ON THE EFFECT OF THE REDISTRIBUTION TIDAL POTENTIAL ON THE ROTATION OF THE NON-RIGID EARTH: DISCREPANCIES AND CLARIFICATIONS

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ABSTRACT. In the last years several works have dealt with the effect of the redistribution tidal potential on the Earth rotation, that is to say on the rotational effects of the additional potential due to the elastic deformations caused on the Earth by gravitational interaction with the Moon and the Sun. However, analytical or numerical results derived by some of these approaches seem to provide significant discrepancies. In this research we compare some of these approaches when considering the motion of the rotational angular momentum axis of a perfect elastic Earth model. To this end, we revise the Hamiltonian formulation of the problem, contrasting it with others approaches well suited for the purposes of comparison and determining the source of the discrepancies.

1. INTRODUCTION

The characteristic feature of elastic Earth models is that there is a relative motion of the parts of the Earth with respect to a frame attached to it. This fact gives rise to additional effects that do not appear in the rigid cases and that provide significative contributions to the rotational motion of the Earth. ¿From a variational approach point of view, the elastic deformation affects both to the kinetic and potential energies of the body, or in terms of the Euler-Liouville type formulations, to the angular momentum of the body and the torques exerted on it.

In the last three decades, Earth rotation studies based on the above mentioned formulations have focused their attention on studying the effects on the rotation due to the elastic variations in the kinetic energy (or angular momentum) of the Earth, since this increment provides the main part of the elastic contribution. This is the case (among others) of the investigations by Sasao *et al.* 1980; Mathews *et al.* 2002; Getino and Ferrándiz 2001; Krasinsky 2003, etc. On the contrary, the effects of the elastic variations in the potential energy (or exerted torque) on the rotation of the Earth, specially on nutation and precession, have only been tackled recently.

In this note, we analice the influence on the rotation of the Earth of the variation of the gravitational potential energy due to the deformation caused on the Earth by the tidal interactions with the Moon and the Sun. This additional potential energy produced by an elastic redistribution of mass will be named as redistribution tidal potential. This name aims to avoid confusions with other parts of the gravitational potential considered in the rigid cases.

2. HAMILTONIAN TREATMENT

Next we sketch the procedure to account for the contributions on the rotation of the Earth of the redistribution tidal potential. A detailed explanation can be found in Escapa *et al.* 2005.

A convenient way to obtain the analytical expression of the redistribution tidal potential is to relate it with the variation that the deformation causes in the inertia matrix of the Earth. By so doing, we reduce our problem to compute the increment in the moments and products of inertia due to the deformation. To this end, it is necessary to find a solution of the Earth elastic problem, that is to say, to know the expression of the displacement vector due to the tidal interactions with the external bodies (the Moon and the Sun). On the basis of different simplifying hypothesis that model the elastic response of the Earth (Takeuchi 1950, Jeffreys and Vicente 1957, Sasao *et al.* 1980, etc.), it is possible to provide the analytical expression for the displacement vector which, for the purposes of computation, is usually written as a sum of spheroidal and toroidal terms. From this elastic solution, we can compute the increment of the inertia matrix, which can be splitted as

$$\Delta\Pi = \Delta\Pi^Z + \Delta\Pi^T + \Delta\Pi^S, \tag{1}$$

accordingly they contain zonal (Z), tesseral (T) or sectorial (S) spherical harmonics of the second degree in the geocentrical coordinates of the external bodies causing the tidal deformation, which are assumed to be known functions of time. Likewise, this increment depends on some rheological parameters which specify the elastic properties of the Earth. It is also expedient to separate the effects of the permanent tide (p), which are only present in $\Delta\Pi^Z$, so, finally, we have

$$\Delta\Pi = \Delta\Pi^Z_p + \Delta\Pi^Z_{np} + \Delta\Pi^T + \Delta\Pi^S. \tag{2}$$

Once we have obtained the variation of the inertia matrix, we can take advantage of MacCullagh formula to write out the analytical expression of the redistribution tidal potential, which in a similar way is put as

$$\Delta U = \Delta U^Z_p + \Delta U^Z_{np} + \Delta U^T + \Delta U^S. \tag{3}$$

To evaluate the contributions of redistribution tidal potential on the nutational and precessional motions of the Earth, we employ the Hamiltonian formalism developed by Getino and Ferrándiz. To this end, and following a standard procedure (see Getino and Ferrándiz 2001) we construct the Hamiltonian of a two–layer Earth model composed of an elastic mantle that encloses a fluid core, incorporating the part relative to the redistribution tidal potential. By applying the Hori's perturbation technique we obtain analytical expressions for the nutations in longitude and obliquity and the precession in longitude of the angular momentum axis.

