Long term solutions for the insolation quantities of the Earth

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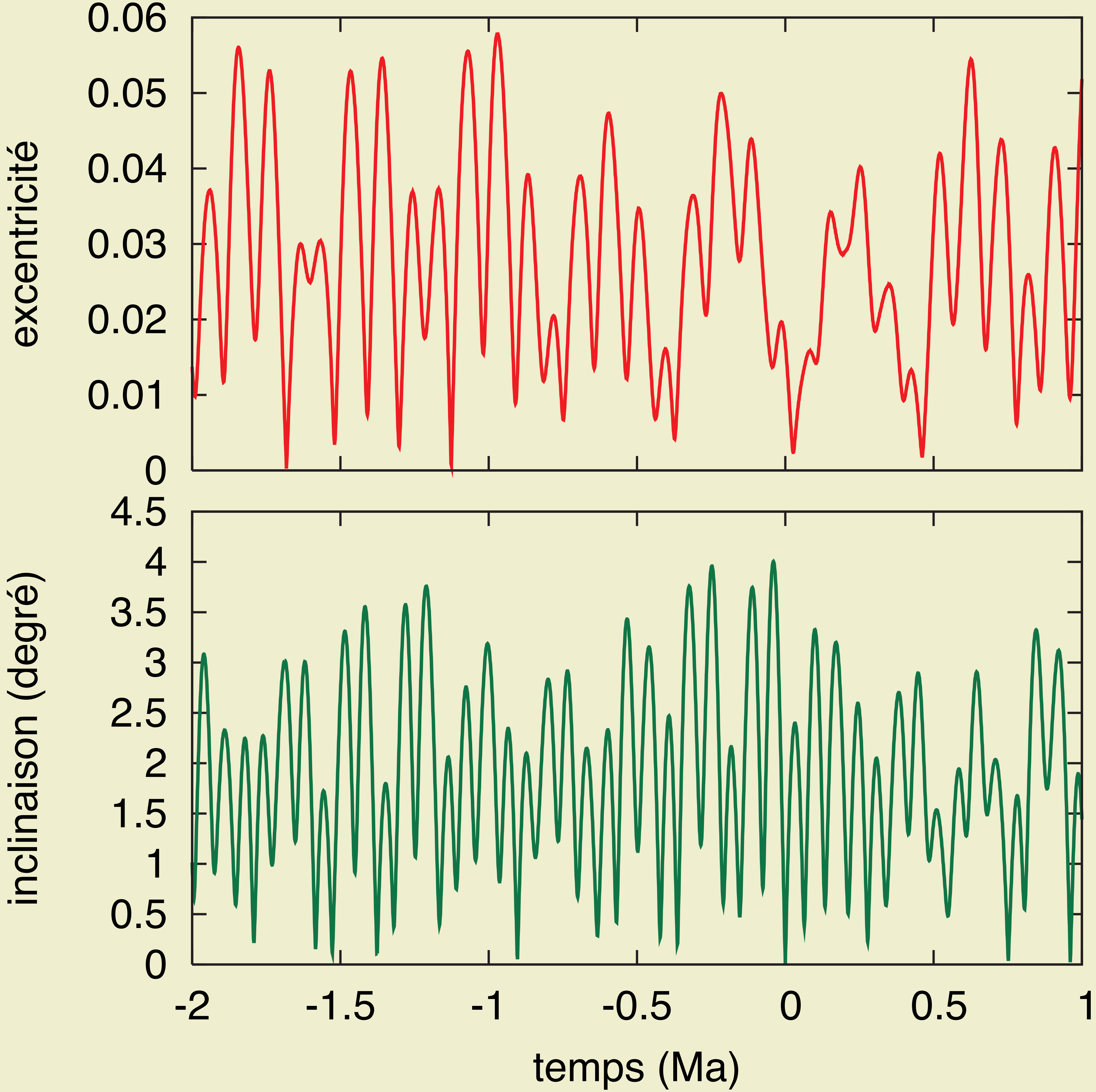
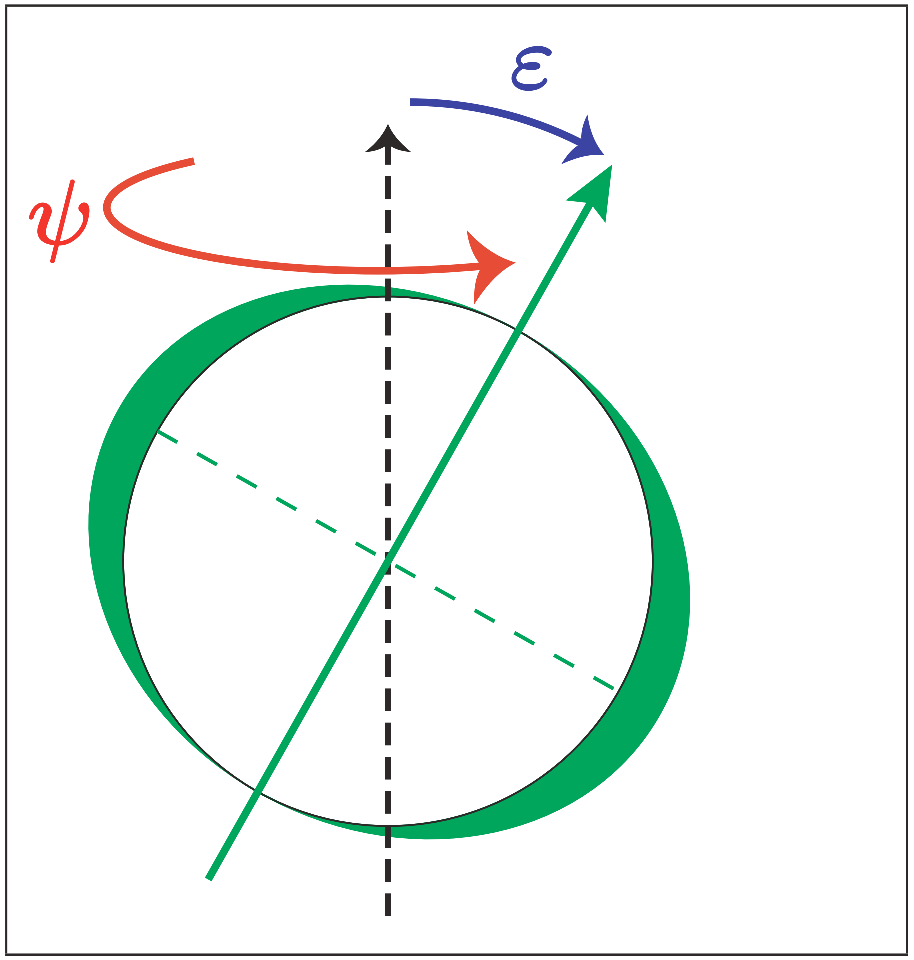
Astronomical theory of paleoclimates

