ESTIMATION OF EARTH INTERIOR PARAMETERS FROM A BAYESIAN INVERSION OF NUTATION TIME SERIES

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ABSTRACT. The nutation response of the Earth to the external gravitational torque is affected by the Earth internal structure. Because the torque is known very accurately, the high precision nutation observations allow to constrain Earth interior parameters. A non-rigid Earth nutation model has been proposed by Mathews et al. (2002). This model is semi-analytical: the analytical equations of the model depend on parameters, related to the Earth interior, which are fitted to the observations. In this paper, we develop a new fit procedure relying on an estimation of the geophysical parameters directly from the nutation data in time series. This allows to use all the information of the time-domain data and to take into account the time dependent uncertainties on the data. We use the Bayesian inversion method, which is well-suited for the non-linear nutation model and easily accommodates the possibility of having uncertainties in the nutation model itself. Finally, the results are compared with those obtained by Mathews et al. (2002).

1. INTRODUCTION

The rotation of the Earth is affected by the gravitational torque exerted on its equatorial bulge by the Moon, the Sun and, to a lesser extent, the other planets. As a response to this torque, the rotation axis of the Earth changes its orientation and describes a large circle around the perpendicular to the ecliptic plane, called the "precession" motion, while small periodic variations of this motion -due to periodic changes in the relative positions of the celestial bodies- are called "nutations". The precession/nutations motion is observed very precisely with the Very Long Baseline Interferometry (VLBI) technique. The main interest of nutation for studying the Earth's interior relies on the fact that the gravitational torque, at the origin of nutation, is known very precisely from celestial mechanics. Since the nutation response of the Earth to this torque depends on its internal structure, the accurate VLBI observations of nutation allow to constrain parameters of the Earth interior model.

2. EARTH INTERIOR AND NUTATION MODELS

The Earth interior model used for nutation modeling by Mathews et al. (2002) is that of a three layered Earth (mantle, liquid outer core and solid inner core) spheroïdally stratified, meaning that the density, the potential and the elastic parameters are constant on the same spheroïdal surfaces. The Earth model is assumed to be deformable and the deformations are due, on the one hand, to the external tidal potential and, on the other hand, to the rotational potentials of the Earth and of the fluid and solid cores. The Earth deforms anelastically, leading to a small time delay between the forcing and the deformational response. The model also includes the effects of the external geophysical fluids: ocean tides and atmospheric seasonal cycles.

The dynamical equations describing the nutation of the Earth are the equations of the angular momentum budget for each part of the Earth. They relates the time variation of the angular momentum of each part to the total torque applied to it. Those torques are due to the external tidal potential and to the interactions between the different layers of the Earth model. Those interactions are from several origins: gravitational (attraction between the masses of the different regions), inertial (pressure on the elliptical boundaries), friction, topographic, and electromagnetic.

The angular momentum of each of the three regions inside the Earth is a function of the inertia tensor and its instantaneous rotation vector. The inertia tensor depends on the way the masses are distributed inside the Earth. It can be written as the sum of the inertia tensor of the unperturbed ellipsoidal steadily rotating reference Earth and small increments due to the deformations and to the ocean tides. The increments due to the deformations are expressed as a linear combination of the potentials at the origin of the deformations (namely the tide generating potential and the three centrifugal potentials). The complex constants of proportionality, describing how each potential increments the inertia tensor, are called the "compliances". Ocean tides also perturb the inertia tensor due to the redistribution of mass in the ocean itself, and to the deformations induced by the surface load. Relative motions in the ocean with respect to the Earth create additional angular momentum. Ocean tides effects on nutation are obtained from an ocean tide model derived from the satellite altimetry data TOPEX/Poseidon.

The geophysical parameters of the above described Earth model are (1) the dynamical ellipticities, defined as a ratio between the equatorial and radial principal moments of inertia, (2) the compliances, describing the deformations, and (3) coupling constants characterizing all the interactions between the regions except gravitational and inertial couplings which are taken into account explicitly. Some of those parameters which have a large influence on nutation can be estimated from the VLBI nutation data.

3. PARAMETERS ESTIMATION METHOD

The first main aspect of the inversion procedure we have developed is that we use the VLBI data time series. This is new with respect to the classical procedure, used by Mathews et al. (2002), which consists in the extraction of the amplitudes and phases of the 21 main terms in frequency. Using the data directly in the time domain has the advantage that (1) it allows us to use all the available data, not only the 21 main periodic terms and (2) it better takes into account the time variability of the error on the VLBI data, which is due to the improvements in the measurement technique and to the additional stable radiosources considered over time.

