SPECTRAL ANALYSIS OF THE NUMERICAL THEORY OF THE RIGID EARTH ROTATION

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ABSTRACT. The discrepancies of the comparison between the high-precision numerical solutions of the rigid Earth rotation problem and the semi-analytical solutions SMART97 (Bretagnon *et al.*, 1998), (Brumberg and Bretagnon, 2000) are processed by means of the spectral analysis methods for the kinematical and dynamical cases over the time interval of 2000 years. The secular trends in the discrepancies are processed by the least squares method and the new temporal polynomials of the 6-th degree for the precessional parameters and for GMST are presented. The power spectra of the periodic parts of the discrepancies are constructed by using the arguments of the nutational harmonics of SMART97 solutions. Starting from the maximum term of the spectra, the coefficients of the harmonics are determined successively. The set of these harmonics and those of SMART97 solutions represent the new nutational series consisting of more than 9000 periodic terms. The comparison between the quadruple precision numerical solutions of the problem and the new rigid Earth rotation series reveals that the discrepancies do not surpass 10μas over 2000 years in Euler angels.

1. INTRODUCTION

The previous papers (Eroshkin *et al.*, 2003, 2004) were devoted to the problem of the construction of the high-precision long-term numerical theory of the rigid Earth rotation dynamically adequate to the ephemerides DE404/LE404. The high-precision semi-analytical solutions of the rigid Earth rotation problem SMART97 were used for calculating the initial conditions, and also for the control and the refinement of the algorithm of the numerical solution. The comparison between the numerical and semi-analytical solutions over 2000 year time span revealed the discrepancies of both secular and periodic characters. The main part of these discrepancies was explained by not sufficiently long time span of the validity of SMART97 solutions: only for several centuries. Besides, it was discovered in the result of the numerical experiments that the linear part of the secular trend in the discrepancies in the angle of the proper rotation was related to an insufficient accuracy of the SMART97 solutions for calculating the initial conditions (Eroshkin *et al.*, 2003, 2004). For the further development of the high-precision numerical theory of the rigid Earth rotation it is very useful to construct alternative semi-analytical solutions of the problem, which are more adequate to the ephemerides DE404/LE404 than SMART97 solutions.

2. ALGORITHM

The problem is solved both for the Newtonian case (Dynamical case) and for the relativistic one (Kinematical case) in which the geodetic perturbations representing the most essential relativistic perturbations in the Earth rotation are taken into account. The numerical solutions of the rigid Earth rotation problem are constructed in the result of the numerical integration with the quadruple precision. The initial conditions are calculated from SMART97 solutions. The numerical solutions are compared with SMART97 solutions over 2000 year time span with one day spacing and the arrays of the discrepancies are constructed.

In (Eroshkin *et al.*, 2003, 2004) only the secular terms and the main periodic ones were determined in the discrepancies of the comparison. For the systematic determination of all periodic terms the spectral analysis methods have to be used.

The investigation of the discrepancies is carried out by the least squares (LSQ) method and by the spectral analysis (SA) methods:

1. The secular parts of the discrepancies are processed by the LSQ method and then removed from the discrepancies. The new precessional and GMST polynomials are constructed by summing the determined secular terms and the precessional and GMST polynomials of SMART97.

2. For the periodic parts of the discrepancies the power spectra are constructed by using the arguments of the nutational harmonics of SMART97 solutions. The power spectra are bounded by the periods from 1.0003 days till 1000 years. The determination of the coefficients of the harmonics is accomplished successively starting from the maximum term in the power spectra (Roberts *et al.*, 1987) by the SA methods. The new nutation series are constructed in the result of summing the determined periodic terms and the corresponding harmonics of SMART97. This procedure is accomplished in the following manner.

a) The amplitude spectra of the discrepancies are computed by the LSQ method for each nutational harmonic of SMART97.

