# HARMONIC MODELS OF TIDE-GENERATING POTENTIAL OF THE TERRESTRIAL PLANETS

S.M. KUDRYAVTSEV
Sternberg Astronomical Institute of Moscow State University
13, Universitetsky Pr., Moscow, 119992, RUSSIA
e-mail: ksm@sai.msu.ru

ABSTRACT. High-accurate harmonic development of the tide-generating potential (TGP) of Mercury, Venus and Mars is made. For that the planets TGP values were first calculated on the base of DE/LE-406 numerical ephemerides over a long period of time and then processed by a new spectral analysis method. A feature of the method is the development is directly made to Poisson series where both amplitudes and arguments of the series' terms are high-degree polynomials of time. The new harmonic development of Mars TGP is made over the time period 1900-2100 and includes 767 second-order Poisson series' terms of minimum amplitude equal to $10^{-7}$ m$^2$/s$^2$. Similar series composing the Mercury and Venus TGP models are built over the time period 1000-3000 and include 1,061 and 693 terms, respectively. A modification of the standard HW95 format for representation of the terrestrial planets TGP is proposed. The number of terms in the planetary TGP models transformed to the modified HW95 format is 650 for Mercury, 422 for Venus, and 480 for Mars.

## 1. INTRODUCTION

Study of the tide generating potential (TGP) of the terrestrial planets is an actual task now. Several space missions scheduled to our neighbor planets, in particular to Mercury and Mars (e.g. Messenger by NASA, BepiColombo, NetLander by ESA and others) will perform in-situ measurements of the planetary nutation and tidal variations of the planets' gravity field. These effects are the result of the planet's body response to the perturbing TGP. Comparison of this response observed at different wave frequencies with the TGP harmonic model helps one to study the planet's internal structure.

There are several developments of the TGP for the terrestrial planets. The latest are the harmonic expansion of Mercury TGP made by Van Hoolst and Jacobs (2003) and expansion of Mars TGP done by Roosbeck (1999) and Van Hoolst et al. (2003). All these developments used VSOP87 analytical theory of the major planets motion (Bretagnon and Francou, 1988) as the source of planetary coordinates, and made a number of analytical transformation of VSOP87 harmonic series to obtain the planets TGP expansion. However, the accuracy of VSOP87 analytical theory is lower than that of the modern numerical ephemerides of the major planets. In particular, the current IERS Conventions (McCarthy and Petit, 2004) recommend the DE/LE-405 planetary/lunar numerical ephemerides (Standish, 1998) for precision studies.

In this paper we present a new harmonic development of the TGP for three terrestrial planets: Mercury, Venus and Mars. [A development of the Earth TGP is done in (Kudryavtsev, 2004).] The latest long-term planetary/lunar numerical ephemerides DE/LE-406 (they are the extension of DE/LE-405 over 6,000 years) are used as the source of the Sun, Moon and major planets coordinates. The expansion of the planetary TGP is made with use of a new modification of the spectral analysis method.

The expansion procedure and obtained results on harmonic development of the three terrestrial planets TGP are described in the following sections.

## 2. FORMULATION OF THE PLANETARY TGP

The TGP generated by external perturbing bodies (the Sun, major planets, etc.) at an arbitrary point $P$ on the planet's surface is represented in our study as

$$V(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{n} V_{nm}(t) = \sum_{n=1}^{\infty}\left(\frac{r}{R_{pl}}\right)^{n}\sum_{m=0}^{n}\bar{P}_{nm}\left(\sin\varphi'\right)\left[C_{nm}(t)\cos m\theta(t) + S_{nm}(t)\sin m\theta(t)\right] \quad (1)$$

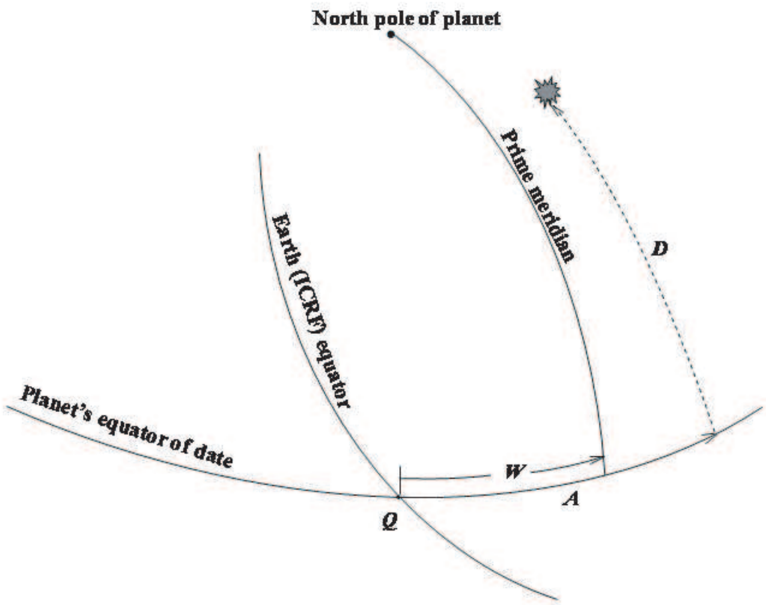


Figure 1: Spherical coordinates used when developing the terrestrial planets TGP

where $V(t)$ is the instantaneous value of the TGP at the point $P$ at epoch $t$, and
if $n = 1$