- Adhémar, 1845
- Croll, 1890
- Pilgrim, 1904
- Milankovitch, 1920, 1941
- Hays, Imbrie, Shackleton, 1976


excentricité
0.06
0.05
0.04
0.03
0.02
0.01
0
inclinaison (degré)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
temps (Ma)

Precession (ψ) and obliquity $(X = \cos\varepsilon)$



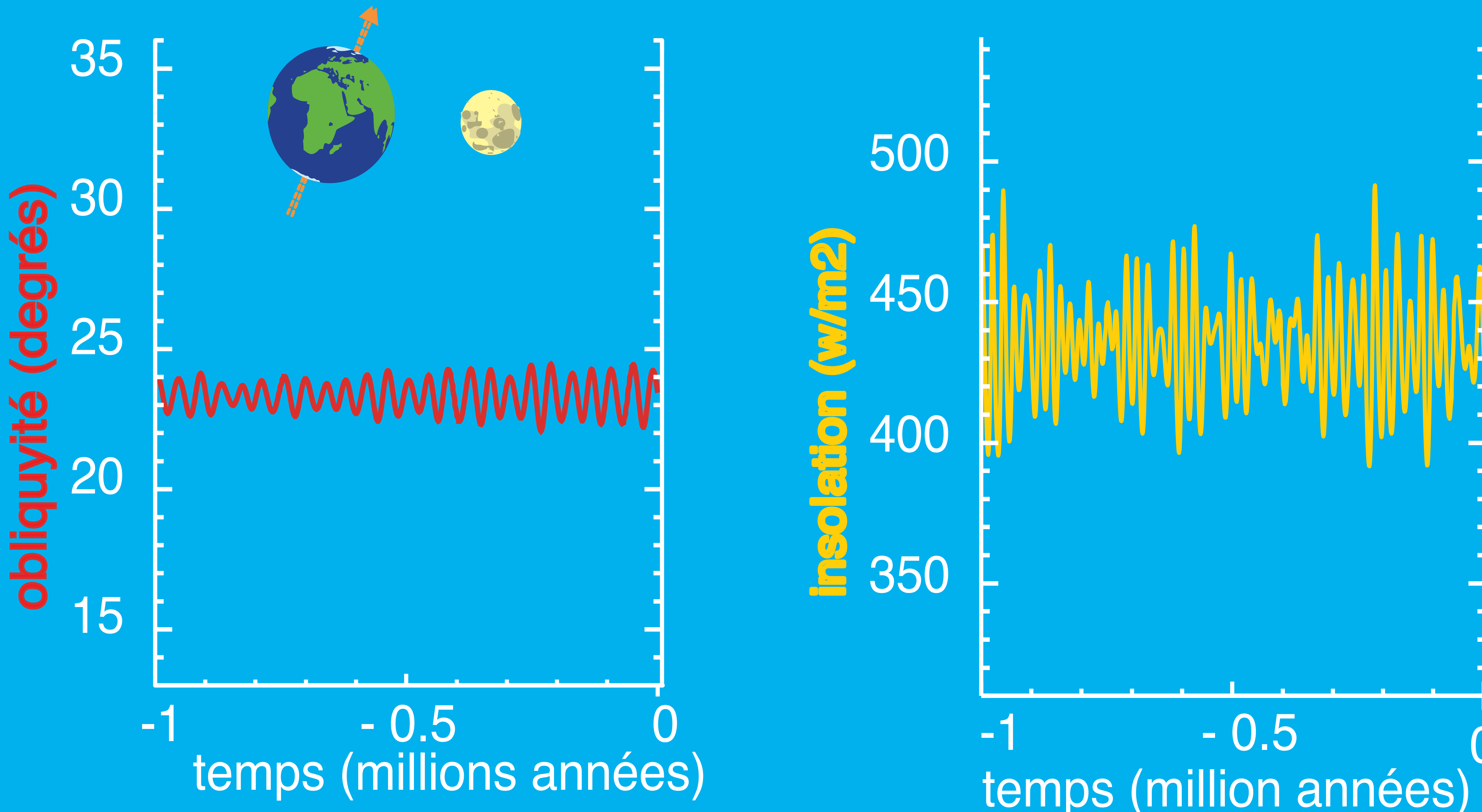
$$H = \frac{1}{2}\alpha X^2$$

$$\begin{cases} \dfrac{d\psi}{dt} = \dfrac{\partial H}{\partial X} = \alpha X_0 \\ \dfrac{dX}{dt} = -\dfrac{\partial H}{\partial \psi} = 0 \end{cases}$$

