

RECENT ADVANCES IN MODELING PRECESSION-NUTATION

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ABSTRACT. Nutations are computed from modeling the gravitational forcing acting on the Earth from the Moon, the Sun and, to a minor extend, the other planets of the Solar system, and from modeling the interior of the Earth. One computes the Earth response to the gravitational forcing and then adds the geophysical effects from the surface liquid layers (atmosphere and ocean). The models for the interior of the Earth generally consider the existence of a liquid core, of a solid deformable inner core, and of a solid deformable inelastic mantle; the boundaries between the core and the inner core (ICB for Inner Core Boundary) and the core and the mantle (CMB for Core-Mantle Boundary) are flattened and can deform; the coupling mechanisms between the inner core, the outer core, and the mantle involve inertial coupling, gravitational coupling, pressure coupling, electromagnetic coupling, and viscous coupling. The last adopted model by the IAU and the IUGG, the so-called MHB2000 model from Mathews et al. (2002), considers all these coupling mechanisms except the viscous coupling, which is further considered in Guo et al. (2004). The rigid Earth nutations that are considered for this adopted model are those of Souchay et al. (1999).

In the present paper we concentrate on the steps performed within the group related to the Royal Observatory of Belgium, toward the next decimal of nutation theory and observation. In particular, we examine the improvements on observations, on atmospheric and oceanic contributions to nutation (for the free and forced nutations), and on the computation of dissipative and excitation mechanisms for nutation.

1. IMPROVEMENTS IN OBSERVATION

The improvement in VLBI observation is mainly coming from the consideration of criteria for radio source stability and from a better ground station network choice. The influence of the ground station networks (R1 versus R4) has been studied by Lambert and Gontier (2007). The influence of radio source instability on VLBI results has been addressed at ROB in Lambert and Dehant (2007) and Lambert et al. (2007). In summary, we have considered various strategies constraining and estimating differently the radio source position following the stability index of the considered source (e.g., application of the No-Net-Rotation (NNR) either on the 212 ICRF defining sources or on the 247 most stable sources).

We have used these criteria for source stability at ROB. Lambert and Dehant (2007) and Lambert et al. (2007) address them in full detail. In summary, we have considered not only the sources from the ICRF (the so-called defining sources), but the stable sources, studying the structure index, the elevation cutoff considered, the number of observations of the same radio source (e.g., sources observed in more than 40 sessions), considering or not the no-net-rotation condition, and/or treating the sources with the addition of local/global parameters.

These different strategies applied to VLBI data have allowed us to determine a set of residuals with re-

spect to the MHB2000 model mentioned above. Because we wanted to study the effects not well modeled, we considered the MHB2000 model without the only atmospheric correction therein (the contribution from the atmosphere on the prograde annual nutation). The residuals represent thus the difference between VLBI observations with the different strategies and the MHB2000 model without atmospheric correction. In these residuals, we should then see the atmospheric contributions. Figure 1 shows these residuals for the main nutations. We have determined the FCN period from fitting on our new data with the different strategies for the choice of the radiosources and found that the period and quality factor are very stable. Our determination of the FCN period for instance lead to a value of 430.31 days (± 0.02), very close the MHB2000 value of 430.20 days (± 0.28).

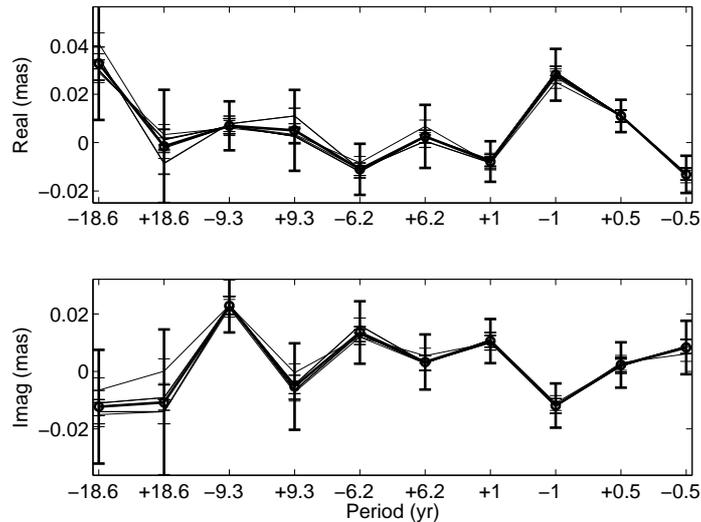


Figure 1: Residuals between VLBI observations with different strategies for the choice of the radiosources and MHB2000 without the annual atmospheric correction.

