IMPROVEMENTS IN THE PRECESSION-NUTATION MODELS

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ABSTRACT. This paper reviews a number of effects that must be taken into account in developing improved expressions for the precession of the Earth's equator in order to be dynamically consistent and compliant with up to date models for the ecliptic and the non-rigid Earth. These effects have been considered in the P03 and the parameterized P04 precession solutions of Capitaine et al. (2003, 2004b); they are related to the ecliptic precession, to parameters of the non-rigid Earth such as the J_2 rate (Bourda & Capitaine 2004) and to the expressions of the precession rates. We also report on similar effects to be considered in nutation, as well as on the influence, on precession-nutation, of the variations in Earth rotation that was recently clarified (Lambert & Capitaine 2004). Some recommendations are given on the best form of the precession-nutation expressions and the appropriate parameters to be fitted to observations.

1. INTRODUCTION

The IAU 2000 precession-nutation model was adopted by IAU 2000 Resolution B1.6 to replace the IAU 1976 Precession and the IAU 1980 Nutation. The nutation series was generated by the convolution of the MHB 2000 transfer function (Mathews et al. 2002) with the nutation series of Souchay et al. (1999) obtained by solving the equations of Earth's rotation for a rigid-Earth. The transfer function was based upon basic Earth parameters estimated from VLBI observations, one of them being the Earth's dynamical flattening H. The precession component of the IAU 2000 model consists simply of VLBI MHB-estimated corrections to the precession rates in longitude and obliquity of the IAU 1976 precession and is therefore not dynamically consistent. Thus, IAU 2000 Resolution B1.6 encouraged the development of new expressions for precession consistent with the IAU 2000A model; then, in 2003, the 25th IAU General Assembly established a WG on "Precession and the Ecliptic" to recommend a new model.

The P03 precession provided by Capitaine et al. (2003) was proposed as a possible replacement for the IAU 1976 ecliptic precession and for the IAU 2000 equator precession. In this paper, we report on the various effects that contribute to the P03 equator precession and make it dynamically consistent as well as on similar contributions to nutation; we also discuss the most appropriate form for the precession expressions and parameters.

2. VARIOUS EFFECTS ON THE PRECESSION OF THE EQUATOR

A number of effects influence the dynamical solution for the precession of the equator. They are related to the expressions of precession rates, to the ecliptic precession and to parameters of the non-rigid Earth. These effects have been considered in the P03 solution and each of them has been evaluated by Capitaine et al. (2004a) and compared with the difference between P03 and other precession solutions. Additionally to IAU 1976 and IAU 2000, these solutions are those obtained by Williams (1994), Bretagnon et al. (2003) and Fukushima (2003), which will be denoted W94, B03 and F03, respectively.

(i) *Effects of the integration constants*

The integration constants to be used for solving the equations for the equator precession are directly related to the precession rate values ψ_1 and ω_1 (at J2000) in longitude and obliquity, respectively. Table 1 provides the values corresponding to the various precession models as well as the precession rates differences with respect to IAU 2000. The precession rate values in longitude are all "observed quantities", the IAU 1976 value being determined by optical astronomy and the other ones by VLBI observations. The obliquity rate values are either theoretical values without any observational constraint (i.e. rigid Earth values for IAU 1976 and B03, and non-rigid Earth value for W94), or VLBI-estimated values. The IAU 2000 values are based on the VLBI MHB-estimated corrections to IAU 1976 (i.e. −299.65 mas/cy in longitude and −25.24 mas/cy in obliquity), whereas the P03 precession rates were obtained by correcting the MHB estimated values for some pertubing effects; the largest of these effects, of 2.8 mas/cy in the precession rate in longitude, is due to the fact that the actual "estimated quantity" is not ψ_1 itself, but is $X_1 \approx \psi_1 \sin\epsilon_0$, ϵ_0 being the obliquity at J2000 of the associated precession model (i.e. IAU 1976 for the MHB estimates). Table 1 also provides the corresponding values for ϵ_0 and X_1 and the X_1 differences with respect to the IAU 2000 value, which are the relevant differences when fitting the various models to VLBI observations. The differences with respect to

Table 1: Comparison between precession rates values (unit: mas).