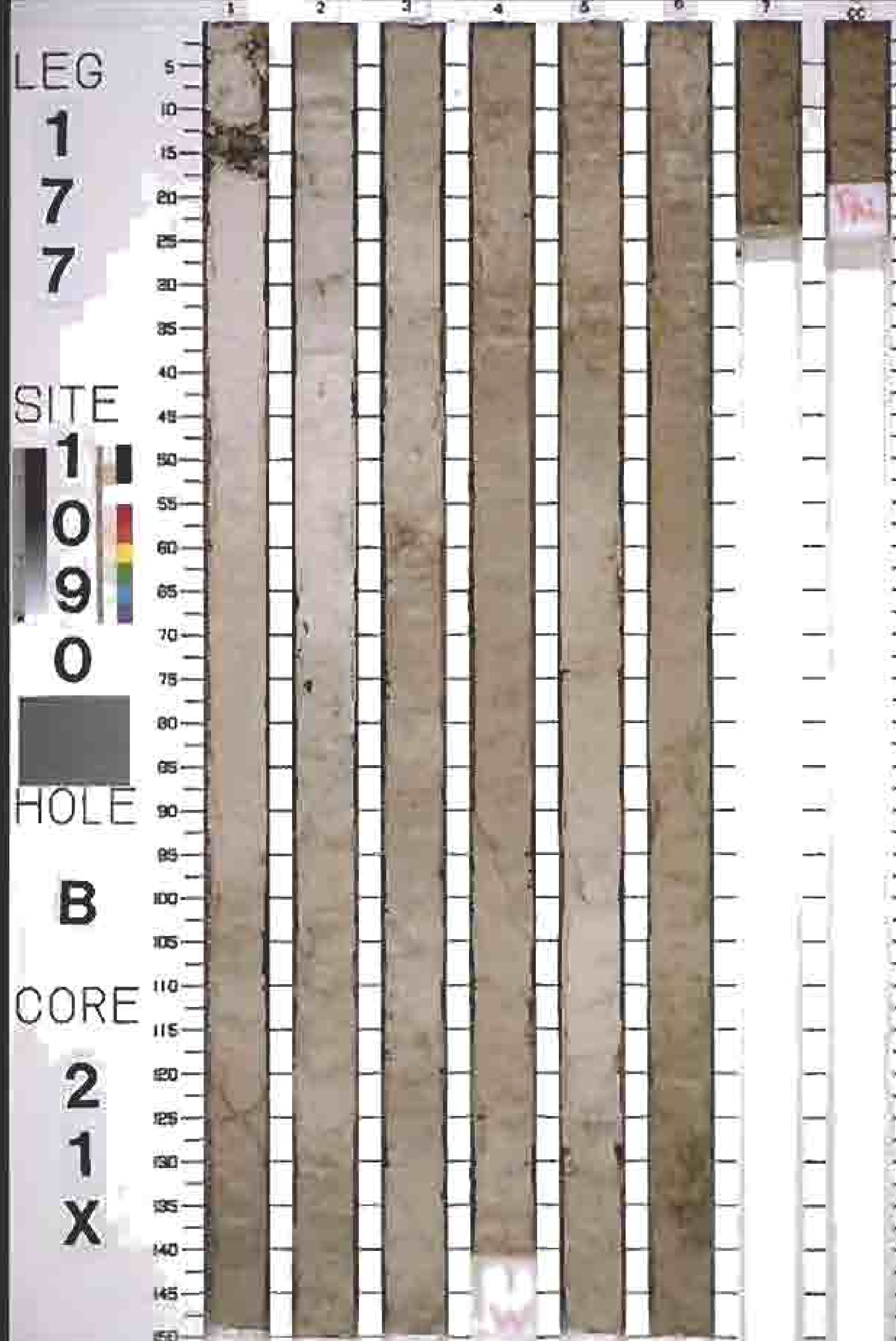
$$\alpha = \frac{3}{2}\frac{k^2}{\nu}\frac{C-A}{C}\left[\frac{m_M}{a_M^3} + \frac{m_\odot}{a_\odot^3}\right]$$