The different strategies applied to reduce the VLBI data are mainly influencing the 18.6 and 9.3 year nutations. This is to be expected as the strategies change mainly the long-term behaviour of the VLBI nutation time series.

Our aim is to better understand the interior of the Earth and therefore to obtain better results for the deep interior modeling from the new data series. More data are now handy with respect to the computation of MHB2000. We suspect that the increase of the time series with respect to the data set used for the MHB2000 construction would be seen in the evaluation of the inner core basic Earth parameters, i.e. the FCN period and quality factor. For these reasons we have decided to perform an experiment with the data and instead of analyzing the residuals with respect to MHB2000, we have analyzed the residuals with respect to MHB2000 when the inner core contributions are ignored. The new residuals represent then thus the difference between the VLBI observations with the different strategies and the modified MHB2000 model with atmospheric corrections at the prograde annual period but without inner core contributions. Figure 2 shows these residuals for the main nutations and for one of the strategies applied for the construction of Figure 1. The residuals are taken for a longer time span than MHB2000 as our data are going to 2007. The difference with MHB2000 is thus considered to be mainly the inner core contribution. This difference is mainly observed on the 18.6 year nutations (and mainly the retrograde 18.6 year nutation), and on the semi-annual nutation.

2. IMPROVEMENTS ON THE THEORY; BASIC GEOPHYSICAL PARAMETERS

As explained in Section 1, many nutation models are constructed from the convolution of rigid Earth nutation series with transfer function incorporating basic geophysical parameters that can either be approximate from our knowledge of the Earth interior or determined from the VLBI observation themselves. Improvements in the theory came recently from the determination of all sorts of coupling at the second order into this approach. In particular, Lambert and Mathews (2006) have considered all coupling mechanism between tides and nutations; Folgueira et al. (2007) have considered all couplings between the

Poisson terms of the tidal potential and the Earth response. These kinds of improvements are at the level of the microarcsecond. Further improvements will come from a better determination of the basic geophysical parameters involved in the transfer function of the Earth. These parameters, as defined in MHB2000, are the parameters entering in the Earth transfer function, and, in particular, in the FCN and FICN periods and quality factors. For an analytical approach such as the one developed in Mathews et al. (1991, see also Dehant et al. 1993), the values of the eigenfrequencies determined from the Liouville equations are approximated by:

$$\sigma_{FCN}^{\Re} = -\Omega \left(1 + \frac{A}{A_m} \left[\alpha_f - \kappa_f + K_{CMB}^{\Re} + \frac{A_s}{A_f} K_{ICB}^{\Re} \right] \right) \quad (1)$$

$$\sigma_{FCN}^{\Im} = -\Omega \frac{A}{A_m} \left(K_{CMB}^{\Im} + \frac{A_s}{A_f} K_{ICB}^{\Im} \right) \quad (2)$$

$$\sigma_{FICN}^{\Re} = -\Omega (1 - [\alpha_s \alpha_2 + \kappa_s - K_{ICB}^{\Re}]) \quad (3); \quad \sigma_{FICN}^{\Im} = -\Omega K_{ICB}^{\Im} \quad (4)$$