Model	ψ_1	$\psi_1 \sin\epsilon_0$	$\mathrm{d}\psi_1$	$\mathrm{d}X_1$	ϵ_0	ω_1	$\mathrm{d}\omega_1$
IAU1976	5038778.4	2004310.94	299.65	+119.19	84 381.448	0.0	25.24
IAU2000	5038478.75	2004191.747	0.00	0.00	84 381.448	−25.24	0.00
W94	5038456.501	2004182.023	−22.25	−9.72	84 381.409	−24.4	−0.84
B03	5038478.750	2004190.869	0.00	0.88	84 381.4088	−26.501	0.00
F03	5038478.143	2004190.569	−0.61	−1.18	84 381.406	−21.951	−51.74
P03	5038481.507	2004191.903	+2.76	−0.16	84 381.406	−25.754	−0.51

the IAU 2000 precession rates are responsible for differences in the precession expressions which can be derived from the theoretical developments of the coefficients of the precession quantities provided in Table 7 of the P03 paper (i.e. Capitaine et al. 2003), as for example:

$$\frac{\partial\psi_2}{\partial r_{01}} \approx \frac{c_1(\cot\epsilon_0 - \tan\epsilon_0)}{2}; \; \frac{\partial\psi_2}{\partial u_{01}} \approx -\frac{r_{01}\tan\epsilon_0}{2}; \; \frac{\partial\omega_2}{\partial r_{01}} = \frac{s_1}{2}; \; \frac{\partial\omega_2}{\partial u_{01}} = -\frac{u_{01}\tan\epsilon_0}{2}, \qquad (1)$$

r_{01} and u_{01} being the first order terms in r_0 $(= \psi_1)$ and u_0 $(= \omega_1)$, which are the actual integration constants to be used in the precession equations, and c_1 and s_1 the coefficients of the linear terms in the ecliptic precession expressions P_A and Q_A, respectively. The largest differences with respect to the IAU 2000 equator precession due to the differences dr_{01} and du_{01} in the integration constants can be written as:

$$d\psi_A = dr_{01}\, t - (212\, dr_{01} + 5297\, du_{01}) \times 10^{-6}\, t^2; \; d\omega_A = du_{01}\, t + (10\, dr_{01} - 4\, du_{01}) \times 10^{-6}\, t^2. \quad (2)$$

Relation (2) shows that, additionally to their direct effect on the linear terms, the MHB corrections to the IAU 1976 precession have a dynamical effect of the order of 200 μas/cy^2 in ψ_A and 2 μas/cy^2 in ω_A. This relation also shows that the largest uncertainty that can be expected in the precession expressions due to the uncertainty in the VLBI-estimated precession rates (which is of the order of 1 mas/cy), is of a few μas/cy^2 in the quadratic term of the expression for ψ_A, coming from the du_{01} term only.

(ii) *Effects of the model for the ecliptic precession*

The dependence of the ψ_A and ω_A expressions on the ecliptic are provided in Table 7 of the P03 paper. The largest terms due to variations dc_1 and ds_1 in the linear terms of the P_A and Q_A quantities, respectively and $d\epsilon_0$ in the obliquity value at J2000 are:

$$d\psi_A = -0.001059\, d\epsilon_0 \ + 0.02288\, dc_1\, t^2\,; \quad d\omega_A = (0.01221\, dc_1 - 0.00530\, ds_1)\, t^2. \tag{3}$$

Relation (3) shows that, additionally to their direct effect on the precession angles referred to the ecliptic of date (i.e. obliquity and planetary precession), changes in the ecliptic precession have dynamical effects which, given the dc_1 and ds_1 values reported in the P03 paper, are of the order of 100 μas/cy^2 and 25 μas/cy^2 in the t^2 terms in ψ_A and ω_A, respectively.