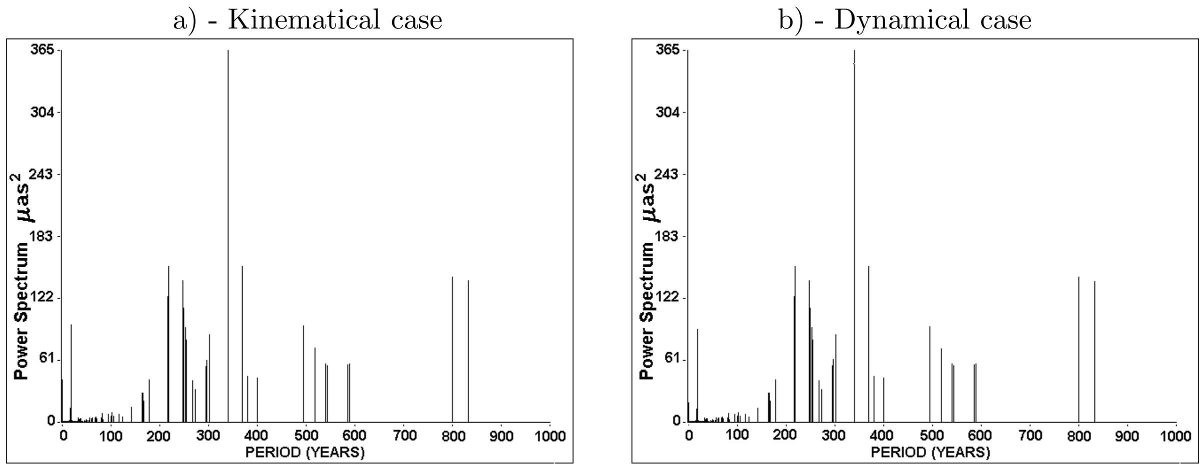

Figure 1: Complete spectra of the periodic discrepancies of the comparison between the numerical solutions and SMART97 solutions in the proper rotation angle

b) The amplitude of the largest harmonic is determined by the LSQ method.

c) If the absolute value of the amplitude of a harmonic exceeds the absolute value of its mean square error then this harmonic is removed from the discrepancies. The new nutational term is determined as a sum of the calculated periodic term and the corresponding nutational term of SMART97.

d) The steps b) and c) are performed up to the end of the spectra.

The spectra for all Euler angles are similar. The spectra for the Kinematical and for the Dynamical cases are similar for the long-period parts but have some difference for the short-period parts (Figures 2a, 2b).

Figures 1–2 demonstrate the main groups of the harmonics with the periods close to 1 day, 1 year, 20 years and so on. The maximum amplitude of the periodic terms has a harmonic with the period 341.3724 years. There are altogether around 9000 periodic terms which are determined. The new precession, GMST parameters and the nutational terms form the new high-precision rigid Earth rotation series S9000.

a) - Kinematical case b) - Dynamical case

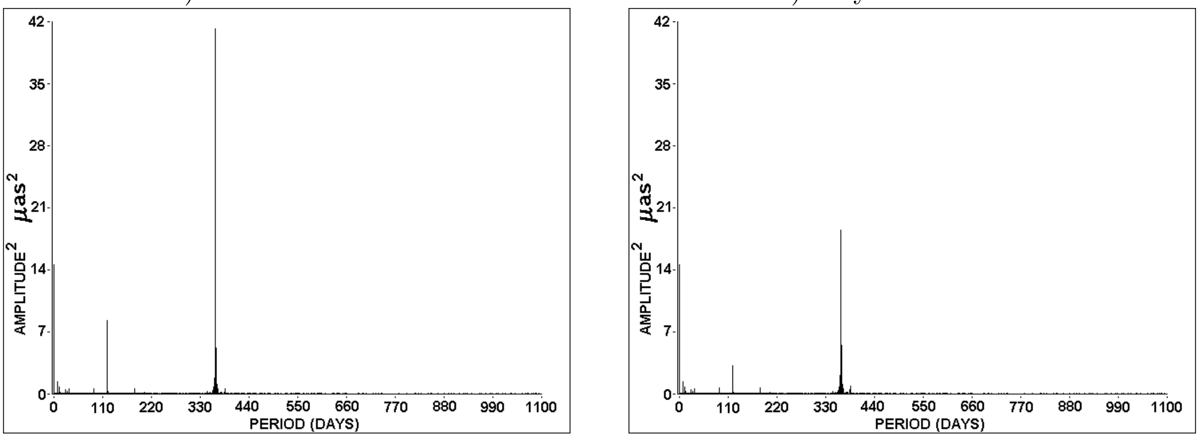

Figure 2: Spectra of the short-period part of the discrepancies of the comparison between the numerical solutions and SMART97 in the proper rotation angle

