ON ASTRONOMICAL CONSTANTS

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ABSTRACT. The set of astronomical constants conceptually fall into different categories; they are classified as natural constants, body constants or initial values. These categories are discussed in detail as is the problem of consistency and accuracy of such constants.

1. ASTRONOMICAL CONSTANTS FALL INTO DIFFERENT CATEGORIES

One of the central tasks of the IAU working group 'Numerical Standards in Fundamental Astronomy' (chair: Brian Luzum, USNO) is a selection of constants related with fundamental astronomy and to provide current best estimates of these constants together with their realistic errors. For that reason we thought it would be of general interest to discuss the relevance of these various constants that clearly fall into different categories that we classify as natural constants, body constants and initial values.

2. NATURAL CONSTANTS

Astronomical constants appear when the dynamics of an astronomical system is under discussion. From a fundamental point of view the dynamics of any physical system can be described by means of just a few fundamental physical interactions: gravity, electromagnetism, the weak or the strong force. These interactions are described by means of certain fields (the metric field g in the case of gravity; a vector potential A in the case of electromagnetism) that obey certain field equations: Einstein's equations in the case of General Relativity (GRT) or Maxwell's equations in the case of electromagnetism. These fundamental laws of nature contain certain natural constants describing certain properties of the interaction such as strength, propagation velocities etc. In GRT the Newtonian gravitational constant G plays a central role, in electromagnetism and Special Relativity (SRT) it is the vacuum speed of light c. In the microscopic world Planck's reduced constant $\hbar$ is of similar importance. As is well known the numerical values of three natural constants can be chosen arbitrarily (i.e., by law) thereby fixing the basic physical units. A well-known theoretical choice are geometrized units where $G = \hbar = c = 1$; the unit of time then is the Planck time $T_\mathrm{P} = (\hbar G/c^5)^{1/2} = 5.4 \times 10^{-44}\,\mathrm{s}$, the unit of length is the Planck length $L_\mathrm{P} = cT_\mathrm{P} = 1.6 \times 10^{-35}\,\mathrm{m}$ and the unit of mass is $(\hbar c/G)^{1/2} = 2.2 \times 10^{-8}\,\mathrm{kg}$. For practical purposes such units obviously are quite inconvenient.

Historically the basic physical units for time (the second), length (the meter) and mass (the kilogram) have been chosen by means of physical prototypes or properties of astronomical bodies. E.g., before 1956 the second was defined as the fraction of 1/86 400 of a mean solar day and then until 1967 as 1/31 556 925.8747 of the tropical year 1900. The meter in 1793 was defined as a fraction of 10^{-7} of the Earth's quadrant passing through Paris and in 1889 it was given by the international prototype of platinum-iridium rod kept at BIPM, Paris. Still the actual definition of the kilogram is through the platinum-iridium prototype of mass, also kept at BIPM. Such prototypes clearly have several disadvantages: precise copies have to be manufactured; they might change their properties due to interactions with the environment. There are indications that copies of the kilogram prototype became heavier in course of time; mass differences of up to 50 μg have been reported. Possibly the prototype has lost mass because of cleaning procedures.

For these reasons one tries to define the basic units through natural constants. This has been done for the second and the meter; soon this goal will also be realized for the kilogram, e.g., by counting the number of atoms of a macroscopic silicon-sphere (the Avogadro method; see e.g., Becker et al., 2001).

2.1. Defined and measurable natural constants

Today, the SI second is defined as: 'duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom'. The SI meter is defined as 'length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second. Note that these two definitions fix the vacuum speed of light once and forever; it has become a *defined* natural constant. In contrast to this the value for G is not defined; it is a *measurable natural constant*.

Clearly multiples of the SI second, SI meter or SI kilogram can be introduced for convenience without any problems. E.g., the Astronomical Unit might be defined by a fixed value in terms of the SI meter rather than by relation to a certain ephemeris (see, e.g., Klioner, 2007 for a discussion).

2.2. Problems related with natural units

The definition of a natural unit related with a defined natural constant is related with a corresponding fundamental law of nature (e.g., Special Relativity). In case a violation of that law will be detected in the future the definition might have to be abandoned because of serious problems. Consider, e.g., the definition of the meter and the isotropy of space. Using an old definition of the meter the famous experiment by Michelson and Morley from 1887 showed that the speed of light is independent of the propagation direction in space (isotropy of space). This fundamental experiment has been repeated over and over again. In recent time Stanwix et al. (2006) performed a Michelson-Morley experiment that uses two orthogonally orientated cryogenic sapphire resonator oscillators rotating in the laboratory. The result is that a possible upper limit for $\Delta c/c$ is of order 1.2×10^{-16} (see also Müller et al., 2007). Suppose that one day a violation of the isotropy of space will be found. In that case with the present definition of the meter the length of a meter stick would depend upon its orientation in space, really an unpleasant situation that would require a change in the definition of meter.

