A NEW METHOD FOR DYNAMICAL ANALYSIS OF ORIENTATION ERRORS FROM NON REGULAR SAMPLES

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ABSTRACT. The main aim of this paper is the analysis of the orientation errors of a celestial reference system from the differences between observed and calculated positions for a set of selected minor planets. In this paper a new numerical method not based on the Gauss-Markov will be presented.

1. INTRODUCTION

A traditional method to study the orientation error of the star catalogues is the analysis of the residual observation minus calculus for a set of selected minor planets. Usually, there are two problems: first, the distribution of the minor planet positions is not homogeneous in a band around the equator or the ecliptic, and second, the means in residuals are not null. Traditional methods based on least squares do not run well because the hypotesis of Gauss-Markov theorem is not allowed (López et al. 2005).

In this work we present an alternative numerical method based on a new class of spatial estimators and a reconstruction is proposed. This method is more suitable in this case.

2. FUNCTIONAL MODEL

Let $\{\{(\alpha_i^r, \delta_i^r)\_i = 1^{n_r}\}_{r=1}^N\}$ be a set of observed positions of the minor planet r at the epoch t_i^r (Marsden 1999). Let $(\sigma_{r,1}^0, \cdot, \cdot, \cdot, \sigma_{r,6}^0)$ the orbital elements of the asteroid r at the occultation epoch t^0 taked from I.T.A. tables (Batrakov 1997). The topocentric calculated positions can be obtained from the integration of the planetary equations of Lagrange. For the integration, the planetary theory VSOP87 of Bretagnon & Francou (1988) has been used.

To analyse the O-C errors, a previous process of improvement of asteroids elements is necessary. The component due to the errors in the reference frame can be modelized by means of three infinitesimal rotations $\varepsilon_x, \varepsilon_y, \varepsilon_z$ arround the OX, OY, OZ axis.

The residuals O-C can be written from the hypothesis as:

$$\triangle\alpha_{Rot} = \alpha_{Cat} - \alpha_D = \varepsilon_x \tan\delta\cos\alpha + \varepsilon_y \tan\delta\sin\alpha - \varepsilon_z + r_\alpha$$
$$\triangle\delta_{Rot} = \delta_{Cat} - \delta_D = -\varepsilon_x \sin\alpha + \varepsilon_y \cos\alpha + r_\delta$$

where r_α, and r_δ are random variables. The residual function Ψ can be written over a spherical domain D as:

$$\Psi(\varepsilon_x, \varepsilon_y, \varepsilon_z) = \int\int_D \left[r_\alpha^2 \cos^2\delta + r_\delta^2\right] \cos\delta d\alpha d\delta$$

and from them we can obtain $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ by a minimization proccess. In order to arrange the minimum, it is necessary to evaluate the integrals countaining the functions $r_\alpha(\alpha,\delta), r_\delta(\alpha,\delta)$, but these funtions are know only for the point belonging to D covered with the observations. To evaluate the integral we propose the following method:
1. Take as D the domain defined by a band around the equator (or the ecliptic) defined as: $B = \{(\alpha, \delta | \alpha \in [0, 2\pi], \delta \in [-\delta_{max}, \delta_{max}\}$.
2. Discretize the spatial domain B by means of a rectangular lattice defined as: $B : \{(\alpha_i, \delta_j) \in S_2 | \alpha_i = ih, \quad \delta_j = jk, \quad i = 0, ..., N_\alpha, \quad j = -M, ..., M\}$; $h = \frac{2\pi}{N}$, $k = \frac{\delta_{max}}{M}$.
3. Estimation of a function of $\triangle\alpha(\alpha,\delta)$, $\triangle\delta(\alpha,\delta)$ quantities from the sample done by the set of observations. For this purpose, we define for a generic function g, the quantities:

$$\overline{g_i}(\delta) = \frac{1}{h}\int_{(i-\frac{1}{2})h}^{(i+\frac{1}{2})h} g(\alpha,\delta)d\alpha \qquad \overline{\overline{g_{i,j}}} = \frac{1}{hk}\int_{\alpha_i-\frac{h}{2}}^{\alpha_i+\frac{h}{2}}\int_{\delta_j-\frac{k}{2}}^{\delta_j+\frac{k}{2}} f(\alpha,\delta)d\alpha d\delta = \frac{1}{k}\int_{\delta_j-\frac{k}{2}}^{\delta_j+\frac{k}{2}} \overline{g_i}(\delta)d\delta$$

To give the value $\overline{\overline{f_{i,j}}}$ from the sample, we use an efficient unbiased linear estimator of order s (López et al. 2005) and to take the values of $g(\alpha,\delta)$ from their means values, we use a reconstruction operator of order r (Casper & Atkins 1993).

3. NUMERICAL RESULTS

The numerical results obtained from this methods are: $\triangle\varepsilon_x = -0''003$, $\triangle\varepsilon_y = -0''002$, $\triangle\varepsilon_z = 0''046$ and from them, we obtain the value $\triangle A = 0''.041$ for the zero point of the FK5 calalogue, which is a value compatible with other determinations.

4. REFERENCES

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