3. NEW HIGH-PRECISION RIGID EARTH ROTATION SERIES — S9000

The constructed new high-precision rigid Earth rotation series S9000 are compared with the semi-analytical solution SMART97 over 2000 year time interval from AD 1000 to 3000.

a) - Kinematical case b) - Dynamical case

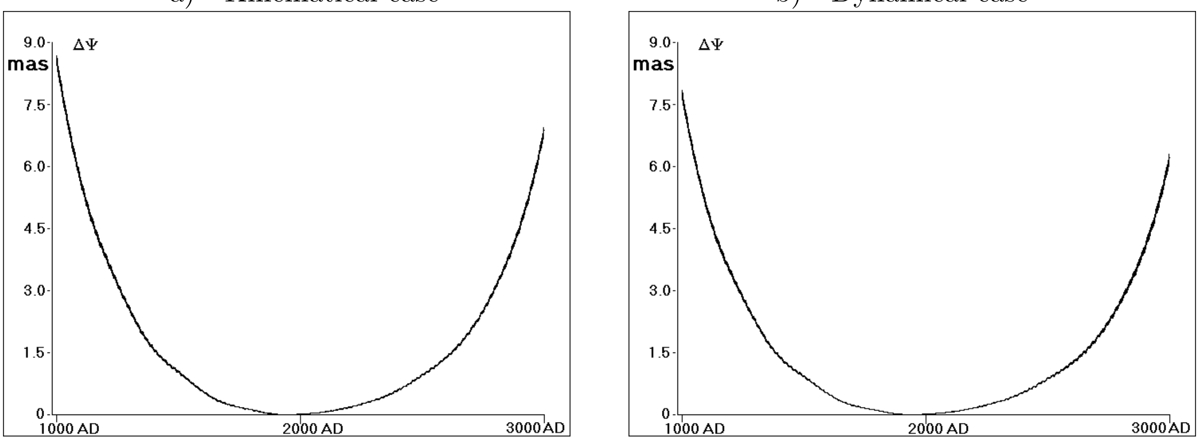

Figure 3: S9000 series minus solutions SMART97 in the angle ψ

In Figures 3—5 the comparison between SMART97 and S9000 is presented in Euler angles: the longitude of the ascending node ψ (Figures 3a, b), the proper rotation angle ϕ (Figures 4a, b) and the inclination angle θ (Figures 5a, b), (a - Kinematical case, b - Dynamical case). It is necessary to notice that the Figures 3, 5 are very similar to the Figures 1, 3 in (Eroshkin *et al.*,

