# THE COUPLING EQUATIONS BETWEEN THE NUTATION AND THE GEOMAGNETIC FIELD IN GSH EXPANSION

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ABSTRACT. In the studies of Earth nutation involving electro-magnetic coupling at the core boundaries inside the Earth, it is convenient to express the magnetic induction equation and the Lorentz force density, as well as the magnetic field ($\mathbf{B}$) itself, in generalized spherical harmonics expansion (GSH). This is especially the case when the ellipticity of the interior structures and boundaries are considered. In this work, the magnetic induction equation and the Lorentz force density in the motion equation are derived to scalar format in GSH. In the resulted induction equation of the perturbed magnetic field $\mathbf{b}$ caused by nutation, it is shown that the spheroidal and toroidal part of $\mathbf{b}$ are decoupled with each other, although both of them involve the steady part of $\mathbf{B}$ and the nutational displacement $\mathbf{s}$ (or velocity $\mathbf{v}$) field. This theoretical result allows one to solve $\mathbf{b}$ and $\mathbf{s}$ (or $\mathbf{v}$) simultaneously.

## 1. INTRODUCTION

When interpreting the variation of earth rotation, like the decadal variation of the length-of-day and the nutation, there are four candidates of coupling mechanics between the earth layers interior, they are gravitational coupling, viscous coupling, topography coupling and electromagnetic coupling between the liquid core and the mantle, and/or between the inner core and the liquid core. In the nutation studies, using the angular momentum equation of the different layers of the Earth (*Buffett*, 1992, 1993; *Buffett et al.*, 2002; *Mathews et al.*, 2002), the electromagnetic coupling is regarded as a very good interpretation to the gap between the observed retrograde annual nutation and its theoretical value obtained by previous works without considering the contribution of the geomagnetic field. This paper discusses the coupling between the geomagnetic field and the nutational motion in numerical integration approach in which generalized spherical harmonics expansion (GSH) is used to provide a group of natural basis for the representation of tensor of any order.

The magnetic field exists throughout the Earth, but the coupling effects may be considered in the regions only near the two boundaries of fluid outer core (FOC) as discussed by *Buffett et al.* for simplicity. When the contribution of the magnetic field is considered, the Lorentz force density, $\mathbf{L}$, is to be added to the right hand of the motion equation. Meanwhile, the nutational motion of the fluid interior will also cause an incremental magnetic field (as will be seen in the induction equation) (*Huang et al.*, 2004). Both the nutational motion and the incremental magnetic field are time-varying. Consequently, the magnetic field and the velocity field are coupled in both equations. It is therefore necessary to calculate the induced magnetic field and the velocity field together.

The traditional motion equation for nutation without the contribution of magnetic field has been given in integrable format in generalized spherical harmonics expansion (GSH), which is used to provide a group of natural basis for the representation of tensor of any order, by *Smith*(1974) and are used in the numerical integration approach for nutation (e.g., *Huang et al.*, 2001). Accordingly, it is needed to represent both the additional term ($\mathbf{L}$) and the induction equation by GSH too.

In the next section, after having introduced the vectorial induction equation in frequency domain, we represent it and all the related variables in GSH, and the induction equation for the toroidal and spheroidal parts of the perturbed magnetic field $\mathbf{b}$ is derived to a scale form which is explicit and integrable; the new Lorentz force density in the motion equation is also developed by the scalars of the magnetic field in Section 3. Then, the two equations, induction equation and motion equation, are theoretically solvable for $\mathbf{b}$ and the displacement field $\mathbf{s}$.

## 2. ON THE VECTORIAL MAGNETIC INDUCTION EQUATION

### *2.1. In frequency domain*

The induction equation links the changes in the magnetic field to existing magnetic field in the presence of flux velocities (enhancing) and of diffusion which tends to decrease the field. This equation is often simplified in the frozen flux hypothesis when diffusion is ignored. This approximation is perfectly valid at diurnal timescale. The general form of this equation, without the pre-hypothesis of frozen flux (see for instance *Moffatt*, 1978) can be written as:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \tag{1}$$

where $\mathbf{B}$ is the magnetic field and $\mathbf{v}$ is the differential flux velocity.

In Earth nutation study, the flow relative to the mantle is assumed to be mainly a small rigid rotation about an axis in the equatorial plane due to the free slip nutational motion of the mantle.