$$C_{10}(t) = \sqrt{\frac{15}{7}}\frac{\bar{J}_{2_{pl}}}{R_{pl}}\sum_j \mu_j \left(\frac{R_{pl}}{r_j(t)}\right)^4 \bar{P}_{30}\left(\sin D_j(t)\right), \tag{2}$$

$$C_{11}(t) = \sqrt{\frac{10}{7}}\frac{\bar{J}_{2_{pl}}}{R_{pl}}\sum_j \mu_j \left(\frac{R_{pl}}{r_j(t)}\right)^4 \bar{P}_{31}\left(\sin D_j(t)\right)\cos A_j(t), \tag{3}$$

$$S_{11}(t) = \sqrt{\frac{10}{7}}\frac{\bar{J}_{2_{pl}}}{R_{pl}}\sum_j \mu_j \left(\frac{R_{pl}}{r_j(t)}\right)^4 \bar{P}_{31}\left(\sin D_j(t)\right)\sin A_j(t) \tag{4}$$

if $n \geq 2$

$$C_{nm}(t) = \frac{1}{2n+1}\sum_j \frac{\mu_j}{R_{pl}}\left(\frac{R_{pl}}{r_j(t)}\right)^{n+1} \bar{P}_{nm}\left(\sin D_j(t)\right)\cos mA_j(t), \tag{5}$$

$$S_{nm}(t) = \frac{1}{2n+1}\sum_j \frac{\mu_j}{R_{pl}}\left(\frac{R_{pl}}{r_j(t)}\right)^{n+1} \bar{P}_{nm}\left(\sin D_j(t)\right)\sin mA_j(t); \tag{6}$$

$r$, $\varphi'$ and $\lambda$ are respectively the planetocentric distance, latitude and longitude of the point $P$; $R_{pl}$, $\bar{J}_{2_{pl}}$ are respectively the planet's mean equatorial radius and normalized dynamical form factor; $\mu_j$, $r_j(t)$, $A_j(t)$, $D_j(t)$ are respectively the gravitational parameter, planetocentric distance and angular spherical coordinates of $j^{th}$ perturbing body referred to the planet's equator of epoch $t$ with an origin point $Q$ - that being the node of the planet's equator of date on the standard Earth (ICRF) equator (Fig.1); $\bar{P}_{nm}$ are the normalized associated Legendre functions; the angle $\theta$ specifies the position of the local meridian of the point $P$ relative to the origin point $Q$ so that

$$\theta(t) = \lambda + W(t), \tag{7}$$

and $W(t)$ is the rotation angle of the planet's prime meridian reckoned from the node $Q$ as defined by Seidelmann et al. (2000).

The coefficients $C_{nm}(t)$, $S_{nm}(t)$ contain all the necessary information about instantaneous positions of the perturbing bodies at every epoch $t$ at which one calculates the TGP value $V(t)$.

Having harmonic expansions for $C_{nm}(t)$, $S_{nm}(t)$ done, one can further calculate time-dependent values of the TGP at an arbitrary point $P(r, \varphi', \lambda)$ on the planet's surface by using (1).

The tidal acceleration along the planet's radius (or 'the gravity tide') is defined as the radial derivative of the TGP

$$g(t) \equiv \frac{\partial V(t)}{\partial r} = \sum_{n=1}^{\infty} \frac{n}{r} \sum_{m=0}^{n} V_{nm}(t). \tag{8}$$

## 3. EXPANSION OF TGP FOR THE TERRESTRIAL PLANETS

We expand the coefficients $C_{nm}(t)$, $S_{nm}(t)$ of the planetary TGP model to finite second-order Poisson series of the following form:

$$C[S]_{nm}(t) = \sum_{k=1}^{N} \left[ \left( A_{k0}^c + A_{k1}^c t + A_{k2}^c t^2 \right) \cos \omega_k(t) + \left( A_{k0}^s + A_{k1}^s t + A_{k2}^s t^2 \right) \sin \omega_k(t) \right] \tag{9}$$

where $A_{k0}^c$, $A_{k1}^c, \cdots$, $A_{k2}^s$ are constants and arguments $\omega_k(t)$ are forth-degree polynomials of time

$$\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \nu_{k3} t^3 + \nu_{k4} t^4 \tag{10}$$

[$\nu_k, \nu_{k2}, \ldots$ are constants].

