THOUGHTS ABOUT ASTRONOMICAL REFERENCE SYSTEMS AND FRAMES

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ABSTRACT. The various theoretical and practical structures necessary for a definition of the astronomical reference systems (BCRS, GCRS, ICRS) and frames (ICRF) are discussed. It is argued that with increasing accuracy the distiction between astronomical reference systems and corresponding frames becomes increasingly problematic.

For the description of precise astronomical observations various astronomical reference systems have been introduced: the BCRS, GCRS, ICRS and ITRS. At least for the ICRS and the ITRS corresponding frames, the ICRF and ITRF have been realized. This article tries to sketch the various constructive elements related with the definition of these systems and frames. Naively speaking a coordinate system is defined by a set of formal rules about mathematical structures and how to interact with the universe to produce a corresponding frame. A frame is thought to be the practical realization of the corresponding system by means of observations and by attributing coordinates to certain material elements. In Table 1 I have tried to keep this distinction between systems and frames. In the left column I have listed theoretical concepts whereas observations, experiments or tests appear in the right column.

It is obvious that due to the high precision needed, e.g., for future astrometric measurements or spacecraft navigation the definitions of these systems and frames is necessarily very complex. Here I will start from simple elementary concepts and then will work my way to higher and higher complexity. I have devided this complexification into various levels simply for didactical purposes.

A reference system is a coordinate system or a chart in a manifold $\mathcal{M}$ giving n numbers to a set of points in an n-dimensional manifold endowed with some abstract metric tensor. Hence we start with a purely mathematical construction (Level 1).

In the next Level we relate the manifold picture with time and space of our universe, i.e., we consider 4-dimensional space manifold and a metric tensor with (non degenerate) metric tensor g obeying Einstein's field equations. To have a natural relation with nature physical time and space should have a variety of properties and the gravitational interaction should be described by Einstein's theory of gravity. This already is related with a huge complex of experiences with nature based upon the usual scientific interactions of observers with the universe including simplifications of various kinds. E.g., the 3 dimensionality of physical space can be experienced by simple observations; the local Euclidean topology certainly is an useful idealization since the treatment of differential equations is simpler than that of difference equations. In any case for distances smaller then about 10^{-33} cm (the Plack length) one expects the classical manifold picture to break down and a quantum mechanical picture will become necessary. Tests of Special Relativity and Einstein's theory of gravity (General Relativity Theory, GRT) present a huge subject for itself and will not be discussed further here (see e.g., Will 1993).

On the next Level we will come to the definition of the BCRS and the GCRS. First we will use a certain approximation to Einstein's theory of gravity, the first post-Newtonian approximation. On the experimental side we face the various tests of metric theories related with the first post-Newtonian approximation. Here the so-called paramatrized post-Newtonian framework where a set of formal PPN-parameters is introduced is of great value since measurements of them provide not only tests of GRT (where the most important parameters β and γ both take the value 1) but also serve is indicator for the measurement accuracy.

One may wonder why the reduction of Einstein's theory of gravity to its first post-Newtonian approximation is so important. The answer lies in the complexity of GRT that would not even allow for a reasonable definition of the mass of a body. For present accuracies the first post-Newtonian approximation is sufficient for the definition of astronomical reference system. Actually the first post-Newtonian framework is much simpler than the full GRT and the equations for the gravitational potentials are not more complex than Maxwell's equations of electromagnetism. In this approximation relativistic masses and higher multipole-moments (potential coefficients) of the various bodies in the gravitational N-body problem can be defined (Damour et al., 1991). This is by non means trivial; e.g., the post-Newtonian center of mass of some matter distribution like the solar system is based upon the vanishing of the corresponding mass dipole-moment $M_i = \int d^3x\, x^i\sigma + (1/10c^2)(d^2/dt^2)\int d^3x\, x^i\mathbf{x}^2\sigma - (12/10c^2)(d/dt)\int d^3x\, \hat{x}_{ij}\sigma^j$ where $\hat{x}_{ij} = x^ix^j - (1/3)\mathbf{x}^2\delta_{ij}$ and the gravitational mass density σ and mass-current density σ^i are determined from the components of the energy-momentum tensor by $\sigma = (T^{00} + T^{ss})/c^2$ and $\sigma^i = T^{0i}/c$.