When discussing the coupling between $\mathbf{B}$ and nutational velocity $\mathbf{v}$, one can decompose $\mathbf{B}$ into initial main part $\mathbf{B}_0$ and time-varying part $\mathbf{b}$:

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0(\mathbf{r}, t) + \mathbf{b}(\mathbf{r}, t), \tag{2}$$

where $\mathbf{B}_0$ varies very slowly in comparison with the nutation periods (in diurnal band) and can be regarded as a steady part, while $\mathbf{b}$ is induced by $\mathbf{v}$ (or by the nutational deformation, $\mathbf{s}$). $\mathbf{b}$ can, in turn, perturb $\mathbf{v}$ and $\mathbf{s}$. $\mathbf{b}$ can be expressed by a sum of Fourier series.

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0(\mathbf{r}) + \sum_\omega \mathbf{b}(\mathbf{r}, \omega) e^{i\omega t}, \tag{3}$$

$$\partial_t \mathbf{B}(\mathbf{r}, t) = \partial_t \mathbf{b}(\mathbf{r}, t) = \sum_\omega i\omega \mathbf{b}(\mathbf{r}, \omega) e^{i\omega t}, \tag{4}$$

Without lost of generality, the flow velocity field $\mathbf{v}(\mathbf{r}, t)$ (or $\mathbf{s}(\mathbf{r}, t)$) is also composed of a steady flow part and a time dependent part. We also use the Fourier series of $\mathbf{s}(\mathbf{r}, t)$ with the same frequency dependence as $\mathbf{B}$, but allow for a time lag with respect to $\mathbf{B}(\mathbf{r},t)$, i.e., the velocity is proportional to $e^{i\omega(t-t_0)}$.

$$\mathbf{s}(\mathbf{r},t)=\sum_{\omega}\mathbf{s}(\mathbf{r},\omega)e^{i\omega t}, \tag{5}$$

$$\mathbf{v}(\mathbf{r},t)=\partial_t\mathbf{s}(\mathbf{r},t)=i\omega\mathbf{s}(\mathbf{r},t)=\sum_{\omega}i\omega\mathbf{s}(\mathbf{r},\omega)e^{i\omega t}. \tag{6}$$

where $\mathbf{b}(\mathbf{r},\omega)$ and $\mathbf{s}(\mathbf{r},\omega)$ (or $\mathbf{v}(\mathbf{r},\omega)$) are complex vectors, which allows us to deal with a phase lag between $\mathbf{s}$ (or $\mathbf{v}$) and $\mathbf{b}$.

The induction equation (1) can then be simplified to

$$\partial_t\mathbf{b}(\mathbf{r},t)=\nabla\times(\mathbf{v}(\mathbf{r},t)\times\mathbf{B}_0(\mathbf{r}))-\nabla\times(\eta(r)\nabla\times\mathbf{b}(\mathbf{r},t)), \tag{7}$$

or in frequency domain

$$i\omega\mathbf{b}(\mathbf{r},\omega)=i\omega\nabla\times(\mathbf{s}(\mathbf{r},\omega)\times\mathbf{B}_0(\mathbf{r}))-\nabla\times(\eta(r)\nabla\times\mathbf{b}(\mathbf{r},\omega)), \tag{8}$$

### *2.2. Representation in GSH*

All the vectors in the equation above, $\mathbf{B_0}(\mathbf{r},\omega)$, $\mathbf{b}(\mathbf{r},\omega)$ and $\mathbf{s}(\mathbf{r},\omega)$ (as well as $\mathbf{v}(\mathbf{r},\omega)$) can be represented by GSH (rather than the ordinary spherical harmonics) as:

$$\mathbf{B}_0(\mathbf{r})=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}B_{lm}^{\alpha}(r)Y_{lm}^{\alpha}(\theta,\phi)\hat{e}_{\alpha}, \tag{9}$$

$$\mathbf{b}(\mathbf{r},\omega)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}b_{lm}^{\alpha}(r,\omega)Y_{lm}^{\alpha}(\theta,\phi)\hat{e}_{\alpha}, \tag{10}$$

$$\mathbf{s}(\mathbf{r},\omega)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}s_{lm}^{\alpha}(r,\omega)Y_{lm}^{\alpha}(\theta,\phi)\hat{e}_{\alpha}, \tag{11}$$

where $Y_{lm}^{\alpha}(\theta,\phi)$ are the GSH, and $\hat{e}_{\alpha}$ are the corresponding canonical unit basis vector (*Dahlen & Tromp*, 1998; *Phinney & Burridge*, 1973; *Huang & Liao*, 2003).