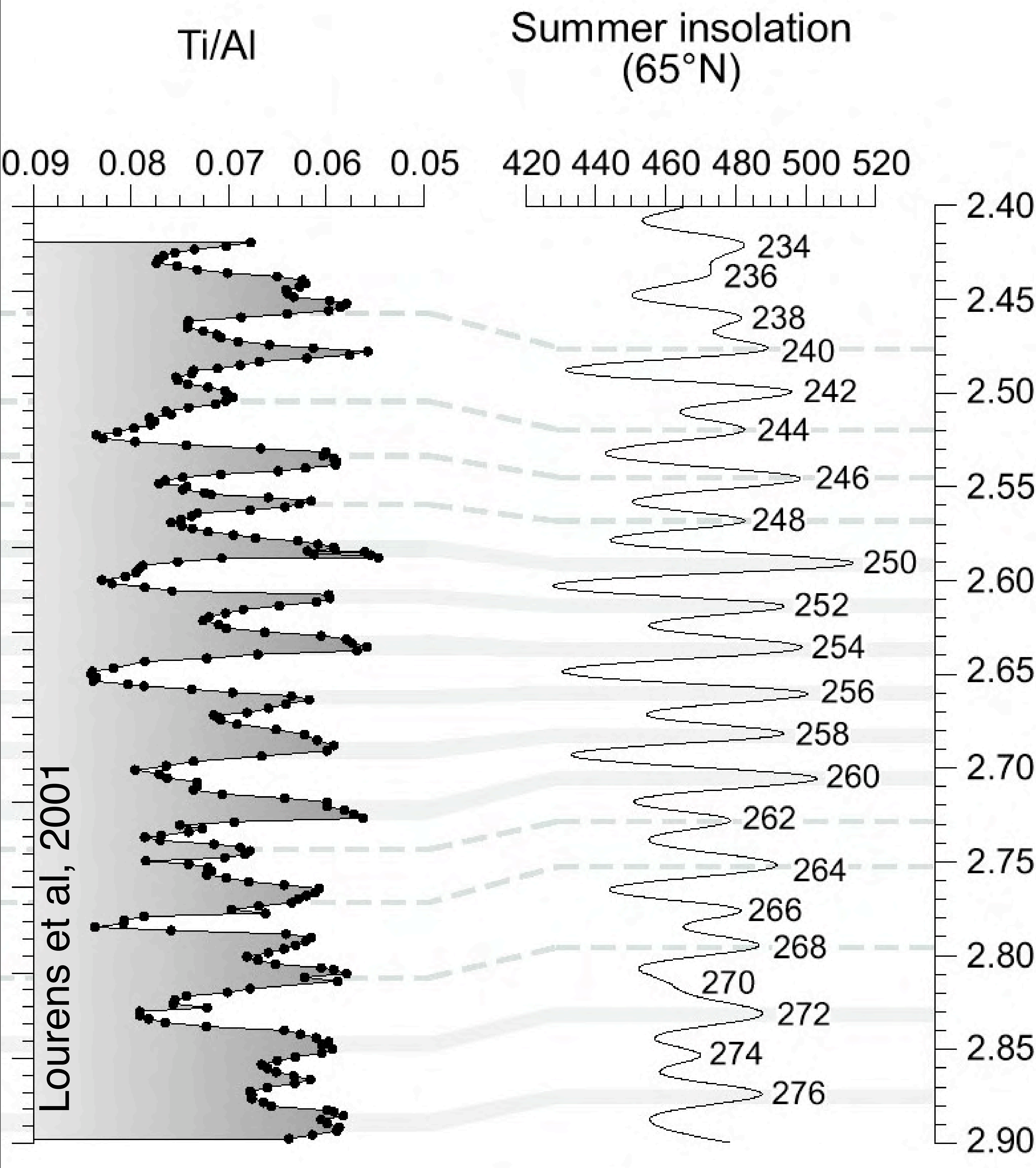
Planetary perturbations

$$H(\psi, X) = \frac{1}{2}\alpha X^2 + \sqrt{1-X^2}\sum_k \alpha_k \sin(\nu_k t + \phi_k + \psi)$$



obliquyité (degrés)
35
30
25
20
15
-1
- 0.5
0
temps (millions années)
insolation (w/m2)
500
450
400
350
-1
- 0.5
0
temps (million années)


LEG
177
SITE
1090
HOLE
B
CORE
21X
1
2
3
4
5
6
7
CC


Ti/Al
Summer insolation (65°N)
0.09 0.08 0.07 0.06 0.05
420 440 460 480 500 520
2.40 2.45 2.50 2.55 2.60 2.65 2.70 2.75 2.80 2.85 2.90
234 236 238 240 242 244 246 248 250 252 254 256 258 260 262 264 266 268 270 272 274 276
Lourens et al, 2001

INTERNATIONAL STRATIGRAPHIC CHART

ICS

IUGS

International Commission on Stratigraphy

Eonothem Eon	Erathem Era	System Period	Series Epoch	Stage Age	Age Ma	GSSP
Phanerozoic	Cenozoic	Neogene	Holocene		0.0115	
			Pleistocene	Upper	0.126	
				Middle	0.781	
				Lower	1.806	
			Pliocene	Gelasian	2.588	
				Piacenzian	3.600	
				Zanclean	5.332	
			Miocene	Messinian	7.246	
				Tortonian	11.608	
				Serravallian	13.65	
				Langhian	15.97	
				Burdigalian	20.43	
				Aquitanian	23.03	
		Paleogene	Oligocene	Chattian	28.4 ±0.1	
				Rupelian	33.9 ±0.1	
			Eocene	Priabonian	37.2 ±0.1	
				Bartonian	40.4 ±0.2	
				Lutetian	48.6 ±0.2	
				Ypresian	55.8 ±0.2	
			Paleocene	Thanetian	58.7 ±0.2	
				Selandian	61.7 ±0.2	
				Danian	65.5 ±0.3	
	Mesozoic	Cretaceous	Upper	Maastrichtian	70.6 ±0.6	
				Campanian	83.5 ±0.7	
				Santonian	85.8 ±0.7	
				Coniacian	89.3 ±1.0	
				Turonian	93.5 ±0.8	
				Cenomanian	99.6 ±0.9	
			Lower	Albian	112.0 ±1.0	
				Aptian	125.0 ±1.0	
				Barremian	130.0 ±1.5	
				Hauterivian	136.4 ±2.0	
				Valanginian	140.2 ±3.0	
				Berriasian	145.5 ±4.0	

Eonothem Eon	Erathem Era	System Period	Series Epoch	Stage Age	Age Ma	GSSP
					145.5 ±4.0	
Phanerozoic	Mesozoic	Jurassic	Upper	Tithonian	150.8 ±4.0	
				Kimmeridgian	155.7 ±4.0	
				Oxfordian	161.2 ±4.0	
			Middle	Callovian	164.7 ±4.0	
				Bathonian	167.7 ±3.5	
				Bajocian	171.6 ±3.0	
				Aalenian	175.6 ±2.0	
			Lower	Toarcian	183.0 ±1.5	
				Pliensbachian	189.6 ±1.5	
				Sinemurian	196.5 ±1.0	
				Hettangian	199.6 ±0.6	
		Triassic	Upper	Rhaetian	203.6 ±1.5	
				Norian	216.5 ±2.0	
				Carnian	228.0 ±2.0	
			Middle	Ladinian	237.0 ±2.0	
				Anisian	245.0 ±1.5	
			Lower	Olenekian	249.7 ±0.7	
				Induan	251.0 ±0.4	
	Paleozoic	Permian	Lopingian	Changhsingian	253.8 ±0.7	
				Wuchiapingian	260.4 ±0.7	
			Guadalupian	Capitanian	265.8 ±0.7	
				Wordian	268.0 ±0.7	
				Roadian	270.6 ±0.7	
			Cisuralian	Kungurian	275.6 ±0.7	
				Artinskian	284.4 ±0.7	
				Sakmarian	294.6 ±0.8	
				Asselian	299.0 ±0.8	
		Carboniferous	Pennsylvanian Upper	Gzhelian	303.9 ±0.9	
				Kasimovian	306.5 ±1.0	
			Middle	Moscovian	311.7 ±1.1	
			Lower	Bashkirian	318.1 ±1.3	
			Mississippian Upper	Serpukhovian	326.4 ±1.6	
			Middle	Visean	345.3 ±2.1	
			Lower	Tournaisian	359.2 ±2.5	