where σ_{FCN}^{\Re} , σ_{FCN}^{\Im} , σ_{FICN}^{\Re} , and σ_{FICN}^{\Im} are the real (\Re) and imaginary (\Im) parts of the FCN and FICN frequencies; Ω is the uniform Earth rotation frequency; A , A_m , A_f , and A_s are the principal moments of inertia of the whole Earth, the mantle, the outer core, and the inner core, respectively; α_f and α_s are the dynamical flattenings of the outer core and inner core; κ_f and κ_s are the compliances for the boundary deformations at the CMB and ICB respectively; K_{CMB}^{\Re} , K_{CMB}^{\Im} , K_{ICB}^{\Re} , and K_{ICB}^{\Im} are the real (\Re) and imaginary (\Im) parts of the coupling constants at the CMB and ICB; and $\alpha_2 = A'e'/A_s\alpha_s(1 + \alpha_g) - \alpha_g$ where α_g is a measure of the strength of the gravitational torque between a tilted inner core and the rest of the Earth (see Mathews et al., 1991), and A' and e' are the equatorial moment of inertia and dynamical ellipticity, respectively, of an homogeneous ellipsoid of the same dimensions as the inner core but having the same density ρ_f as the core fluid at the ICB.

As seen from Eq. (3), the determination of the real and imaginary parts of the FICN frequency σ_{FICN}^{\Re} and σ_{FICN}^{\Im} is perfectly equivalent to the determination of the real and imaginary parts of the ICB coupling constant, K_{ICB}^{\Re} and K_{ICB}^{\Im} , provided that we consider that the inner core is in hydrostatic equilibrium and thus knowing the hydrostatic dynamical flattening α_s . Knowing the real and imaginary parts of the ICB coupling constant, K_{ICB}^{\Re} and K_{ICB}^{\Im} , the imaginary part of the CMB coupling constant K_{CMB}^{\Im} can be determined from σ_{FCN}^{\Im} by using Eq. (4). Eq. (1) relates σ_{FCN}^{\Re} to a combination of the real part of the CMB coupling constant K_{CMB}^{\Re} and the core flattening α_f that can thus not be uniquely determined from σ_{FCN}^{\Re} . One way to solve the problem is to suppose a theoretical relation between the real and imaginary parts of the CMB coupling constant, K_{CMB}^{\Re} and K_{CMB}^{\Im} . This is equivalent to a perfect understanding of the coupling mechanisms at the CMB, which is far to be the case. A first step in the direction of a solution has been performed by Mathews et al. (2002) who supposed that the coupling at the CMB was mainly due to the reaction to stretching by the nutation of the CMB magnetic field. The electromagnetic coupling was further examined together with viscous coupling by Guo et al. (2004). The topographic coupling has been estimated by Wu and Wahr (1997) with a numerical approach and by Folgueira and Dehant (2007) with an analytical approach. Using these kinds of assumption, it is possible to estimate the departure from hydrostatic equilibrium flattening at the CMB.

Figure 2 represents the determination of the real and imaginary parts or equivalently the period and quality factor of the FICN. The arrows indicate the values proposed in MHB2000. The contributions from the different forced nutations and for the possible changes arising from the different strategies for the VLBI data analysis as explained in the previous section are shown in different colors.

We have then computed the contribution to the forced nutations from the different possibilities for the FICN period and quality factor. We have observed that the contribution at the 18.6 retrograde nutation, one of the most sensitive nutation to the FICN parameters as shown in Section 2, as computed from MHB2000 is very close to our best determination, i.e. -0.16 mas (milliarcsecond) on the real part of 18.6 year retrograde nutation and 0.21 mas on the imaginary part.

3. TIME DOMAIN APPROACH FOR A BETTER DETERMINATION OF THE BASIC GEOPHYSICAL PARAMETERS

The geophysical parameters presented here above have been fitted from the amplitudes of the forced nutations, which have been, in turn, determined from the VLBI data (the delays). This procedure involves three steps: (1) a determination of the nutation in longitude and in obliquity as a function of time from