$B_{lm}^{\alpha}$ is for $\mathbf{B}_0$ rather than $\mathbf{B}$ and assumed independent with time (or frequency), while all other terms have been expressed in the nutational frequency domain by Fourier expansion. So these terms depend on $(\mathbf{r},\omega)$ (or $(r,\omega)$). In the following text, for simplifying the writing, we express the variables in the frequency domain and we only discuss their amplitudes, i.e. we speak about functions of $(r,\theta,\phi,\,\omega)$ and ignore the $e^{i\omega t}$ part.

### *2.3. The explicit scalar form*

We define

$$\left\{\begin{array}{l}U_{lm}=s_{lm}^{0}\\ V_{lm}=s_{lm}^{+}+s_{lm}^{-}\\ W_{lm}=s_{lm}^{+}-s_{lm}^{-}\end{array}\right.,\quad\left\{\begin{array}{l}F_{lm}=B_{lm}^{0}\\ G_{lm}=B_{lm}^{+}+B_{lm}^{-}\\ H_{lm}=B_{lm}^{+}-B_{lm}^{-}\end{array}\right.,\quad\left\{\begin{array}{l}f_{lm}=b_{lm}^{0}\\ g_{lm}=b_{lm}^{+}+b_{lm}^{-}\\ h_{lm}=b_{lm}^{+}-b_{lm}^{-}\end{array}\right.. \tag{12}$$

where $(U,V,W)$, $(F,G,H)$ and $(f,g,h)$ are the so-called radial, poloidal and toroidal scalars of $\mathbf{s}$, $\mathbf{B}_0$ (rather than the total field $\mathbf{B}$) and $\mathbf{b}$.

Using the properties of the GSH, the magnetic induction equation (8) then can be derived to following explicit scalar form at diurnal timescale in terms of spheroidal and toroidal solutions as:

$$\begin{aligned}0=\;&-\eta\frac{\Omega_l^0}{r}\left[\partial_r g_{lm}+\frac{1}{r}g_{lm}-\frac{2\Omega_l^0}{r}f_{lm}\right]\\&+i\omega\left\{f_{lm}+\frac{\Omega_l^0}{r}\sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k}f_l^{l'k}\times\tilde{h}_{lm}^{0}\right\}.\end{aligned} \tag{13}$$

$$
\begin{aligned}
0 = \quad & -\eta\left[\partial_r^2 g_{lm} + \tfrac{2}{r}\partial_r g_{lm} - \tfrac{2\Omega_l^0}{r}\partial_r f_{lm}\right] \\
& -\partial_r\eta\left[\partial_r g_{lm} + \tfrac{1}{r} g_{lm} - \tfrac{2\Omega_l^0}{r} f_{lm}^0\right] \\
& +i\omega\left\{g_{lm} + \sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k} f_l^{l'k}\times\tilde{h}_{lm}^P\right\}.
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
0 = \quad & -\eta\left[\partial_r^2 h_{lm} + \tfrac{2}{r}\partial_r h_{lm} - 2(\tfrac{\Omega_l^0}{r})^2 h_{lm}\right] \\
& -\partial_r\eta\left[\partial_r h_{lm} + \tfrac{1}{r} h_{lm}\right] \\
& +i\omega\left\{h_{lm} + \sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k} f_l^{l'k}\times\tilde{h}_{lm}^T\right\}.
\end{aligned} \tag{15}
$$