3. DISCUSSION

The numerical computation of the former analytical formula is performed by considering the numerical values of Earth parameters given in Getino and Ferrándiz 1995, 2001. The values derived for the long period terms of the nutation in obliquity for the angular momentum axis are displayed in Table 1. We have splitted the total contribution in different parts according to eq. 3. Similar results are obtained for the nutation and precession in longitude. ¿From these values, we can stress two fundamental conclusions. First, under the elastic hypothesis considered the total effect of the redistribution tidal potential on nutation and precession of the angular momentum axis is zero. This result have been confirmed by means of analytical developments based on some properties fulfilled by the trigonometric expansions of the perturbing bodies coordinates.

l_m	l_S	F	D	Ω	ΔU_p^Z	ΔU_{np}^Z	ΔU^T	ΔU^S	Total
+0	+0	+0	+0	+1	+72.244	-7.535	-118.197	+53.488	+0.000
+0	+0	+0	+0	+2	-0.706	-1.603	+3.060	-0.752	+0.000
+0	+1	+0	+0	+0	+0.000	-0.000	+0.000	+0.000	+0.000
+0	-1	+2	-2	+2	-0.072	+0.068	+0.010	-0.006	+0.000
+0	+0	+2	-2	+2	+4.309	+0.002	-9.515	+5.205	-0.000
+0	+1	+2	-2	+2	+0.168	+0.023	-0.421	+0.230	-0.000
+1	+0	+0	+0	+0	+0.000	+0.000	+0.000	-0.000	-0.000
+0	+0	+2	+0	+2	+0.693	-0.005	-1.534	+0.846	+0.000
+0	+0	+2	+0	+1	+0.143	-0.015	-0.237	+0.109	-0.000
+1	+0	+2	+0	+2	+0.089	+0.026	-0.253	+0.138	-0.000

Table 1: Nutations in obliquity in μas: angular momentum axis

Second, the different parts of the redistribution tidal potential (see eq. 3) give raise to numerical contributions of the same order of magnitude, therefore to explain properly the influence of this effect in the rotation of the Earth it is necessary to take all the contributions of the redistribution tidal potential (zonal, tesseral and sectorial), treating them in an homogeneous way.

It may be not out of place to compare our results with other investigations that have also considered the same issue. In particular, we will consider the approaches worked out by Souchay and Folgueira 1998; Mathews *et al.* 2002; Krasinsky 2003 and Lambert and Capitaine 2004. In Souchay and Folgueira 1998 the contribution of the redistribution tidal potential is also investigated under a Hamiltonian approach. The authors only consider the part of the potential related to ΔU_{np}^Z providing analytical and numerical values for the nutations by means of an expression of $\Delta\Pi_{np}^Z$ taken from Melchior (1978).

Mathews *et al.* 2002 treat the problem with the aid of an Euler-Liouville equation for the whole Earth, as a part of a set of effects named "nonlinear terms". However, the treatment is quite opaque since details of the derivations and justifications used are omitted. The torque due to the deformation of the Earth is constructed through the redistribution tidal potential, although the contribution due to the tesseral part ΔU^T, coming from $\Delta\Pi^T$, seems to be omitted[1]. Besides, the term relative to the permanent tide ΔU_p^Z also seems not to be considered here but included as a part of the ordinary nutation and precession amplitudes. The contributions to the nutations and precession are computed numerically, obtaining the values from two different models of deformation: the terms arising from $\Delta\Pi_{np}^Z$ are taken from the tables of the axial spin rate variation (IERS Conventions 1996) and those ones arising from $\Delta\Pi^S$ are obtained with the help of the tables of tides provided by Cartwright and Tayler (1971).

Lambert and Capitaine 2004 tackle the issue together with the effects that the Earth's rotation rate variations due to zonal tides have on precession-nutation. As in the investigation by Mathews *et al.* 2002, the authors evaluate the influence of the redistribution tidal potential on nutations and precession by computing its torque and inserting it in the equations of Sasao *et al.* 1980. To this end, only the part ΔU_{np}^Z of the redistribution tidal potential is considered and the torque is numerically evaluated, taking for $\Delta\Pi_{np}^Z$ the values given in IERS Conventions 2003 and using ELP2000 (Chapront-Touzé and Chapront 1983) and VSOP87 (Bretagnon and Francou 1988) orbital theories.

Krasinsky 2003 works out the problem in a comprehensive framework which generalices the work of Sasao *et al.* 1980. With respect to the effect of the redistribution tidal potential on the rotation, the author considers a classical expression of the redistribution tidal potential in terms

[1]N. Capitaine has informed us that this part was included by the authors in a work presented at 2003 AGU Fall Meeting, but as far as we know it has not been incorporated to MHB 2000 tables.

of the Love number k_2. By taking the vectorial product of the gradient of the redistribution tidal potential, it is shown that in the elastic case the torques vanish due to the proportionality of the vectors entering in the vectorial product and to the cancellation of the crossed effect between the Moon and the Sun. Therefore, since the total torque is zero, there is no net effect on the rotation of the Earth, that is to say, there is no contribution to the nutation and precession of the angular momentum axis arising from this effect.

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