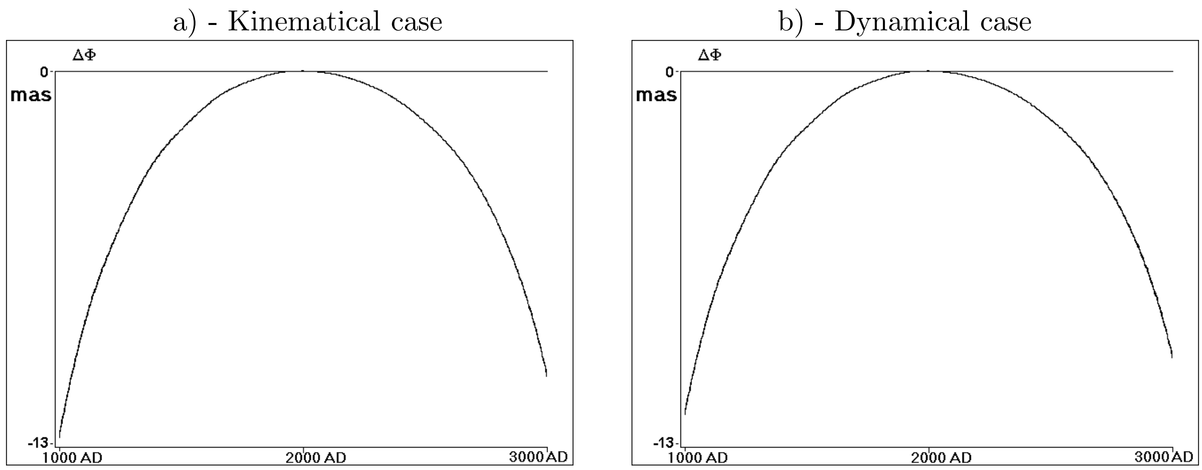

Figure 4: S9000 series minus solutions SMART97 in the angle ϕ

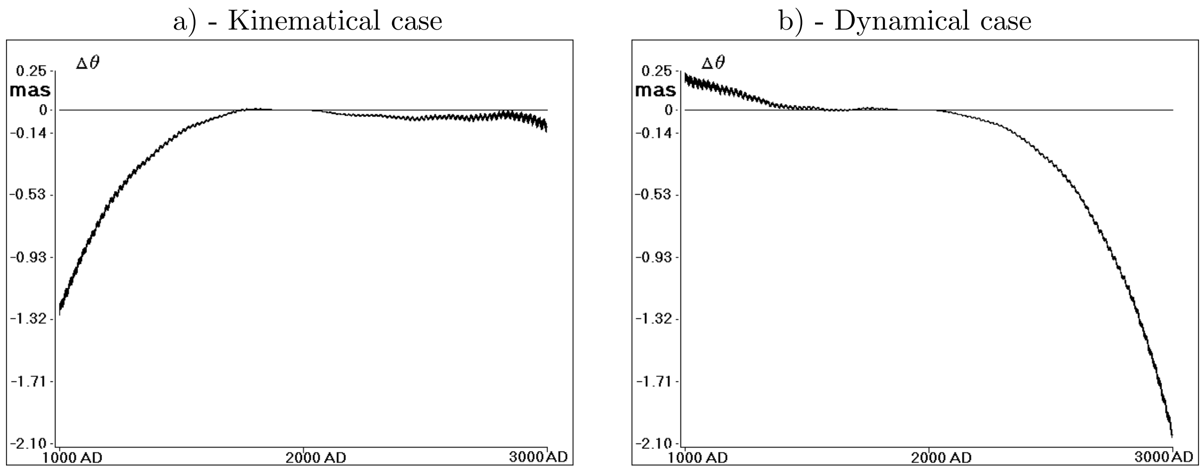

Figure 5: S9000 series minus solutions SMART97 in the angle θ

2003) representing the comparison between the numerical solutions of the rigid Earth rotation problem and the semi-analytical solutions SMART97.

The numerical solutions of the rigid Earth rotation are constructed over 2000 year time span from AD 1000 to 3000, with the quadruple precision of the calculations. The initial epoch of numerical integration is January 1, 2000. The initial conditions are calculated by means of S9000 series. The results of the numerical integration are compared with S9000 series in Euler angles (Figures 6a, b). There are no secular trends in the residuals in the angles ψ and θ but they present in the angle ϕ. The secular trend in the Dynamical case is smaller than that in the Kinematical case. As it was stated in (Eroshkin *et al.*, 2003, 2004) the linear part of the secular trend of the discrepancies in the angle of the proper rotation was due to an insufficient accuracy of the SMART97 series for calculating the initial conditions. In the present investigation the initial conditions are calculated by S9000 series and consequently the new series are also not sufficiently accurate. However, the values of the linear parts of the trends in the residuals for the proper rotation angle ϕ (51mas/tjy in Kinematical case and 32mas/tjy in Dynamical case) are smaller than in the same residuals (Eroshkin *et al.*, 2003, 2004), when comparing the results of the numerical integration and SMART97 solutions (90mas/tjy in both cases).

Figures 7a, b demonstrate the behaviour of the residuals of the comparison between the nu-