where,

$$
\begin{aligned}
\tilde{h}_{lm}^0 \quad = & \begin{pmatrix} l & l' & k \\ + & 0 & + \\ m & m-n & n \end{pmatrix} U_{l'm-n}\left\{\begin{array}{c} G_{kn} \\ H_{kn} \end{array}\right\} \\
- & \begin{pmatrix} l & l' & k \\ + & + & 0 \\ m & m-n & n \end{pmatrix} F_{l'm-n}\left\{\begin{array}{c} V_{kn} \\ W_{kn} \end{array}\right\} \begin{array}{l} \text{if } l+l'+k \text{ even} \\ \text{if } l+l'+k \text{ odd} \end{array},
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\tilde{h}_{lm}^P \quad & \equiv \tilde{h}_{lm}^+ + \tilde{h}_{lm}^- \\
& = \begin{pmatrix} l & l' & k \\ + & 0 & + \\ m & m-n & n \end{pmatrix}\left\{\begin{array}{c} U_{l'm-n}\partial_r G_{kn} + G_{kn}\partial_r U_{l'm-n} + \frac{1}{r}U_{l'm-n}G_{kn} \\ U_{l'm-n}\partial_r H_{kn} + H_{kn}\partial_r U_{l'm-n} + \frac{1}{r}U_{l'm-n}H_{kn} \end{array}\right\} \\
& - \begin{pmatrix} l & l' & k \\ + & + & 0 \\ m & m-n & n \end{pmatrix}\left\{\begin{array}{c} V_{l'm-n}\partial_r F_{kn} + F_{kn}\partial_r V_{l'm-n} + \frac{1}{r}V_{l'm-n}F_{kn} \\ W_{l'm-n}\partial_r F_{kn} + F_{kn}\partial_r W_{l'm-n} + \frac{1}{r}W_{l'm-n}F_{kn} \end{array}\right\}.
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\tilde{h}_{lm}^T \quad & \equiv \tilde{h}_{lm}^+ - \tilde{h}_{lm}^- \\
& = \begin{pmatrix} l & l' & k \\ + & 0 & + \\ m & m-n & n \end{pmatrix}\left\{\begin{array}{c} U_{l'm-n}\partial_r H_{kn} + H_{kn}\partial_r U_{l'm-n} + \frac{1}{r}U_{l'm-n}H_{kn} \\ U_{l'm-n}\partial_r G_{kn} + G_{kn}\partial_r U_{l'm-n} + \frac{1}{r}U_{l'm-n}G_{kn} \end{array}\right\} \\
& - \begin{pmatrix} l & l' & k \\ + & + & 0 \\ m & m-n & n \end{pmatrix}\left\{\begin{array}{c} W_{l'm-n}\partial_r F_{kn} + F_{kn}\partial_r W_{l'm-n} + \frac{1}{r}W_{l'm-n}F_{kn} \\ V_{l'm-n}\partial_r F_{kn} + F_{kn}\partial_r V_{l'm-n} + \frac{1}{r}V_{l'm-n}F_{kn} \end{array}\right\} \\
& + \frac{\Omega_l^0}{r}\begin{pmatrix} l & l' & k \\ 0 & + & - \\ m & m-n & n \end{pmatrix}\left\{\begin{array}{c} W_{l'm-n}G_{kn} - V_{l'm-n}H_{kn} \\ V_{l'm-n}G_{kn} - W_{l'm-n}H_{kn} \end{array}\right\}.
\end{aligned} \tag{18}
$$

where the coefficients in the two braces above, as well as in the following text, take the upper (or lower) values if $l + l' + k$ is even (or odd); the $3 \times 3$ matrix symbol, named J-square, is a compact form of the product of two Wigner 3-j symbols, arising from the product of two GSHs, and its nine indices subject to some selection rules (see *Smith* (1974) for detail); and

$$
\Omega_l^n \equiv \sqrt{(l+n)(l-n+1)/2}. \tag{19}
$$

$$
f_l^{l'k} \equiv \sqrt{\frac{(2l'+1)(2k+1)}{4\pi(2l+1)}}. \tag{20}
$$

## 3. THE LORENTZ FORCE DENSITY IN THE MOTION EQUATION

As mentioned above, when the contribution of the magnetic field is considered, the Lorentz force density $\mathbf{L}$ is added to the right hand of the motion equation, i.e.,

$$\begin{aligned}\rho D_t^2\mathbf{s} + 2\rho\mathbf{\Omega_0}\times D_t\mathbf{s} = \; & -\rho\mathbf{\Omega_0}\times(\mathbf{\Omega_0}\times\mathbf{s}) + \nabla\cdot\mathbf{T}^e - \nabla(\gamma\nabla\cdot\mathbf{s}) \\ & -\rho\nabla\phi_1 - \rho\mathbf{s}\cdot\nabla\nabla\phi + \nabla\cdot[\gamma(\nabla\mathbf{s})^T] + \mathbf{L},\end{aligned} \tag{21}$$

where all the notations are the same as in *Smith* (1974), and $\mathbf{L}$ is defined as

$$\mathbf{L} \equiv \mathbf{J}\times B = \frac{1}{\mu}\mathbf{B}\cdot\nabla\mathbf{B} = \frac{1}{\mu}\nabla\cdot\mathbf{BB} \equiv \frac{1}{\mu}\nabla\cdot\mathbf{M}, \tag{22}$$

where, $\mu$ is the magnetic permeability (rather than rigidity $\mu$ in the motion equation for nutation), $\mathbf{J}$ is the induced current, the dyad (or co-vector) $\mathbf{BB}$ is the Maxwell magnetic stress tensor $\mathbf{M}$, and the solenoidal condition about the magnetic field, $\nabla\cdot\mathbf{B} = 0$, is used in the Equation above.