Eonothem Eon	Erathem Era	System Period	Series Epoch	Stage Age	Age Ma	GSSP
					359.2 ±2.5	
Phanerozoic	Paleozoic	Devonian	Upper	Famennian	374.5 ±2.6	
				Frasnian	385.3 ±2.6	
			Middle	Givetian	391.8 ±2.7	
				Eifelian	397.5 ±2.7	
			Lower	Emsian	407.0 ±2.8	
				Pragian	411.2 ±2.8	
				Lochkovian	416.0 ±2.8	
		Silurian	Pridoli		418.7 ±2.7	
			Ludlow	Ludfordian	421.3 ±2.6	
				Gorstian	422.9 ±2.5	
			Wenlock	Homerian	426.2 ±2.4	
				Sheinwoodian	428.2 ±2.3	
			Llandovery	Telychian	436.0 ±1.9	
				Aeronian	439.0 ±1.8	
				Rhuddanian	443.7 ±1.5	
		Ordovician	Upper	Hirnantian	445.6 ±1.5	
					455.8 ±1.6	
					460.9 ±1.6	
			Middle	Darriwilian	468.1 ±1.6	
					471.8 ±1.6	
			Lower		478.6 ±1.7	
				Tremadocian	488.3 ±1.7	
		Cambrian	Furongian			
				Paibian	501.0 ±2.0	
			Middle			
					513.0 ±2.0	
			Lower			
					542.0 ±1.0	

Eonothem Eon	Erathem Era	System Period	Age Ma	GSSP GSSA
			542	
Precambrian Proterozoic	Neo-proterozoic	Ediacaran	~630	
		Cryogenian	850	
		Tonian	1000	
	Meso-proterozoic	Stenian	1200	
		Ectasian	1400	
		Calymmian	1600	
	Paleo-proterozoic	Statherian	1800	
		Orosirian	2050	
		Rhyacian	2300	
		Siderian	2500	
Archean	Neoarchean		2800	
	Mesoarchean		3200	
	Paleoarchean		3600	
	Eoarchean	Lower limit is not defined		

Subdivisions of the global geologic record are formally defined by their lower boundary. Each unit of the Phanerozoic interval (~542 Ma to Present) and the base of the Ediacaran is defined by a Global Standard Section and Point (GSSP) at its base, whereas the Precambrian Interval is formally subdivided by absolute age, Global Standard Stratigraphic Age (GSSA).

This chart gives an overview of the international chronostratigraphic units, their rank, their names and formal status. These units are approved by the International Commission on Stratigraphy (ICS) and ratified by the International Union of Geological Sciences (IUGS).

The Guidelines of ICS (Remane et al., 1996, Episodes, 19: 77-81) regulate the selection and definition of the international units of geologic time. Many GSSP's actually have a 'golden' spike ( ) and Stage and/or System name plaque mounted at the boundary level in the boundary stratotype section, whereas a GSSA is an abstract age without reference to a specific level in a rock section on Earth. Updated descriptions of each GSSP and GSSA are posted on the ICS website (*www.stratigraphy.org*).

Some stages within the Ordovician and Cambrian will be formally named upon international agreement on their GSSP limits. Most intra-stage boundaries (e.g., Middle and Upper Aptian) are not formally defined. Numerical ages of the unit boundaries in the Phanerozoic are subject to revision. Colors are according to the United States Geological Survey (USGS). The listed numerical ages are from 'A Geologic Time Scale 2004', by F.M. Gradstein, J.G. Ogg, A.G. Smith, et al. (2004) with Cambridge University Press.

This chart was drafted and printed with funding generously provided for the GTS Project 2004 by ExxonMobil, Statoil Norway, ChevronTexaco and BP. The chart was produced by Gabi Ogg.

Eonothem Eon	Erathem Era	System Period	Series Epoch	Stage Age	Age Ma	GSSP
Phanerozoic	Cenozoic	Neogene	Holocene		0.0115	
			Pleistocene	Upper	0.126	
				Middle	0.781	
				Lower	1.806	✓
			Pliocene	Gelasian	2.588	✓
				Piacenzian	3.600	✓
				Zanclean	5.332	✓
			Miocene	Messinian	7.246	✓
				Tortonian	11.608	✓
				Serravallian	13.65	
				Langhian	15.97	
				Burdigalian	20.43	
				Aquitanian	23.03	✓
		Paleogene	Oligocene	Chattian	28.4 ±0.1	
				Rupelian	33.9 ±0.1	✓
			Eocene	Priabonian	37.2 ±0.1	
				Bartonian	40.4 ±0.2	
				Lutetian	48.6 ±0.2	
				Ypresian	55.8 ±0.2	✓
			Paleocene	Thanetian	58.7 ±0.2	
				Selandian	61.7 ±0.2	
				Danian	65.5 ±0.3	✓
				Maastrichtian		✓

