$ (moments of inertia of the whole Earth, its mantle, solid and fluid core, respectively), $K^{CMB}$ and $K^{ICB}$ (electromagnetic couplings at core-mantle and inner core boundaries). When we use our value $\mathrm{Re}(s_2) = -0.00231406 \pm 103$, corresponding to NCEP(IB)+ECCO solution of Tab. 3, the numerical values from PREM Earth model (Mathews et al. 1991) $\tilde{\beta} = 6.160 \times 10^{-4}$, $A_m/A = 0.88621$, $A_s/A_f = 0.0064616$ and MHB paper $\mathrm{Re}(K^{CMB}) = 2.32 \times 10^{-5}$, $\mathrm{Re}(K^{ICB}) = 1.11 \times 10^{-3}$ (the latter two being known to only 10%), we arrive at $e_f = 0.0026364 \pm 23$, whose uncertainty is however fully limited by the imprecision of electro-magnetic couplings. The dynamical ellipticity of the whole Earth can be calculated from our $N_\circ$ and rigid Earth value $H_{dR} = e_R/(1 + e_R)$ by a simple relation $H_d = (C - A)/C = N_\circ H_{dR} = 0.0032738041 \pm 110$.

## 4. CONCLUSIONS

Unlike in the recent study by Lambert and Dehant (2007), all VLBI solutions studied yield similar values of estimated parameters $N_\circ, s_2$, consistent within their standard errors. Forced nutations due to the excitation by the atmosphere and oceans are significant at annual and semi-annual frequencies; they are similar for different oceanic models used, and the prograde annual term is in a good agreement with the value of the MHB empirical Sun-synchronous correction. The application of geophysical excitations yields a slightly longer period of retrograde FCN, and a larger value of a common multiplier $N_\circ$, than the MHB Sun-synchronous correction. Determination of the flattening of the core and of the whole Earth, based on these values, is possible, but the precision of the former is severely limited due to the not well known electro-magnetic couplings.

*Acknowledgments.* This study was supported by the grant LC506 awarded by the Ministry of Education, Youth and Sports of the Czech Republic.

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