In a next step we consider an idealization of our solar system as an isolated N-body problem. Locally that means that we consider only N bodies, the Sun, Moon, planets, certain asteroids etc., of constant post-Newtonian mass subject to their mutual gravitational action and nothing else. Nongravitational forces, mass losses etc. are neglected. Similarly we neglect all matter outside the solar system such as neighbouring stars other matter in our Milky Way or other galaxies. In addition to that we assume to be asymptotically flat, i.e., the metric potentials w and w^i to vanish asymptotically for $|\mathbf{x}| \to \infty$ and $t = \text{const.}$ ('spacelike infinity').

Next we choose special conditions for our coordinates by assuming a special form of the metric tensor. If we denote the flat space Minkowski metric tensor in inertial Cartesian coordinates by η ($\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$) we require *local* conditions: at the origin of our Barycentric Celestial Reference System we require that $g = \eta$ in the limit $T^{\mu\nu} = 0$, that is for vanishing masses in our N-body problem. Next we require an *asymptotic* condition that $g \to \eta$ if we approach spacelike infinity. Finally we relate the local with the asymptotic conditions by choosing the harmonic gauge for the BCRS metric tensor.

This finally defines the BCRS by the corresponding choice of the metric tensor that fixes the space-time coordinates up to certain symmetry transformations. Especially the orientation of spatial coordinates is not fixed and it can be done in many different ways, e.g., by observations of solar-system bodies or remote astronomical objects such as quasars.

A special coordinate transformation from barycentric coordinates (t, x^i) to suitably chosen geocentric coordinates (T, X^a) then defines the GCRS. Here one requires local conditions for the GCRS metric tensor: if we neglect the matter from the Earth itself then $G \to \eta$ at the geocenter. The gravitational action of other bodies appears only in form of tidal terms in the GCRS metric tensor. Moreover, we require the GCRS coordinates to be harmonic. Since the geocenter is accelerated one can show that such geocentric coordinates loose their meaning far from the geocenter (typically at distances of order c^2/a, where a is the acceleration of the geocenter; see, e.g., Misner et al., 1972). This implies that without the BCRS the GCRS cannot be defined. The orientation of spatial GCRS coordinates is fixed by choosing them to be kinematically non-rotating with respect to the spatial BCRS coordinates.

These constructions can be introduced theoretically in the frame of a post-Newtonian formalism. However, we want the BCRS to be related with the astronomical bodies of our solar system. This relation requires solar-system data with hight precision. Theoretically the dynamical equations of motion for a system of mass monopoles ('point masses') are the well known relativistic Einstein-Infeld-Hoffmann equations of motion that form the basis of the JPL DE-ephemerides. The theoretical framework features several parameters such as masses and initial conditions for positions and velocities that have to be fitted to observational data. Here a whole complex of practical problems how to deal with observational solar system data comes into play. For each specific kind of observations or measurements, e.g., optical observations on CCD-frames, radar ranging to spacecrafts or planets, LLR etc. a whole set of rules, recipies or models exist that tells the observer how to proceed.

In the next Level we want to approach the ICRS and the ICRF. Naively we might consider the ICRS as a special version of the BCRS with spatial orientations fixed by VLBI observations of quasars. This standpoint ignores the actual large scale structure of our universe including its global expansion. Since the redshifts observed in spectral lines of quasars usually are significant this cosmic expansion should not be neglected and the BCRS should be modified to account for that. Present work in that direction is described e.g., in Klioner and Soffel, 2004.

Obviously there is a basic concept related with the idea for the ICRS. The ICRS should represent some sort of cosmic global quasi-inertial coordinate system with respect to rotational motion defined by means of observations of very remote cosmic objects showing almost no proper motions. This is the vague concept behind the ICRS and the real problem is if or how it can be realized in our actual universe. In GRT a coordinate system is determined by the choice of the metric tensor that itself is related with the cosmic distribution of energy and momentum in the universe by Einstein's field equations. The ICRS concept in that manner is related with *cosmic assumptions* on the distribution of matter on very large distance scales and the corresponding world model. In our approach to the BCRS we neglected all cosmic matter outside the solar system and the field equations then imply that a corresponding 'world model' is asymptotically flat. In addition we could then assume that the distribution of quasars is such that apart from small random proper motions they are at rest with respect to our asymptotically Minkowskian coordinate system.