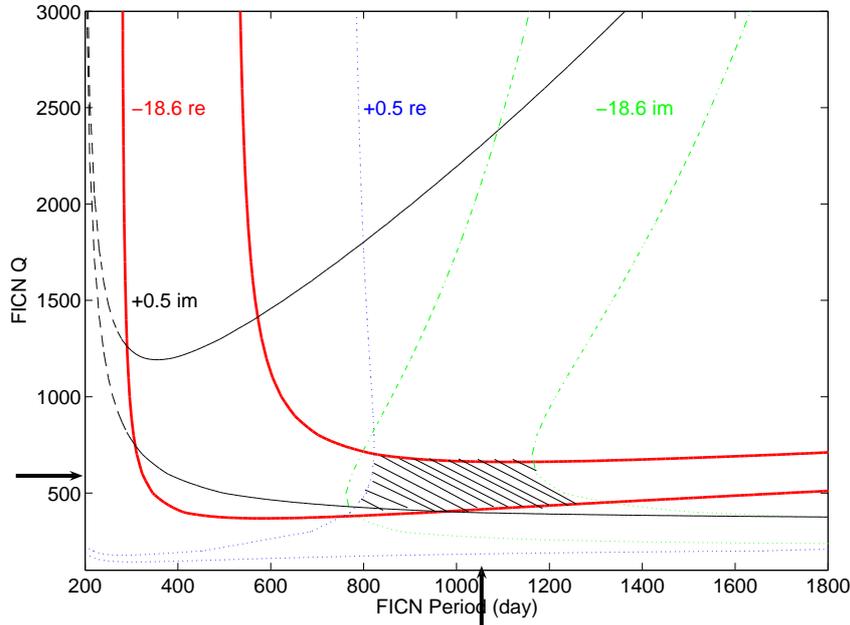


Figure 2: Period and quality factor of the FICN determined from the different forced nutations individually and for the possible uncertainties in the data arising from the different strategies for the VLBI analysis. The numbers on the curves indicate the nutation periods in years; “re” for their real part; “im” for their imaginary part. The hatched area indicates the intersection of the different possibilities for the parameters. The arrows indicate the values proposed in MHB2000.

the VLBI data, (2) a Fourier transform and an estimation of the in-phase and out-of-phase parts of the nutation amplitudes for the main forced nutations, and (3) an estimation of the basic Earth geophysical parameters. Laurence Koot has been working recently on a new approach, determining the basic Earth geophysical parameters directly from the nutation in longitude and in obliquity as a function of time. This new procedure avoids one step of the previous approach in three steps (called the frequency domain approach) as it fits the parameters directly on the nutation data in the time domain. This time domain approach allows accounting for the time variable error on the nutation data. This error on nutation is very much changing with time as the observations have become better and better. Another advantage of the new Koot procedure is that it allows to get probability distribution on the parameters (see Koot et al. 2007).

The frequency domain approach showed correlation between amplitudes of some of the nutations such as the 9.3 and 18.6 year nutations; the new approach developed by Koot et al (2007) is that it uses a Bayesian inversion method which allows dealing with the non-linear nutation model and to get probability distributions on the parameters. With the new Bayesian approach, most of the parameters have the same values as MHB2000 but with a smaller error. However, some parameters have a different estimated value: the global dynamical flattening and the imaginary parts of the coupling constants K_{CMB} and K_{ICB} leading to a different estimation for the quality factors of the FCN and FICN modes.

4. SURFACE FLUID LAYERS

The present-day limitation in the estimation of the geophysical parameters is the contamination from the ocean and atmosphere perturbing the nutation amplitudes. The surface fluid layers transfer their angular momentum to the solid Earth, therewith changing the Earth orientation and rotation. For nutation, the atmosphere and ocean angular momentum must be evaluated at a diurnal timescale in a terrestrial reference frame (corresponding to nutation in a celestial frame). This is not yet a very accurate determination and geophysical fluid corrections need further improvements: the tidal ocean, non-tidal ocean, and atmosphere angular momentum must be computed precisely in the retrograde diurnal

frequency band. The GGOS (Global Geodesy Observing System) project of the IAG aims at better understanding the mass transfers within the Earth with its fluid layers, which will therefore help to better achieve this objective in the future.

5. FREE CORE NUTATION

Last, but not least, the observed nutation series show a large contribution from the FCN free mode. This cannot be modeled yet as it would demand a very precise knowledge of the atmospheric excitation. The FCN free mode contribution to nutations needs to be estimated from epoch to epoch as done by Lambert for the IERS conventions.