(iii) *Effects of the expression for the precession rates*

Expressions for the precession rates include constant terms which are the precession rate values at J2000, $r_0(=\psi_1)$, $u_0(=\omega_1)$, and linear and quadratic terms, r_1, u_1 and r_2, u_2, respectively, to which the quadratic and cubic terms in the ψ_A and ω_A solutions of the precession equations are directly related. The precession rates components used in the P03 solution are provided in Table 3 of the P03 paper; they include rigid-Earth and non-rigid-Earth parts. One improvement in the rigid-Earth part of r_1 with respect to the IAU 1976 one is a change of 0.295 mas/cy in the largest component (due to changes in the the eccentricity of the Earth's orbit); the second improvement is including the 1.074 mas/cy J_2 and planetary tilt effect, firstly considered by Williams (1994), which is also responsible for the main component of the obliquity rate (and a -0.044mas/cy^2 quadratic variation in ω_A). Note that considering this effect is compliant with the IAU 2000 precession in obliquity and also with the nutation series of Souchay et al. (1999) on which the IAU 2000 nutation is based. The change also includes a 3 μas/cy^2 additional contribution in the geodesic precession.

(iv) *Effects of the Earth model*

Additionally to its direct contribution to the precession rates at J2000 that was considered in (i), the influence of the Earth model on the equator precession appears through its contribution to the linear variations of the precession rates; this includes tidals terms in r_1 and a term proportional to the time variation in the Earth's dynamical flattening H to which the main component of r_0 and u_0 are proportional. Using the values provided by Williams (1994) both for the J_2 rate variations (i.e. $\dot{J}_2/J_2 = -2.7774 \times 10^{-6}$) and the tidal terms (i.e. with a total contribution of -235 μas/cy), the largest differences in the expressions for the equator precession relative to the non rigid and rigid Earth are (1) of -7 mas/cy^2 and -118 μas/cy^2 in the t^2 term in longitude due to the J_2 rate effect and the tidal effect, respectively, and (2) of 2.4 mas/cy in the t term in obliquity due to the tidal effect.

The large uncertainties in the theoretical models for the J_2 variations is one of the most important limiting factors in the accuracy of the precession-nutation models (Williams 1994, Capitaine et al. 2003). Bourda & Capitaine (2004) investigated how the use of the variations of J_2 observed by space geodetic techniques can influence the theoretical expressions for precession and nutation. The conclusion was that a realistic estimation of the J_2 rate should rely not only on space geodetic observations over the limited available period but also on other kinds of observations. The uncertainty in this model is such that the expected uncertainty in the t^2 term in longitude is of the order of 1.5 mas/cy^2.

(v) *The various dynamical contributions*

Table 2 provides the dynamical contributions to the equator precession described in the previous sections that have been obtained by solving the P03 equations, based on (i) the IAU 1976 and IAU 2000 integration constants, (ii) the IAU 1976 and P03 ecliptic precessions, (iii) the IAU 1976 and P03 expressions for the precession rates and (iv) the rigid Earth and non-rigid Earth models. Additional tests of dynamical consistency have been made by Capitaine et al. (2004a) by evaluating the differences with respect to a dynamical solution, denoted "P03-like", using exactly the same equations than for the P03 solution (and thus considered as having a perfect dynamical consistency), but based on ecliptics, precession rates and Earth models of the other precession solutions. Discrepancies were found that reached −3 mas/cy and −529 mas/cy2 in obliquity for the F03 model and 488 mas/cy2 in longitude for the IAU 2000 model.

Table 2: Various contributions to be applied to the IAU 2000 equator precession to make it a dynamically consistent solution, compliant with up to date models for the ecliptic and the non-rigid Earth (unit: mas).