From the decomposition (eq.(2)),

$$\mathbf{M} = \mathbf{B}_0\mathbf{B}_0 + \mathbf{B}_0\mathbf{b} + \mathbf{b}\mathbf{B}_0 + \mathbf{bb}, \tag{23}$$

the first term $\mathbf{B}_0\mathbf{B}_0$ does not contribute to nutation; moreover, the perturbed field is far smaller than the main field ($\mathbf{b} \ll \mathbf{B}_0$), then $\mathbf{bb}$ or equivalently $\mathbf{b}\cdot\nabla\mathbf{b}$ can be ignored in comparison with the other terms. Therefore, the terms in $\mathbf{L}$ kept in the motion equation for nutation are

$$\mathbf{L} \cong \frac{1}{\mu}(\mathbf{B}_0\cdot\nabla\mathbf{b} + \mathbf{b}\cdot\nabla\mathbf{B}_0). \tag{24}$$

The solenoidal condition for both $\mathbf{B}_0$ and $\mathbf{b}$ have been used again.

As the gradient of the incremental magnetic field is important and from the dimensional analysis (*Huang et al., 2005, in preparation*), the second term can be ignored in comparison with the first one. The horizontal derivatives in $\mathbf{b}$ is further assumed to be negligible in comparison with the radial derivative ($\nabla_1\mathbf{b} \ll \partial_r\mathbf{b}$) in the boundary layer (this assumption maybe too strong. In fact, what we need is only that $\nabla_r\mathbf{b} \equiv \frac{1}{r}\nabla_1\mathbf{b} \ll \partial_r\mathbf{b}$, this requirement, involving further more a division by $r$ for the horizontal gradient part, is more loose and more reasonable than the first one). The Lorentz force is then written:

$$\mathbf{L} \cong \frac{1}{\mu}B_0^r\partial_r\mathbf{b}. \tag{25}$$

Analogously to the magnetic induction equation and the motion equation, $\mathbf{L}$ can then be also expanded in GSH as

$$\begin{aligned}
L_{lm}^R \equiv \quad [\mathbf{L}]_{lm}^0 \quad &= \frac{1}{\mu}\sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k} f_l^{l'k}\begin{pmatrix} l & l' & k \\ 0 & 0 & 0 \\ m & m-n & n\end{pmatrix} F_{l'm-n}\partial_r f_{kn} \\
L_{lm}^P \equiv [\mathbf{L}]_{lm}^+ + [\mathbf{L}]_{lm}^- &= \frac{1}{\mu}\sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k} f_l^{l'k}\begin{pmatrix} l & l' & k \\ + & 0 & + \\ m & m-n & n\end{pmatrix}\left\{\begin{array}{c} F_{l'm-n}\partial_r g_{kn} \\ F_{l'm-n}\partial_r h_{kn}\end{array}\right\}, \\
L_{lm}^T \equiv [\mathbf{L}]_{lm}^+ - [\mathbf{L}]_{lm}^- &= \frac{1}{\mu}\sum_{k=0}^{\infty}\sum_{n=-k}^{k}\sum_{l'=|l-k|}^{l+k} f_l^{l'k}\begin{pmatrix} l & l' & k \\ + & 0 & + \\ m & m-n & n\end{pmatrix}\left\{\begin{array}{c} F_{l'm-n}\partial_r h_{kn} \\ F_{l'm-n}\partial_r g_{kn}\end{array}\right\}
\end{aligned} \tag{26}$$

These three scalars of $\mathbf{L}$ can then be used directly in the new motion equation as in *Smith*(1974).

## 4. SHORT REMARKS

In section 2, we present the induction equation about the perturbed magnetic field **b** as three scalar ordinary differential equations by GSH, they depends on the initial main magnetic field $\mathbf{B}_0$ and the nutational displacement field **s**; meanwhile, in the new motion equation about **s**, the Lorentz force density **L** is also related to $\mathbf{B}_0$ and **b**; and $\mathbf{B}_0$ can be obtained beforehand from any geodynamo model (so $\tilde{h}^0_{lm}, \tilde{h}^P_{lm}$ and $\tilde{h}^T_{lm}$ are known), therefore **b** and **s** can be theoretically solved from these two equations. In practice, all the upper limits of the sum over $l, l'$ (and/or $k, n$) are set to a definite number (10, for example), because, the magnetic field is given in the harmonic series up to that fixed degree.

Moreover, in the resulted induction equations, the spheroidal part (eq.(13) and (14)) and the toroidal part (eq.(15)) of **b** are decoupled with each other, although both of them involve $\mathbf{B}_0$ and **s**. This result makes the equations solvable more easily.

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