Eonothem Eon	Erathem Era	System Period	Series Epoch	Stage Age	Age Ma	GSSP
					145.5 ±4.0	
Phanerozoic	Mesozoic	Jurassic	Upper	Tithonian	150.8 ±4.0	
				Kimmeridgian	155.7 ±4.0	
				Oxfordian	161.2 ±4.0	
			Middle	Callovian	164.7 ±4.0	
				Bathonian	167.7 ±3.5	
				Bajocian	171.6 ±3.0	✓
				Aalenian	175.6 ±2.0	✓
			Lower	Toarcian	183.0 ±1.5	
				Pliensbachian	189.6 ±1.5	✓
				Sinemurian	196.5 ±1.0	✓
				Hettangian	199.6 ±0.6	
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				Norian	216.5 ±2.0	
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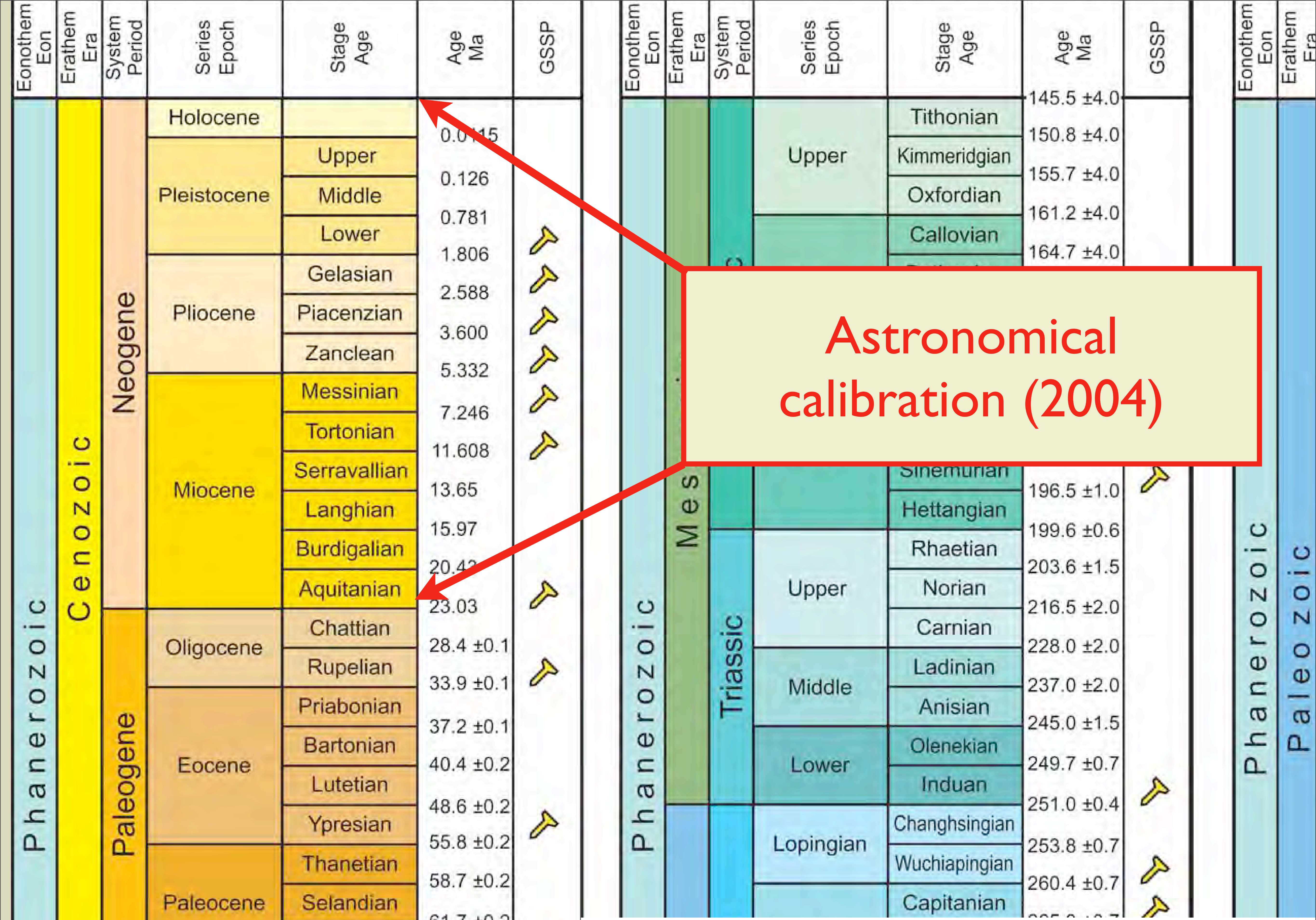
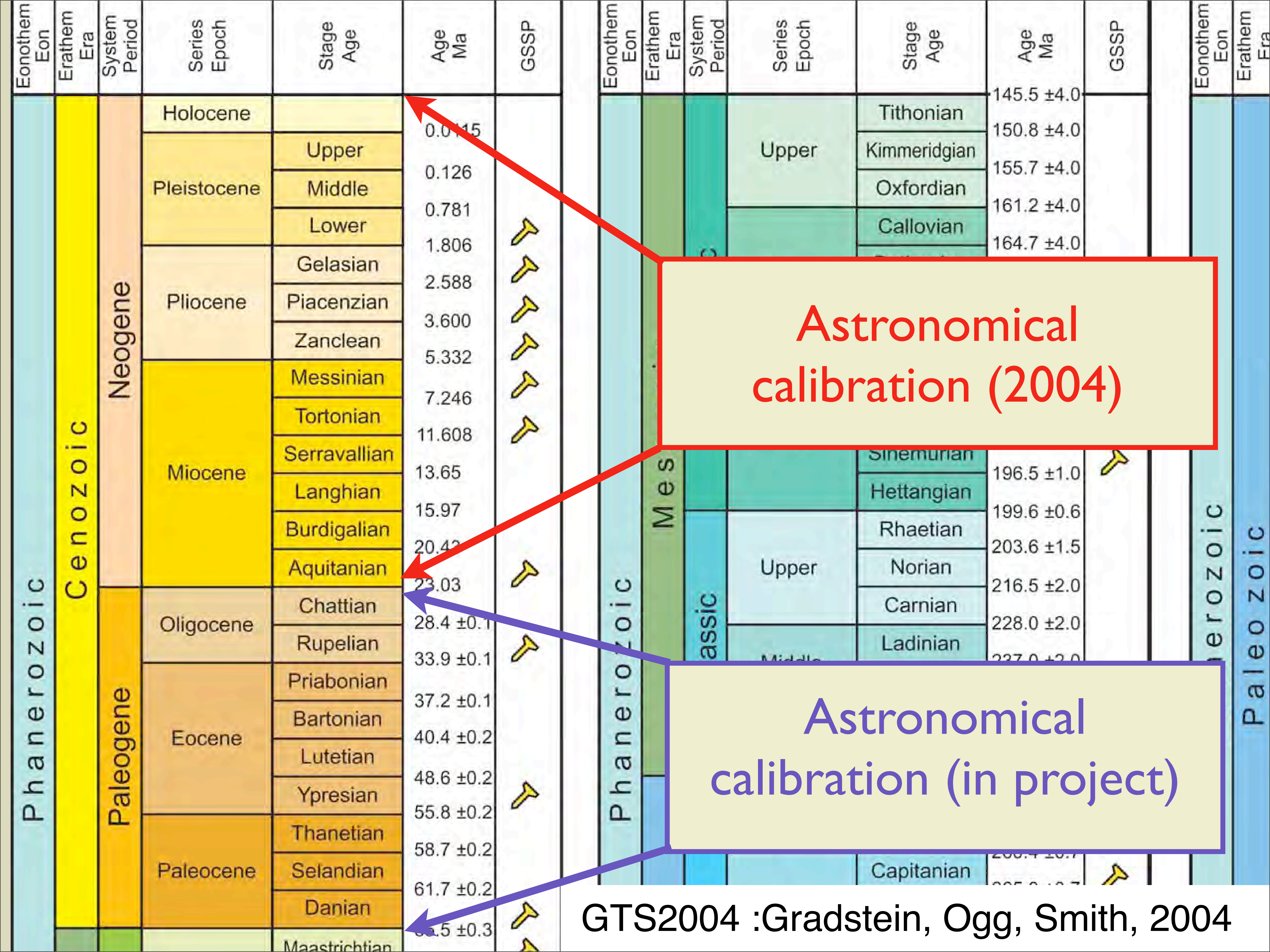
			Middle	Ladinian	237.0 ±2.0	
				Anisian	245.0 ±1.5	
			Lower	Olenekian	249.7 ±0.7	
				Induan	251.0 ±0.4	✓
			Lopingian	Changhsingian	253.8 ±0.7	
				Wuchiapingian	260.4 ±0.7	✓
				Capitanian		✓