3. BODY CONSTANTS AND FRAMEWORK

The two astronomical constants G and c are related with GRT and Maxwell's theory of electromagnetism. In the latter case the properties of astronomically interesting light-rays follow from Maxwell's theory in the case of geometrical optics. On the other hand exact GRT is too complex to treat solar-system problems. Not even the constants describing certain aspects of astronomical bodies could be defined. These constants will be called 'body constants'; examples for such body constants are the mass of a body, its potential coefficients (mass multipole moments), its intrinsic angular momentum (spin) etc. Consider, e.g., the mass of an astronomical body which in Newton's theory of gravity is well defined and given as integral over the density. In Relativity because of the mass-energy equivalence ($E = mc^2$) all kinds of energy contribute to the inertial or gravitational mass even the gravitational field itself. Now, GRT is a non-linear theory and in principle one is not able to separate the gravitational field of a body A from that of another body B. For that reason one resorts to an approximation to GRT or a whole class of metric theories of gravity if one allows for a violation of Einstein's theory of gravity. Such an approximation will be called a 'framework'. For solar system applications usually one employs the (first) post-Newtonian framework (a weak field, slow motion approximation to GRT) or the parametrized post-Newtonian framework (PPN) with a suitable choice of coordinates (e.g., harmonic coordinates). This framework might contain additional constants such as the PPN-parameters β, γ, α_1 etc. whose numerical values are related with tests of GRT (deviations from $\beta = \gamma = 1$ and $\alpha_1 = 0$ indicate a violation of GRT at the post-Newtonian level). As is well known (e.g., Damour et al., 1991) body constants can be defined in the basic (post-Newtonian) framework. For $\beta = \gamma = 1$ and $\alpha_1 = 0$ the mass of a body E (Earth) can be defined in the local co-moving system (GCRS) with coordinates $(T, \mathbf{X})$ as (e.g., Damour et al., 1991)

$$M_E(T) = \int_E d^3\Sigma + \frac{1}{6c^2}\frac{d^2}{dT^2}\left(\int_E d^3X\mathbf{X}^2\Sigma\right) - \frac{4}{3c^2}\frac{d}{dT}\left(\int_E d^3XX^a\Sigma^a\right),$$

where Σ and Σ^a are the gravitational (energy-) mass density and mass current in the GCRS. Though this theoretical post-Newtonian expression for the mass of the Earth looks quite complicated it appears as parameter in the gravitational potential W_E outside the Earth in the simple, quasi-Newtonian form

$$W_E = \frac{GM_E}{R} + \dots .$$

Body constants can be considered as "constants" only within some accuracy limits. In general body constants will be time dependent. This time dependence together with their realistic errors has to be indicated explicitly, especially when our accuracy is close to that where the corresponding constant becomes time-dependent. For example, major sources for such a time dependence for mass variations in the solar system are dust accretion or energy loss and solar wind. For our Sun with luminosity $L = 4 \times 10^{33}$ erg/s the fractional mass variation is of order $\dot{M}/M \sim 10^{-13}$ per year.

4. INITIAL VALUES AND MODEL

Bodies with their body constants plus initial conditions appear in a dynamical model, e.g., for the motion of the gravitational N-body problem (ephemeris equations). Such a model might involve a variety of interactions (not only gravitational) and might contain additional constants describing certain features of them (e.g., a lag angle to describe the tidal friction in the Earth-Moon system). Examples are the basic equations for the DE, INPOP or EPM ephemerides and all the constants required in those equations. Note, that if we start with a certain model and add another interaction (e.g., we consider potential coefficients of higher order for a certain body of the model) we basically face *another model*. Initial values are intimately related with the underlying model. If the values of certain initial values are discussed the full underlying model has to be specified in some way or another. Note that in the relativistic framework this model includes not only physical ideas and assumptions, but also a series of pure conventions concerning the choice of coordinates.

In contrast to this one expects the body constants to have some well defined (time-dependent) values within a certain framework. Clearly given a certain data set different models will imply different values for them with certain errors. However, if realistic errors are given these values should be compatible with each other.

5. THE PROBLEM OF CONSISTENCY AND ACCURACY

To ensure consistency of several models realistic errors should be given. To this end correlations have to be studied, possibly different models or even different branches of science have to be consulted etc. This implies that chasing after the current best estimate of a constant is highly problematic as long as a realistic estimate of the error is not given. There are well known examples in the literature, e.g., related with a determination of $\dot{G}/G$, where five times the formal error is still an order of magnitude smaller than a realistic error (for $\dot{G}/G$ presently a realistic error is of order 6×10^{-13}/yr).

6. SUMMARY AND CONCLUSIONS

A new classification of astronomical constants into natural constants, body constants and initial values is suggested. The set of natural constants is divided into defined and measurable natural constants. They appear in fundamental laws of nature. Body constants can be defined in a certain framework such as the (first) post-Newtonian approximation to Einstein's theory of gravity. If the framework is fixed the body constants have a well-defined meaning and certain numerical values independent of the model used to interpret astronomical data. On the other hand initial values depend upon the subtleties of the concrete model. This implies that together with the numerical values all the details of the corresponding model should be indicated in some way or another. Finally, hunting for current best estimates is a necessary enterprize but meaningful only if realistic errors are given.

7. REFERENCES

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