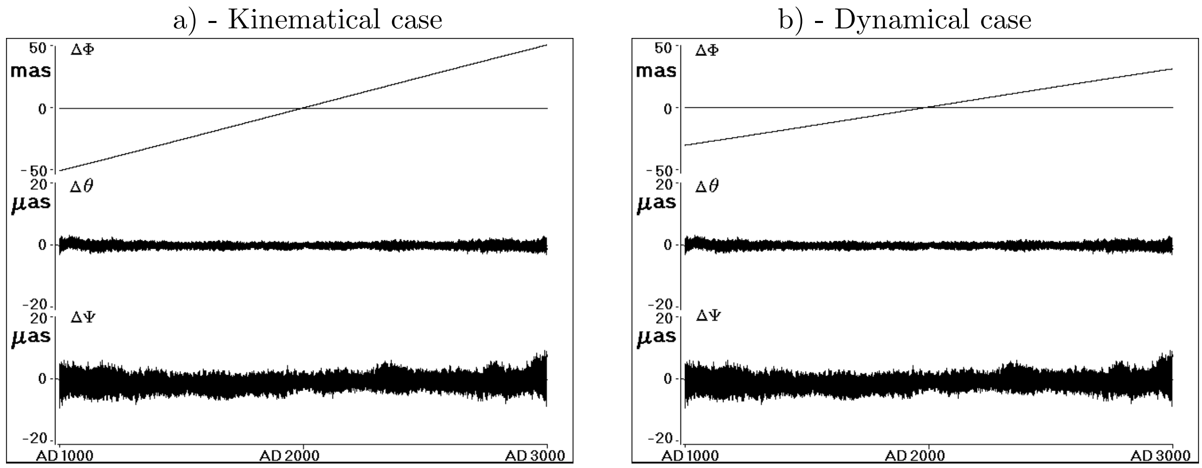

Figure 6: Numerical solutions minus S9000

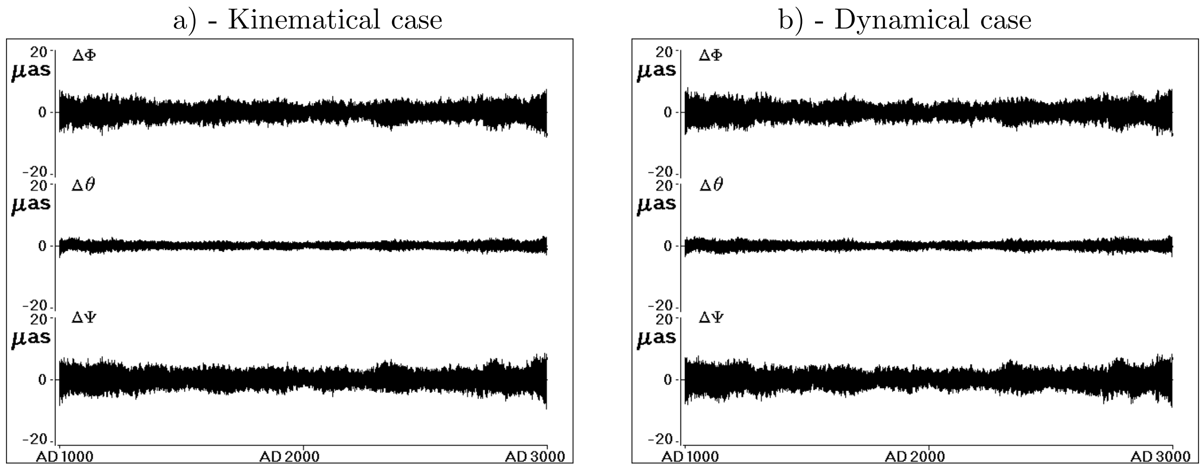

Figure 7: Numerical solutions minus S9000 after the formal removal of the secular trends in the proper rotation angle

merical solutions and S9000, after the formal removal of the secular trends in the proper rotation angle. The discrepancies of the comparison do not surpass 10μas over 2000 year time interval. It means a good dynamical consistency of S9000 series with the ephemerides DE404/LE404. The analogous comparison between the numerical solutions and SMART97 revealed the essentially larger discrepancies (Eroshkin *et al.*, 2003, 2004).

The residuals of the comparison between S9000 series and the high-precision numerical integration have to be processed for the determination of the diurnal and sub-diurnal harmonics. It can be expected that the addition of these short-period terms to S9000 series increase the degree of the adequacy of these series to the ephemerides DE404/LE404 and eliminate the secular trend in the residuals in the proper rotation angle.

4. CONCLUSION

The spectral analysis of the discrepancies of the numerical solutions and SMART97 semi-analytical solutions of the rigid Earth rotation problem is carried out for the Kinematical and Dynamical cases over the time interval of 2000 years.

The new high-precision rigid Earth rotation series S9000, which are dynamically adequate to the ephemerides DE404/LE404, are constructed for Dynamical and Kinematical cases of the problem. The S9000 series are presented by 6 files (3 files for the Kinematical case and 3 files for the Dynamical case). Each file includes around 9000 terms and is compiled quite analogous to SMART97 files.

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