A METHOD FOR ACCURACY AND EFFICIENCY’S INCREASE OF 
GEODETIC-ASTRONOMICAL DETERMINATION OF THE 
VERTICAL’S DEVIATION

O. BĂDESCU\textsuperscript{1,2}, P. POPESCU\textsuperscript{2}, R. POPESCU\textsuperscript{2}

\textsuperscript{1} Technical University of Civil Engineering, Faculty of Geodesy
B-dul. Lacul Tei, no.124, sect.2, Bucharest, Romania
e-mail: octavian@aira.astro.ro

\textsuperscript{2} Astronomical Institute of the Romanian Academy
Str. Cuţitul de Argint, no. 5, sect. 4, RO-040558 , Bucharest, Romania
e-mail: petre@aira.astro.ro, pradu@aira.astro.ro

ABSTRACT. In this paper an algorithm for the simultaneous and at the same time rigorous
determination of the astronomical coordinates is described: the latitude, longitude and astro-
nomical azimuth, by means of an universal device (total electronical station). We have taken as
a basis the azimuthal and zenithal angular observations for a large number of stars, uniformly
distributed on the celestial sphere, without using observation ephemerides. Through the in-
troduction of an adequate matrix of weights, the unequal weights of all direct measurements
are taken into account: the angular measurements and times at the chronometer. By applying
the theory of conditional measurements with unknowns, we obtain one rigorous algorithm for
the determination with maximum efficiency of all three fundamental elements of astronomical
geodesy. This is the typical case of different accuracy measurements, in which the weights pro-
vide uniformity to the final accuracy of the results and to their coherence. It is important to
remark that the determination of the three fundamentals elements of the astronomical geodesy,
with accuracy and rapidity, is a present problem in the context of GPS technology, as a method
of detail studying of both the geoid and the deviation of the geoid from the adopted reference ellipsoid.

1. EQUATION OF OBSERVATION AND THE PROCESSING

The azimuthal observation equation of the star, results from the star’s positional triangle, is
a conditional equation of the following form:

\[ F = \sin H \cot A + \cos \varphi \tan \delta - \sin \varphi \cos H = 0 \quad (1) \]

The unknowns are \( \varphi \), \( \lambda \), \( u \), corrections of initial values (approximate values) \( \varphi^\circ \), \( \lambda^\circ \), \( u^\circ \),
respectively the astronomical latitude, astronomical longitude and south azimuth of the zero
direction of the azimuthal circle. The observed zenithal equation of the star results from the
star’s positional triangle and it is a conditional equation too, of the following form:

\[ F = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H - \cos z = 0 \quad (2) \]
The unknowns are $d\varphi$, $d\lambda$, corrections of initial values $\varphi^\circ$, $\lambda^\circ$, respectively the astronomical latitude and astronomical longitude. In the above relations, $H$ is the hour angle of the star at the moment $t$ of azimuthal or zenithal observation. $\alpha$ and $\delta$ are the computed right ascension and declination of the star at the time of observation. A right rigorous mathematically process of both types of observations (azimuthal and zenithal) is obtained by including them into a single relations system conditional measurements with unknowns from which an unique vector $\mathbf{X}$ of the three unknowns ($d\varphi$, $d\lambda$, $du$) results, under the condition of the minimum $[PVV] = \min$. (P represents the weights of direct measurements, $V$ is the corrections of the measured values). Briefly, one star azimuthally observed, provides one conditional equation of form (1). Next, the same star zenithally observed, provides one conditional equation of form (2). It can be noticed that each observed star generates two conditional equations.

<table>
<thead>
<tr>
<th>Astronomical vertical deviation reported to WGS 84</th>
<th>Number of observations (nights)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>$\xi$</td>
<td>9.985</td>
</tr>
<tr>
<td>$s_\xi$</td>
<td>1.008</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>4.981</td>
</tr>
<tr>
<td>$s_\eta$</td>
<td>1.008</td>
</tr>
<tr>
<td>$\theta'' = (\xi^2 + \eta^2)^{\frac{1}{2}}$</td>
<td>11.158</td>
</tr>
<tr>
<td>$s_\theta$</td>
<td>1.008</td>
</tr>
</tbody>
</table>

Table 1: Notations: $\xi$ - component in meridian of the astronomical vertical deviation; $s_\xi$ - standard deviation of $\xi$; $\eta$ - component in the prime vertical of the astronomical vertical deviation; $s_\eta$ - standard deviation of $\eta$; $\theta$ - total astronomical vertical deviation; $s_\theta$ - standard deviation of $\theta$.

2. RESULTS

Table 1 gives the values of the astronomical vertical deviation referred to the WGS ellipsoid. Further, we intend to improve this method by using a CCD camera adapted to the optical system of LEICA TC2002, a GPS time receiver and eventually another set of weights. We will also try to fully automate the method by writing a software (under the Linux operating system) that provides the value of the vertical astronomical deviation in real time.

This method was tested at Bucharest Technical University of Civil Engineering, by means of a LEICA TC2002 total electronically station. The astronomical pilaster situated on the roof of the Faculty of Geodesy was stationed, and 6 nights of observations were made.

3. REFERENCES

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