Effect		t^2	t^3	t^4
(i) of the MHB precession rate correction (− 299.650)		0.214	0.002	
(ii) of upgrading the ecliptic to P03 ($r_{01} = r_{01}$+ 0.445)	ψ_A	0.090	0.006	0.002
(iii) of upgrading the precession rates expressions to P03		0.685		
(iv) of upgrading the Earth model to P03 (tides + $J2$ rate)		−7.130	0.003	
(i) of the MHB precession rate correction (−25.240)		−0.002	0.001	
(ii) of upgrading the ecliptic to P03 ($\epsilon_0 = \epsilon_0$−42.000)	ω_A	0.018	−0.003	
(iii) of upgrading the precession rates expressions to P03		−0.022	0.002	
(iv) of upgrading the Earth model to P03 (tides + $J2$ rate)		−0.001		

3. EFFECTS ON NUTATION

In order that the IAU 2000 nutation be compliant with the P03 precession, the effects of the change of the obliquity value at J2000 with respect to the IAU 1976 value and of the J_2 rate have to be taken into account. Improvements in the model for the precession-nutation of the equator also requires considering all the effects that may not have been included in the IAU 2000 model, such as the coupling effects appearing in the global equations for Earth's rotation.

(i) *Effects of the obliquity value at J2000 and of the J_2 rate model*

a) The MHB nutation amplitudes in longitude, $\Delta\psi_{\rm IAU2000}$, which were estimated along the IAU 1976 ecliptic, have to be transformed into amplitudes along the P03 ecliptic; similarly to the effect on the observed precession rate mentioned in Sect. 2 (i), this means multiplying $\Delta\psi_{\rm IAU2000}$ by: $\sin\epsilon_{\rm IAU1976}/\sin\epsilon_{\rm P03} = 1.000000470$. The corrections to the IAU 2000 nutation amplitudes larger than 1 μas corresponding to this change are, in μas:

$$d_1\psi = -8.1\ \sin\Omega - 0.6\ \sin(2F - 2D + 2\Omega). \qquad (4)$$

b) The nutation angles being proportional to J_2, the consideration of the same J_2 rate model than in P03 gives the following additional Poisson terms to be added to the IAU 2000 nutation:

$$d_2\psi \;=\; (\dot{J}_2/J_2)\ t\,\Delta\psi_{\rm IAU2000}\,;\ \ d_2\varepsilon = (\dot{J}_2/J_2)\ t\,\Delta\varepsilon_{\rm IAU2000}\,. \qquad (5)$$

The largest terms are of 50 μas/cy and 30 μas/cy, respectively in the 18.6-yr nutation.

(ii) *Effects of the variations in Earth rotation*

Coupling effects between the Earth's rotation rate and precession-nutation were considered to be negligible as compared to the accuracy of the observations. However, recent studies by Bretagnon et al. (2001) predicted that the Earth's rotation rate variations due to zonal tides had noticeable effects on precession-nutation with an amplitude of the order of 700 μas in the 18.6-yr nutation in obliquity and of 4 mas/cy in the precession in longitude. Although this effect was not included in IAU 2000 because it could not be detected in VLBI observations, the need of taking it into account was still questioned. This question was recently clarified by Lambert & Capitaine (2004) who showed that the contribution to precession-nutation coming from the coupling with the rotation rate variations due to zonal tides was an artefact coming from an incomplete way of taking into account the effect of the rotation rate variations; if the zonal variations are also considered in the computation of the external torque in the celestial reference system, such an effect is cancelled out and there is in fact no contribution from the coupling between axial and equatorial components of the rotation vector larger than 0.1 μas. Moreover, this effect was shown to be distinct from the contribution due to the tidal variations in the dynamical ellipticity, although it had sometimes been understood as being so, and this latter effect has to be considered. The contributions of the second-order torque induced by the variations of the dynamical ellipticity due to Earth's zonal deformations were evaluated by Lambert & Capitaine (2004) to be of the order of 200 μas and -10 μas in the 18.6-yr nutation in longitude and obliquity, respectively and -5 mas/cy in precession. However, other second order contributions of the luni-solar torque on precession-nutation should also be considered and the total effect is still under discussion (see Lambert and Escapa et al., this Volume).

