POINCARÉ FLOW IN THE EARTH'S CORE

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ABSTRACT. In this paper, we discuss the motion inside the Earth's core. At low frequency, torsional oscillations occur associated with magneto-hydrodynamic effects. At high frequency, the Poincaré flow is believed to be a good approximation of the fluid dynamics. This simple flow is deduced in a very simplified case, but it appears to be true in a broader range of configurations.

1. INTRODUCTION

The rotation of the Earth is irregular in time: the Earth's rotation rate, and the associated length-of-day, present variations at the millisecond level, and the Earth's rotation axis moves both in the Earth and in space. The new definition of the IAU of polar motion and precession/nutation separate the two in terms of frequency: any motion which is low frequency in the inertial frame, and consequently retrograde diurnal in the Earth fixed frame, is called nutation, while the polar motion is all what remains. The nutation motion is at the level of 600 meters at the Earth's surface, and the polar motion is about 20 meters.

The main causes of the irregularities in the length-of-day and polar motion are the interactions between the solid Earth and the geophysical fluids: the atmosphere, the ocean, the hydrosphere, and the core. The gravitational interactions between the Earth and the other celestial bodies are the main causes of the precession/nutation motion.

Classically, the effect of the fluid layer on the rotation of the Earth is estimated using the angular momentum budget of the (supposedly isolated) Earth-fluid system: any change in the fluid angular momentum is associated with an opposite change in the Earth's angular momentum. As a consequence, when knowing the variation of the fluid angular momentum, we can deduce the variation in the Earth rotation due to the interaction with this fluid. For this reason, the International Earth Rotation and Reference Systems Service (IERS) has created the Global Geophysical Fluids Center, composed of 7 special bureaus in charge of collecting data such as the angular momentum of the major fluids in the Earth system.

In this study, we focus on the effect of the core on the Earth rotation. It is now well accepted that the core is the major cause of variation of the length-of-day for periods ranging between a few years to several dozen years.

Obviously, the estimation of the effect of the core on the Earth rotation is not as easy as the estimation of the atmospheric effect, for instance. Indeed, at present, it is impossible to get direct observations of the core flow. The best option is to use information deduced from the observed surface magnetic field and from hypotheses as meaningful as possible to make reasonable estimates of the core flow and angular momentum. Order of magnitude considerations allow us to see that the core dynamics will be dominated by the effect of magnetism at decadal timescale, while the magnetic effect will be much smaller at high frequency.

In Section 2, we discuss how the effect of the core on the Earth rotation at decadal timescales can be estimated using the observation of the magnetic field at the Earth surface. In Section 3, we present the basic idea of the Poincaré flow in the case of high frequency motion. Section 4 is devoted to discussion and conclusion.

2. DECADAL CORE FLOW AND EARTH ROTATION

As mentioned above, the effect of the magnetic field is very important at decadal frequency. The main part magnetic field observed at the Earth's surface has a core origin, but the mantle screens part of the field. Nevertheless, using the observed magnetic field at the Earth surface, it is possible to infer the poloidal field at the Core Mantle Boundary (CMB). We know that the magnetic field is linked to the motion inside the fluid by the induction equation

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \wedge \left(\vec{B} \wedge \vec{v} \right) + \eta \Delta \vec{v} \tag{1}$$

where $\vec{B}$ is the magnetic field, $\vec{v}$ the velocity of the core flow, and η the magnetic diffusion.

At decadal timescale, the last term, associated with diffusion, is negligible, and the equation simply tells us that the cause of variation of the magnetic field is its advection by the core flow. As a consequence, we can deduce the velocity field from the knowledge of the observed magnetic field and its variations. Unfortunately, this problem is underdetermined. Consequently, we have to make assumptions on the core flow at the CMB in order to be able to get the full flow there. Several different assumptions have been shown to be successful (see, e.g., Ponsar *et al.,* 2002). In order to get the flow inside the core, it is necessary to continue the flow inside the core.

This continuation is classically done using the Taylor column hypothesis: the velocity is supposed to be the same on a set of cylinders coaxial with the Earth's rotation axis. Using that method, we can estimate the core flow and the core angular momentum for every hypothesis. They all give core angular momentum fluctuations and associated length-of-day variations that look more or less similar to the observed variations of the length-of-day, as shown in Figure 1..

Note that this method has some important additional drawbacks. First, the magnetic field is observed with only a limited temporal resolution. The present solution has only independent data every 5 years, limiting the resolution of the core angular momentum data. With the new satellite mission Oersted, the time resolution of the magnetic field will strongly improve. Secondly, the magnetic field coming from the crust and the solar wind is also included in the observed magnetic field. This field is mostly high spatial frequency, it hides consequently the higher spatial resolution of the core induced magnetic field. The limit is classically put at degree 12.

Those last years, some other attempts has been done. They are mostly based on magneto hydrodynamic models, with no observation included. Some attempts have been made to include gravimetric coupling (Mound and Buffet, 2003) or magnetic observations (Kuang, 1999, Kuang and Chao, 2001).

3. HIGH FREQUENCY POINCARÉ CORE FLOW

The Poincaré motion was first derived in the case of a homogeneous, incompressible, inviscid core, with no magnetic coupling. Using those hypotheses, an analytical solution to the flow equations can be obtained (see for instance Hough 1895). The solution method is the following:

- The CMB is supposed to be rotating.