Eonothem Eon	Erathem Era
Phanerozoic	Paleozoic

GTS2004 :Gradstein, Ogg, Smith, 2004



GTS2004 :Gradstein, Ogg, Smith, 2004



GTS2004 :Gradstein, Ogg, Smith, 2004

Insolation solutions for the Earth

analytical

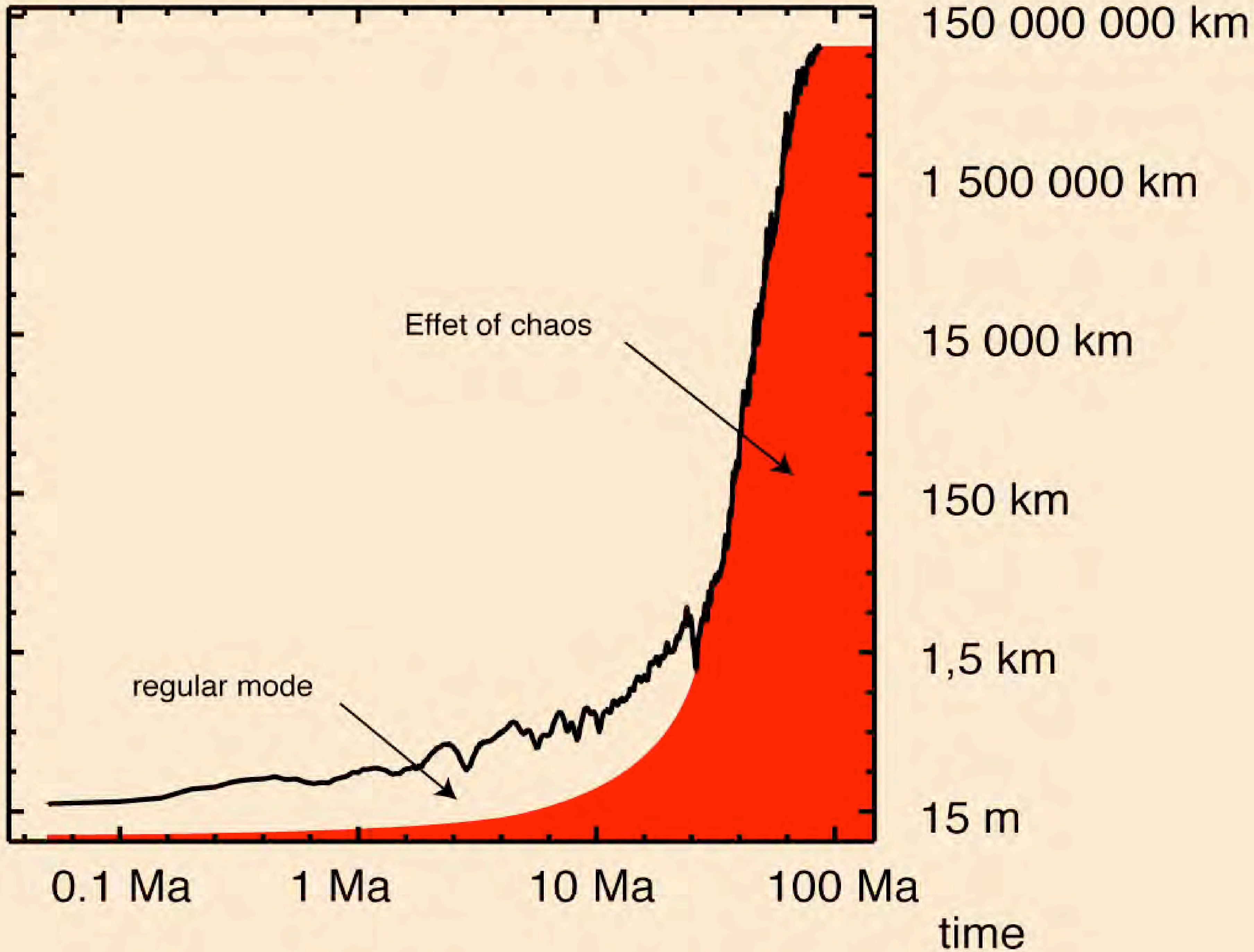
numerical

Orbital Solution	Precession equations
Le Verrier, 1856	Pilgrim, 1904 Milankovitch, 1920, 1941
+Hill, 1897 Brouwer & Van Woerkom, 1950	Sharaf & Budnikova, 1967 Vernekar, 1972
Bretagnon, 1974	Berger, 1978
Laskar, 1988, 1990	Laskar et al, 1993 (-20 Ma -> + 10 Ma)
Quinn, Tremaine, Duncan, 1991 (-3Ma -> + 3Ma)	
Laskar et al, 2004 (-250 Ma -> + 250 Ma orb: ~ 40 Ma pre : ~ 20 Ma)	