In our real world we might proceed with the Cosmological Principle saying that on very large scales of several billion lightyears the universe is homogeneous and isotropic, a picture that is supported by the latest data on the Cosmic Microwave Background Radiation (CMBR). In such a world-model asymptotically the metric would reduce to the Robertson-Walker metric and we might assume the set of quasars to be approximately at rest in suitably chosen Robertson-Walker coordinates.

Clearly any reasonable world-model should be supported by cosmological observations, deep redshift surveys, studies of the CMBR, etc. As solar system observations they present an art for itself related with expert knowledge and know-how.

After having chosen a suitable world-model we idealize again by neglecting e.g., the gravitational action of certain galaxies (i.e., certain gravitational lensing effects) or gravity waves. On the observational side we now study the properties of quasars in detail by means of VLBI observations and corresponding software such as CALC-SOLVE. We study the structures of quasars, variablities, identify fiducial points for coordinization etc. At this level of accuracy all the details of the software enter. How plate tectonics, the topospheric delay, loading effects etc. are modelled might influence the fitting of parameters related with the reference system.

Formally we might then require additional conditions for the spherical angles (α, δ) to fix the origin of coordinates and to ensure historical continuity. We then end up with the ICRS and VLBI observations of certain structural elements of quasars finally yield the ICRF in form of a quasar catalogue.

Considering these various aspects in the construction of astronomical reference systems and frames I would like to point out the following.

1 With increasing accuracy the precise definition of a reference system requires more and more observations. The distinction between a system and its frame becomes increasingly problematic. People frequently have asked: what is the ICRS? The answer might be related to very different possible standpoints between two extremes. Someone preferring the idea of a system to be defined by formal rules might argue that the ICRS is given by the BCRS plus cosmic assumptions. For most astronomers, however, that definition would not be broad enough and fail to characterize what commonly is thought to be the ICRS.

   For someone else the ICRS is defined by the complete set of rules (mathematical and others) for its construction including the treatment of atmospheric delays or the solar-system ephemeris. This other extreme standpoint implies that we devide the set of all observations, experiments and tests related with ICRS and described above into two parts: 1. into those that are related with the definition, that e.g., is based upon Einstein's theory of gravity, and 2. those very dedicated observations necessary for the realization of the corresponding frame.

   For all of that reasons one might suggest to speak about astronomical reference systems only. For a mechanical structure servoing for spatial reference such as a telescope mounting or a wall inside a spacecraft used for the orientation of spatial coordinates the word frame ic clearly appropriate.

2 The ICRS and the BCRS appear at different levels of abstraction. In principle the orientation of spatial axes of the BCRS could be fixed by different techniques. Presently it is determined by the ICRS and as long as this is clear the nomenclature must not necessarily point this out explicitly. However, in case several techniques compete in that respect the nomenclature should account for that and one should write e.g., $BCRS_{[\mathrm{QSO}]}$, $BCRS_{[\mathrm{dyn}]}$ etc.

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Theoretical concepts	**Observations, experiments, tests**
coordinates in manifolds with metric tensor g	
Level 1	
space-time manifolds (dim = 4, sign(g))	general properties of space and time
g satisfies Einstein's GRT	tests of GRT
Level 2	
post-Newtonian framework	specific tests
definitions, e.g., for centers of mass (barycenter)	
idealization, e.g., solar-system as isolated N-body problem	solar-system observations, tests of idealizations (e.g., external tidal forces)
special coordinate conditions	
BCRS [$(x^\mu = (ct, \mathbf{x}))$; g]	
orientation of spatial coordinates not fixed; can principally be done in many ways	
special coordinate transformation leads to **GCRS**, only a local system	solar-system ephemerides
Level 3	
ICRS concept	cosmological observations tests of Cosmol.Principle and world models
cosmic idealizations	detailed QSO observations identify fiducial marks
auxiliary conditions for (α, δ)	observe (relative) positions
ICRS	**ICRF**
Level 4	

Table 1: Various steps necessary for the definition of the BCRS, GCRS, ICRS and ICRF