The second point of our procedure is that, rather than the linearized least-squared method used by Mathews et al. (2002), we use a Bayesian inversion method. This method offers the advantage that the highly non-linear nutation model has not be linearized. Within the Bayesian method, the parameters are considered to be random variables. The joint probability density function (pdf) is inferred from the data and from prior knowledge associated with the parameters. Moreover, the Bayesian framework allows to easily include modeling errors, which takes into account the imperfection in the modeling. Those imperfections come from the simplicity of the Earth interior modeling, the ocean tides effects computed from data, the atmospheric effects,...

4. RESULTS

The probability density functions (pdf) obtained for the Earth interior model parameters are presented on the Figures 1, 2, and 3 for the dynamical ellipticities, compliances and coupling constants respectively. The pdf's are summarized by their expectation value and 3-sigmas credible range. Those values are shown in Table 1 together with the ones obtained by Mathews et al. (2002) for comparison.

4.1. Dynamical ellipticities

The estimated values for the dynamical ellipticities confirm the well-known fact that the flattenings of the whole Earth and fluid core are more important than those predicted by hydrostatic equilibrium (see e.g. Gwinn et al. 1986). Our estimation is in agreement with that of Mathews et al. (2002) in the sense that there is a large overlap between the three-sigma domains. However, we think that, for the whole Earth dynamical ellipticity, the agreement of our result with that of Mathews et al. (2002) is quite fortuitous because, depending on the length of the data series, the estimated value is different. This can be explained by the fact that this parameter is estimated mainly on the trend of the data which represents the precession. This trend is highly correlated with the 18.6 years nutation term, which is poorly known as only 28 years of data are available.

4.2. Compliances

In this work, the complex compliances are estimated from the nutation data, whereas in Mathews et al. (2002), only the elastic contribution to the compliances were estimated from the data while the anelastic contributions were computed theoretically. Figure 2 shows the result for the compliance

Parameter	Definition	This paper	MHB
e	Earth dynamical ellipticity	0.0032845471 ± 4	0.0032845479 ± 13
e^f+Re(K^{CMB})	Core dynamical ellipticity + coupling at the CMB (real part)	0.0026701 ± 7	0.0026680 ± 21
Re(κ)	Compliance for the Earth (real part)	0.0010528 ± 63	0.0010466 ± 99
Im(κ)	Compliance for the Earth (imaginary part)	-0.0000108 ± 70	0.00000529
Re(γ)	Compliance for the fluid core (real part)	0.00198548 ± 90	—
Im(γ)	Compliance for the fluid core (imaginary part)	0.00000933 ± 99	—
Im(K^{CMB})	Coupling at the CMB (imaginary part)	-0.0000205 ± 7	-0.0000185 ± 15
Re(K^{ICB})	Coupling at the ICB (real part)	0.00104 ± 19	0.00111 ± 11
Im(K^{ICB})	Coupling at the ICB (imaginary part)	-0.00185 ± 16	-0.00078 ± 14

Table 1: Numerical values of the Earth interior parameters and comparison with the values of Mathews et al. (2002). The second column gives the values for the inversion performed in this paper. The errors are on the last written digits and correspond to 3 σ. Numerical values obtained by Mathews et al. (2002) are listed in column four.

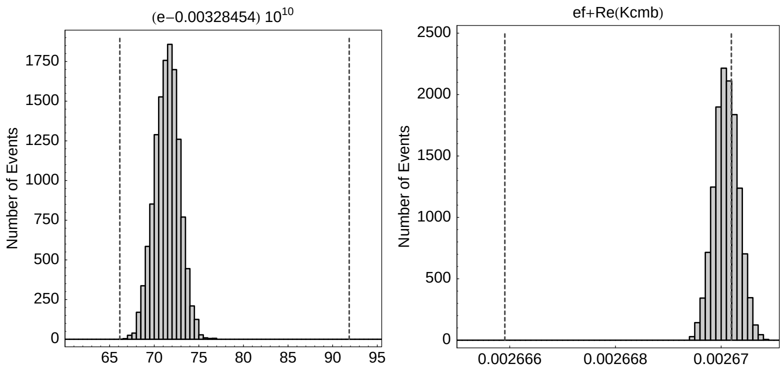

Figure 1: Marginal pdf's for the dynamical ellipticities of the whole Earth and fluid core, obtained from the Bayesian inversion of the nutation data. The dashed vertical lines represent the 3 σ-range of values obtained by Mathews et al. (2002).

describing the deformability of the whole Earth under the action of an external potential. The real part of this compliance is in agreement with Mathews et al. (2002) while the imaginary part is completely different. This difference can be explained by the fact that, the value of Mathews et al. (2002), because it has been computed theoretically from anelasticity models, represents purely the imaginary part of the compliance due to anelasticity. On the other side, our value being estimated from the nutation data, it is contaminated by the mismodeling of the ocean tides effects, which give also rise to imaginary contributions to the compliances. The fact that our estimated value is negative (which is not physically meaningful, as the deformational response of the Earth must lag behind the forcing) reflects the imperfection in the ocean tides modeling. Mathews et al. (2002) also deduced from their computations that the ocean tide contributions must be somewhat erroneous and decided to scale the current oceanic term by a 0.7 factor. This example shows the limitation of the estimation process: the mismodeling of certain effects can be absorbed by some parameters.