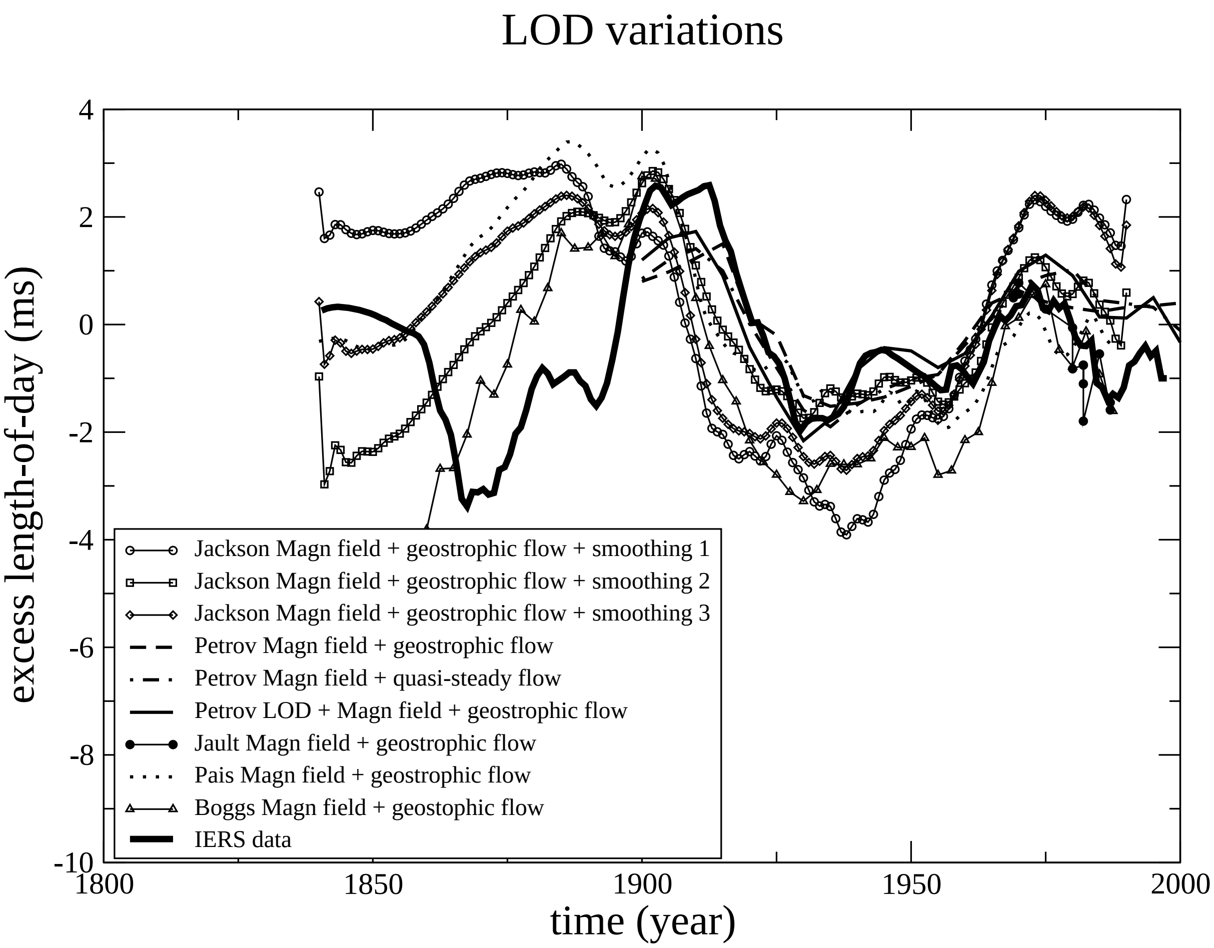

Figure 1: Length-of-day variations for various models and IERS observational LOD data. The data can be found on the SBC website.

- By using some heavy algebra, the core flow is shown to be decomposed into two parts: a rigid rotation of the core and a (small) irrotational flow, verifying the boundary condition (impermeability of the CMB),
- From the Helmholtz vortex equations, the flow in the core, which mainly is a rigid rotation, as a function of time in response to the CMB motion can be obtained.

It is possible to extend the Poincaré flow when some of the hypotheses are dropped. The Poincaré flow seems to be true for slightly viscous compressible flow, in presence of a magnetic field and has also been observed experimentally by Noir et al. (2003). Using a numerical approach, Smith (1974) was able to reproduce this behavior of the core fluid, for the Earth's rotational core normal mode (the Free Core Nutation, FCN). The Poincaré flow is often used in the modelling of the non-rigid Earth nutational response to gravitational forcing (see Mathews et al., 1991, Greff-Lefftz et al., 2000, ...). In the case of the FCN, the differential rigid rotation of the core with respect to the mantle becomes very large, allowing an observable motion of the mantle.

4. DISCUSSION AND CONCLUSION

The dynamics of the core is very complex, as it responds to the magneto-hydrodynamic equations. In addition, the core is not observed directly, and only limited information can be gathered, using indirect methods. From the magnetic field observations, it is possible to infer part of the core flow at decadal period. Nevertheless, this needs some hypotheses (the solution is not unique). The solutions found present variations of the angular momentum of the right order of magnitude, but differences as large as 100% remain.

5. REFERENCES

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