RELATIVISTIC MODELING OF THE ORBIT OF GEODETIC SATELLITES EQUIPPED WITH ACCELEROMETERS

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1. THE CLASSICAL APPROACH: *GINS*

In today's planetary orbitography softwares, as in GINS (Géodésie par Intégrations Numériques Simultanées, developed by CNES [1] and GRGS [2]), the motion of spacecrafts is still described according to the classical Newtonian equations plus the so-called "relativistic corrections", computed with the required precision using the Post-(Post-) Newtonian formalism. Hence, it is the 3-vector acceleration ($i = 1, 2, 3$),

$$\frac{d^2X^i}{dT^2} = \frac{\partial W}{\partial X^i} + \textit{non-gravitational accelerations} + \textit{general relativistic corrections,} \tag{1}$$

which is numerically integrated with respect to coordinate time T. The gravitational potential W includes not only the central planetary potential model but also the Earth-tide (due to the Sun and Moon, corrected for Love number frequencies, ellipticity and polar tides) potential, ocean-tide potential and Newtonian-perturbation potentials from other solar system bodies. The atmospheric drag, the radiation pressure (solar radiation, Earth albedo, thermal emission) are the non-gravitational perturbations considered. The orbitography software GINS also includes, as relativisitic corrections, the Schwarzschild, geodesic and Lense-Thirring precessions [1].

2. THE (SEMI-CLASSICAL) RELATIVISTIC APPROACH: *(SC)RMI*

The classical Newton plus relativistic corrections method faces three major problems. First of all, it ignores that in General Relativity time and space are intimately related. Secondly, a (complete) review of all the corrections is needed in case of a change in conventions (metric adopted), or if precision is gained in measurements. Today with the increase of tracking precision (32 GHz Ka/Ka-Band Doppler radio tracking at at the level of 1 mm/s with respect to a relative motion Earth/spacecraft of 10 km/s, i.e. with a relative accuracy of 10^{-13}), active interplanetary laser tracking (at the level of 10 cm with respect to a distance of 10^8 km, i.e. with a relative accuracy of 10^{-9}) and clock stabilities (Allan deviation of $\sim$4 $10^{-14}\tau^{-1/2}$ for atomic fountains), the classical method is reaching its limits in terms of complexity. The penalty for not taking relativistic effects into account is the risk of polluting very weak geophysical effects, like the polar motion of Mars ($\sim$ 1 m in amplitude at the planet surface), or the signature on the nutations of the liquid core of Mars ($\sim$ a few cm over an amplitude of $\sim$10 m), by unwanted relativistic effects that are at the same period (typically one planetary year, or 687 days for Mars), and, worse, that can be cumulative (up to or larger than 10 m ranging error coming from relativity

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over one Mars orbit, ~150 minutes). Thirdly, with such a classical method, one correction can sometimes be counted twice (for example, the reference frequency provided by the GPS satellites is already corrected for the main relativistic effect), if not forgotten. For those reasons, a new approach, called (SC)RMI ((Semi-Classical) Relativistic Motion Integrator) [2], was suggested.

The relativistic equation of motion, when non-gravitational accelerations encoded in a 4-vector K_β [5] are present, is

$$\frac{dU^\alpha}{d\tau} = -\Gamma^\alpha_{\beta\gamma} U^\beta U^\gamma + K_\beta \left(G^{\alpha\beta} - \frac{U^\alpha}{c^2} \frac{U^\beta}{c^2} \right) \quad \text{with } U^\alpha \equiv \frac{dX^\alpha}{d\tau}, \quad U^\alpha U_\alpha = c^2 \tag{2}$$

where $X^{\alpha=0,1,2,3} \equiv \left(c \cdot T,\ X^i\right)$ are the space-time coordinates; $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols, functions of the derivatives of the space-time metric $G^{\alpha\beta}$; and τ is the proper time. In the relativistic approach, it is those 4-dimensional equations (i.e. $\frac{d^2X^\alpha}{d\tau^2}$) which are directly numerically integrated. For the appropriate metric at the required order, they contain all the gravitational effects at the corresponding order. Indeed, computing the above equations for the Geocentric Coordinate Reference System (GCRS) metric [3,4] will take into account gravitational multipole moment contributions from the central planetary gravitational potential, perturbations due to solar system bodies, the Schwarzschild, geodesic and Lense-Thirring precessions. Non-gravitational forces can be treated as perturbations, in the sense that they do not modify the local structure of space-time (the metric). Moreover, K_β being small, one can safely replace $G^{\alpha\beta}$ by its Minkowskian counterpart in the second term of the right-hand-side of equation (2), hence the terminology "Semi-Classical" in SCRMI. When $K_\beta = 0$, equation (2) reduces to the geodesic equation of the local space-time.

3. THE PRINCIPLE OF ACCELEROMETERS

Last we show how to update the classical equation for accelerometers, in other words, how to measure K_β, or consider introducing a non-gravitational force model in the relativistic framework. Let the satellite center of mass (CM) be located at X^μ; while a test-mass is at $X^\mu + \delta X^\mu$, in a cavity inside the satellite, hence shielded from non-gravitational forces. The test-mass motion is described by geodesic equations ((2) with K_β=0) while that of the satellite is described by (2). Evaluating the difference between those two equations at first order in δX^μ gives a general relativistic equation for accelerometers:

$$\frac{d^2\delta X^\alpha}{d\tau^2} = K^{(CM)}_\beta \left(G^{\alpha\beta} - \frac{dX^\alpha}{d\tau}\frac{dX^\beta}{d\tau} \right) - \frac{\partial \Gamma^\alpha_{\beta\gamma}}{\partial X^\mu} \delta X^\mu \frac{dX^\beta}{d\tau}\frac{dX^\gamma}{d\tau} - 2\Gamma^\alpha_{\beta\gamma}\frac{dX^\beta}{d\tau}\frac{d\delta X^\gamma}{d\tau} \tag{3}$$

Equation (3) reduces to geodesic deviation if $K^{(CM)}_\beta$=0.

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