4.3. Coupling constants

The coupling constants characterize several types of interactions between the layers such as electromagnetic, viscous or topographic couplings. This makes their interpretation rather difficult because the individual effects can not easily be separated. The estimations we obtain for those parameters are not always in agreement with Mathews et al. (2002). For the imaginary part of the coupling constant at the ICB, our result is even very different, putting forward the fact that this parameter is probably the one that can be the less reliably estimated from nutation data. This is not really surprising as the parameters related to the inner core have a smaller effect on nutation than those of the outer core and mantle.

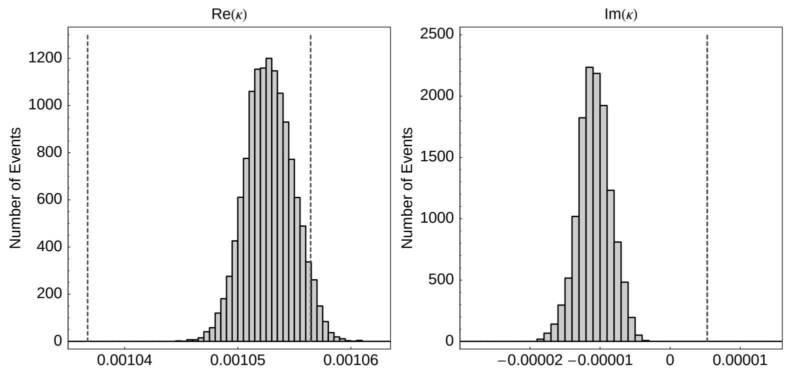

Figure 2: Marginal pdf's for the compliances, obtained from the Bayesian inversion of the nutation data. For Re(κ), the dashed vertical lines represent the 3 σ-range of values obtained by Mathews et al. (2002) for the purely elastic Earth plus the theoretically computed anelastic contribution. The dashed vertical line for Im(κ) represents the theoretical value computed by Mathews et al. (2002).

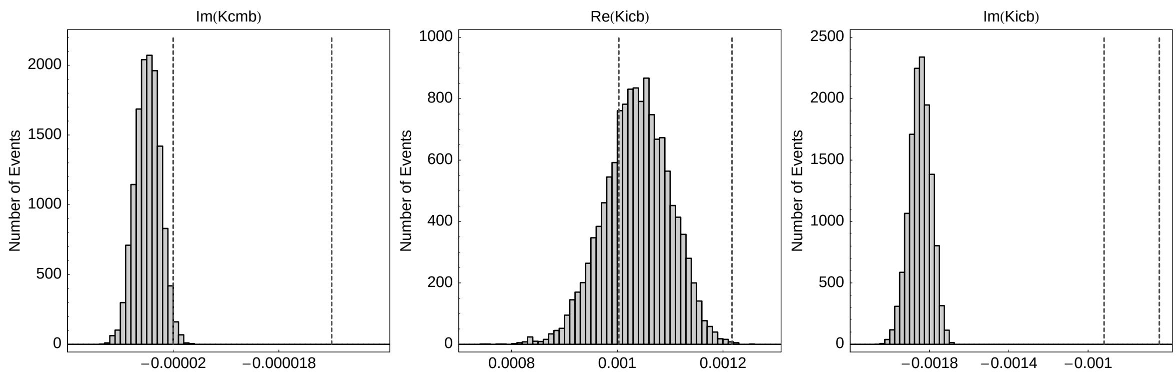

Figure 3: Marginal pdf's for the coupling constants, obtained from the Bayesian inversion of the nutation data. The dashed vertical lines represent the 3 σ-range of values obtained by Mathews et al. (2002).

5. CONCLUSIONS

The VLBI nutation data allow to estimate Earth interior parameters, such as the dynamical ellipticities of the Earth and fluid core, the compliances and the coupling constants. This estimation can be performed directly from the nutation data time series, there is no need of the extraction of amplitudes at some given frequencies, as it is done by Mathews et al. (2002). Some of our estimates are not in agreement with those of Mathews et al. (2002). This is due to the use of a different inversion strategy and the seven years of additional data. In particular, our estimation of the imaginary part of the coupling constant at the ICB is very different from that of Mathews et al. (2002) putting forward the fact that its interpretation in terms of physical couplings must be performed carefully.

6. REFERENCES

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