The expansion is made with use of a new modification of the spectral analysis method described in (Kudryavtsev, 2004). Prior to the expansion procedure we calculated numerical values of the coefficients $C_{nm}(t)$, $S_{nm}(t)$ according to (2)–(6) over a long interval of time. The latest long-term numerical ephemerides DE/LE-406 (Standish, 1998) are chosen as the source of the Sun, Moon and major planets coordinates. These ephemerides cover the time interval 3000BC–3000AD, and our expansion of the planetary TGP is centered at epoch J2000. So, in case of Mercury and Venus the time interval over which we sampled the planetary TGP values is chosen equal to 1000–3000, and the sampling step is one day. In case of Mars the time interval is 1900-2100, and the sampling step is 0.01 days only. It is necessary to set a small sampling step when calculating Mars TGP because of the fast motion of Martian moon Phobos (its orbital period is less than eight hours, and effect of Phobos attraction forms a large constituent in Mars TGP). A drawback of such a small sampling step is that the number of sampled TGP values (and as a consequence the computation time required by the spectral analysis method) dramatically increases, so we had to consider a smaller time interval for Mars than for the other two planets. MARTSAT analytical theory of Phobos and Deimos motion (Kudryavtsev et al., 1997) is chosen as the source of Martian moons coordinates. When calculating sample values for both Mercury and Venus TGP we took in account the attraction of the Sun, Moon, and all major planets (except Uranus and Neptune). For the case of Mars TGP the effect of both Phobos and Deimos attraction is additionally accounted. The radii and masses of the major planets, the Sun and Moon correspond to DE/LE-405/406 solution (Standish, 1998); the masses of Phobos and Deimos are taken from Konopliv et al. (2006).

The final expansion for the coefficients $C_{nm}(t)$, $S_{nm}(t)$ of Mars TGP includes 767 second-order Poisson series' terms of minimum amplitude equal to $10^{-7}$ m$^2$/s$^2$ (the maximum value for $n$ is equal to 8 here), and expansion series for the coefficients of Mercury and Venus TGP include respectively 1,061 and 693 analogous terms (the maximum value for $n$ is equal to 3 for the both planets TGP).

The values for the mean radius of every planet, $R_{pl}$, to be used in expression (1) along with the obtained series for coefficienties $C_{nm}(t)$, $S_{nm}(t)$ are as follows:

$R_{Mercury} = 2439.76$ km, $R_{Venus} = 6052.3$ km, $R_{Mars} = 3397.515$ km.

The expansion series for the coefficients $C_{nm}(t)$, $S_{nm}(t)$ are available on:

- for Mercury TGP: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mercury/Mercury_TGP_coefficients.dat (the description of the data format: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mercury/Readme.pdf);
- for Venus TGP: http://lnfm1.sai.msu.ru/neb/ksm/tgp_venus/Venus_TGP_coefficients.dat (the description of the data format: http://lnfm1.sai.msu.ru/neb/ksm/tgp_venus/Readme.pdf);
- for Mars TGP: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mars/Mars_TGP_coefficients.dat (the description of the data format: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mars/Readme.pdf).

We estimated the accuracy of calculation of the gravity tide at a mid-latitude 'station' (lander) on the planetary surface ($\varphi' = 50^\circ$, $\lambda = 0^\circ$) by means of the obtained TGP models. For every planet we found the maximum difference between the values of the gravity tide given by the relevant TGP harmonic development and 'exact' values of the tide obtained according to (1)–(8) with direct use of the perturbing bodies coordinates from DE/LE-406 numerical ephemerides and MARTSAT theory. The maximum difference between the two sets of values for the gravity tide on every planet is as follows:

- for Mercury: 0.8 nGal over the time interval 1000–3000;
- for Venus: 0.3 nGal over the time interval 1000–3000;
- for Mars: 1.1 nGal over the time interval 1900–2100.

## 4. REPRESENTATION OF THE PLANETARY TGP IN MODIFIED HW95 FORMAT

The coefficients $C_{nm}(t)$, $S_{nm}(t)$ are developed in a slowly precessing reference frame defined by the planet's mean equator of epoch with the node $Q$ as the origin point (Fig.1). To represent the planetary TGP models in a planet-fixed (rotating) reference frame we employed and slightly modified the HW95 format suggested by Hartmann and Wenzel (1995) for the Earth TGP model. The planets TGP series transformed to the modified HW95 format (and description of the modified format) are available on:

- for Mercury: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mercury/K07_Mercury.dat;
- for Venus: http://lnfm1.sai.msu.ru/neb/ksm/tgp_venus/K07_Venus.dat;
- for Mars: http://lnfm1.sai.msu.ru/neb/ksm/tgp_mars/K07_Mars.dat.

The number of terms in the planetary TGP harmonic models represented in the modified HW95 format is 650 for Mercury, 422 for Venus, and 480 for Mars.

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