4. PRECESSION PARAMETERS AND PRECESSION-NUTATION EXPRESSIONS

Suitable precession-nutation parameters would integrate the computation of bias, precession and nutation and provide a transformation between celestial and terrestrial coordinates that involves a minimum number of variables and coefficients. Moreover, as the precession-nutation solution strongly relies on quantities that are fitted to observations, the choice of the parameters that are the best suitable for being used with these observations is essential.

As VLBI observations provide the actual position of the pole in the GCRS, the precession parameters most suitable for use with VLBI observations are based on the x, y coordinates of the CIP unit vector in the GCRS which include precession, nutation, coupling between precession and nutation, and frame biases. IAU 2000A expressions for X and Y have been provided by Capitaine et al. (2003), the polynomial part of which being for the precession of the equator. The polynomial differences between the P03 and IAU 2000 expressions for X and Y are, in μas, with t expressed in Julian centuries of TT since J2000 TT:

$$dX = 155t - 2564t^2 + 2t^3 + 54t^4 \ ; \quad dY = -514t - 24t^2 + 58t^3 - 1t^4 - 1t^5. \qquad (6)$$

Then, linear fits of the P03 and IAU 2000 precessions to VLBI by Capitaine et al. (2004b) provided $dX_1 = -180t$ and $dY_1 = -70t$ for P03 and $dX_1 = -528t$ and $dY_1 = -441t$ for IAU 2000, showing that P03 fits VLBI distinctly better than IAU 2000.

Due to the strong dependence of (i) the P03 precession expressions on the precession rates values in longitude and obliquity, respectively, and (ii) the precession rate in longitude on the J_2 rate model, both of which being handicapped by large uncertainties, a P04 parameterized precession solution has been provided in Capitaine et al. (2004b) as function of these parameters, both for the equinox based and CIO based quantities. The P04 parameterized solution corresponds to expressions of the precession quantities as functions of the corrections dr_0, du_0 to the P03 precession rates r_0 and u_0, respectively, and of $\dot{J}_2/J_2$ and retains only the parameterized terms that, given the expected values for the parameters considered, can contribute to the expressions with amplitudes larger than one microarcsecond.

The parameterized P04 expressions for the X and Y quantities can be expressed as functions of the corrections dX_1 and du_0, and $\mathrm{d}\xi_0$, $\mathrm{d}\eta_0$, $\mathrm{d}(\mathrm{d}\alpha_0)$ to the IAU 2000 frame biases, as:

$$\begin{aligned} X(P04_{par}) &= X(P03) + d\xi_0 + 0.0001\, d(d\alpha_0)\, t^2 + dX_1\, t + \; 0.0203\, du_0\, t^2 \\ &\quad + [0''.002784\, t^2 - 0''.000001\, t^3] + (\dot{J}_2/J_2)\; \times (1002''.5\, t^2 - 0''.4\, t^3) \\ Y(P04_{par}) &= Y(P03) + d\eta_0 + X_1\; d(d\alpha_0)\, t + du_0\, t - 0.0224\, dX_1\, t^2 \\ &\quad - [0''.000062\, t^3] - (\dot{J}_2/J_2)\; \times (22''.5\, t^3). \end{aligned} \quad (7)$$

Such a form of the solution is intended to be used to produce (or check) future precession models based on extended VLBI records and improved geophysical models.

Other formulations are being studied, including various series of Euler angles and use of the "rotation vector" (Capitaine et al. 2003).

5. SUMMARY

In this paper we have reviewed the largest dynamical effects that were taken into account in developing the P03 expressions for the precession of the Earth's equator. We also reviewed the effects to be taken into account in the IAU 2000 nutation to make it compliant with P03. The parameterized P04 solution associated with P03 was shown to be the best form of the precession expression for use for further improvements to produce (or check) future precession models based on extended VLBI records and improved geophysical models.

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