6. PERSPECTIVES FOR NUTATIONS OF OTHER PLANETS

For a planet to have nutation, we must have an inclination from the symmetry axis or mean rotation axis with respect to the ecliptic, we must have a rapidly rotating planet, and we must have a flattening (which is a consequence of the rapid rotation except for dynamic contribution). The Earth is not the only planet to have nutation: Mars has nutation as well. Mars has an inclination of about 25 degree and is rotating at about 24h40m. Mars nutations are mainly due to the Sun as the little moons of Mars, Phobos and Deimos have dimensions of the order of 20km. The main nutation of Mars is thus the semi-annual prograde nutation arising from the Sun as for the Earth nutation forced by the Sun. The amplitudes are roughly at the same level of magnitude for both planets (about 500 mas). Annual for Mars refers to one Martian year, i.e. about 687 terrestrial mean solar days. The semi-annual nutation corresponds to about 30 meters peak to peak at the surface of the Earth and 15 meters peak to peak at the Martian surface. Although the core is thought to be liquid from space geodesy observation (Konopliv et al. 2006), it is only determined from tidal contributions to the orbital motion of a spacecraft and is at the limit of the detection. Geochemical considerations tend to corroborate this statement as well: Mars would be in the phase of having a liquid core without inner core; this is possible if Mars contains a little bit more light elements than Earth in the core alloy. It is nevertheless very desirable to obtain another observation of the core state of Mars. Nutations would provide with this observation as they are enhanced by the FCN if the core is liquid. This enhancement is at the level of one percent for the semi-annual prograde nutation that is far from the FCN resonance (at around 256 days in the retrograde part of the spectrum). The ter-annual retrograde nutation at 229 days is close to the FCN; this is particularly true when the core is large. Indeed in Dehant et al. (2000a) a sequence of Mars interior models have been considered, constrained by the most probable moments of inertia as recently observed. These models have a core radius between 1268 km and 1668 km, corresponding to a 200 km uncertainty interval around the 1468 km radius of model A of Sohl and Spohn (1997). Dehant et al. (2000a and 2000b) have found an FCN period in the range [238, 288] days. The larger the core is, the lower the FCN period is. The value of the tidal Love number provided by space geodesy indicates a large core and therefore it is believed that the resonance on the retrograde ter-annual nutation will be high.

Observing nutation could be performed using the observation of one or a few landers at the surface of Mars as a function of time, for a long period (at least one Martian year). This opportunity will be provided by the future AURORA/ExoMars mission of ESA. The use of an X-band transponder on the surface of Mars will allow us to measure the Doppler effects on the radio signal from the rotation and orientation variations of Mars. The transponder is called LaRa for Lander Radioscience. The Principal Investigator of LaRa are V. Dehant and W. Folkner. The transponder is built by OMP (Orban Microwave Product), a Belgian industry. The mission is expected to be launched in 2013.

7. CONCLUSIONS

In summary, we may consider presently that the rigid Earth nutations are well determined. The FCN frequency is quite well determined from VLBI observations, even if we consider different strategies for computing nutation series, or even considering atmospheric contamination of the amplitudes. However, the same conclusion may not be drawn for the FICN. The 18.6 year retrograde nutation is the key nutation for getting the right FICN parameters (the 0.5 year prograde nutation also helps). The 18.6 year retrograde nutation is though not very well determined and its amplitude changes from one strategy of observation to the other. Different strategies for VLBI observations provide indeed different amplitudes of the 18.6 year retrograde nutation with respect to MHB2000 at the level of 60-70 mas. The FICN

frequency and quality factor are thus not yet very well constrained and VLBI observations still need improvement. The time domain approach looks very promising for a better determination of the basic Earth parameters.

Another important point for the future of nutation is the existence of a FCN free mode. This free mode contribution cannot be computed from atmospheric excitation as this last is not well known. It needs then to be observed and determined.

The atmosphere effects on nutations may be large and their determination from the angular momentum needs to be improved. The effects are important as nutations need to be corrected for, prior to the estimation of the Earth interior parameters.

The perspectives for the planet Mars provided by the future ExoMars mission and in particular the radio science experiment LaRa are very promising for our understanding of the interior of Mars.

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