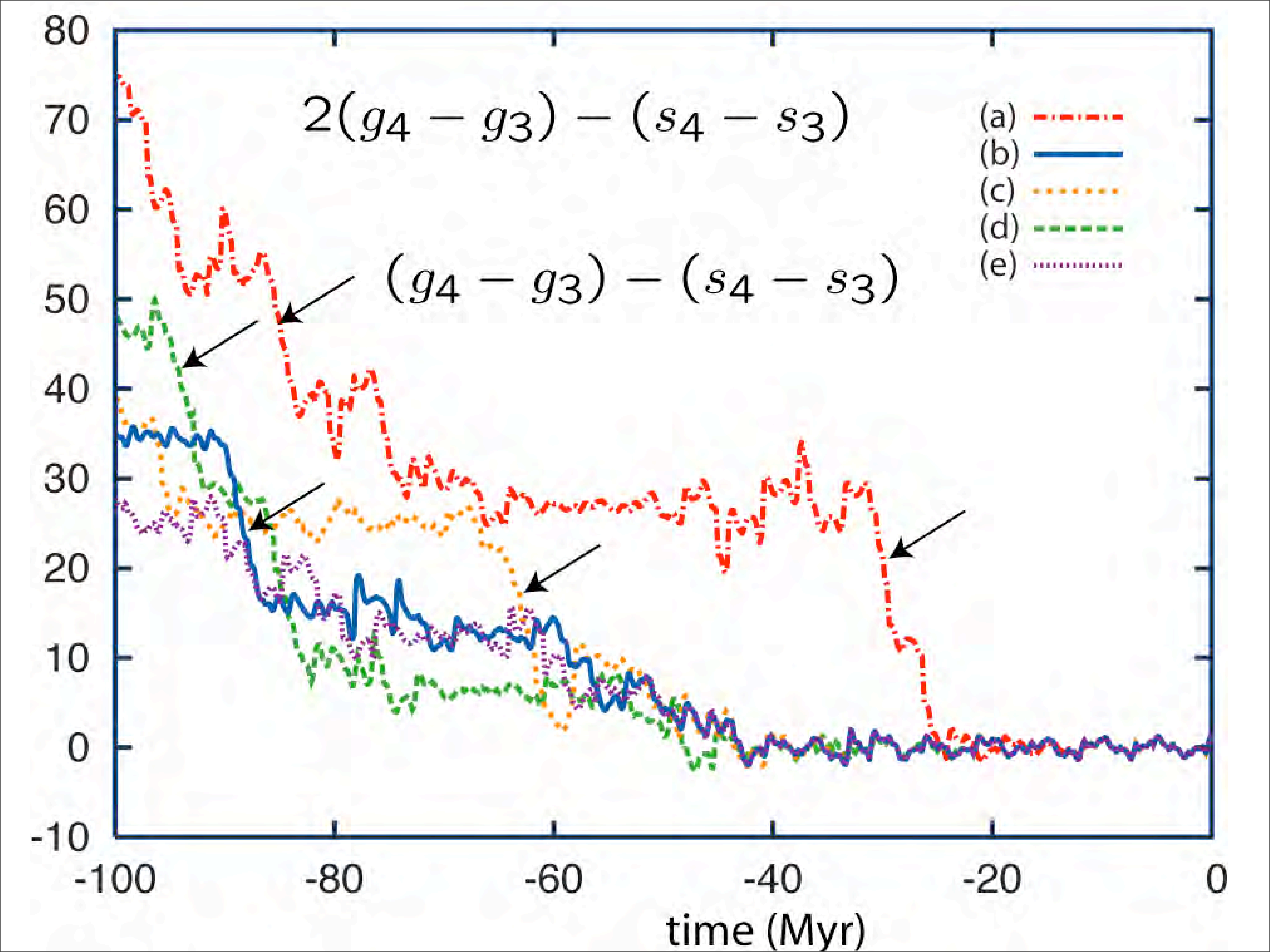
Chaotic motion of the Solar System

Secular equations : 200 Ma : Laskar (1989,1990)

Direct integration : 100 Ma : Sussman and Wisdom (1992)



$$d(T) \approx d_0 10^{T/10}$$



80
70
60
50
40
30
20
10
0
-10
-100
-80
-60
-40
-20
0
time (Myr)
$2(g_4 - g_3) - (s_4 - s_3)$
$(g_4 - g_3) - (s_4 - s_3)$
(a)
(b)
(c)
(d)
(e)

New Challenge :
Orbital solution over ~ 60 Myr

↓

Improve the accuracy
by
2 orders of magnitude !

Development of a New planetary ephemeris

INPOP

(see Agnès Fienga’s presentation)

Interactions in La2004, DE405, INPOP

- Newtonian (planets ↔ planets, planets ↔ asteroïds (300), asteroïds ↔ asteroïds (5))
- Relativistic corrections (planets, asteroïds)
- Non-spherical body ↔ point mass
- Sun (J_2) ↔ Planets
- Earth (J2)↔ (Moon, Sun, Venus, Jupiter)
- Moon (J2, C21, C22, S21, S22) ↔ (Earth, Sun, Venus, Jupiter)
- Deformation of extended bodies (tides) ↔ point mass
- Earth (Sun, Moon) ↔ (Moon, Sun, Venus, Jupiter)
- Moon (Spin,Earth, Sun) ↔ (Earth, Sun, Venus, Jupiter)
- Earth Shape ↔ Moon shape (torque exerted by the Moon)

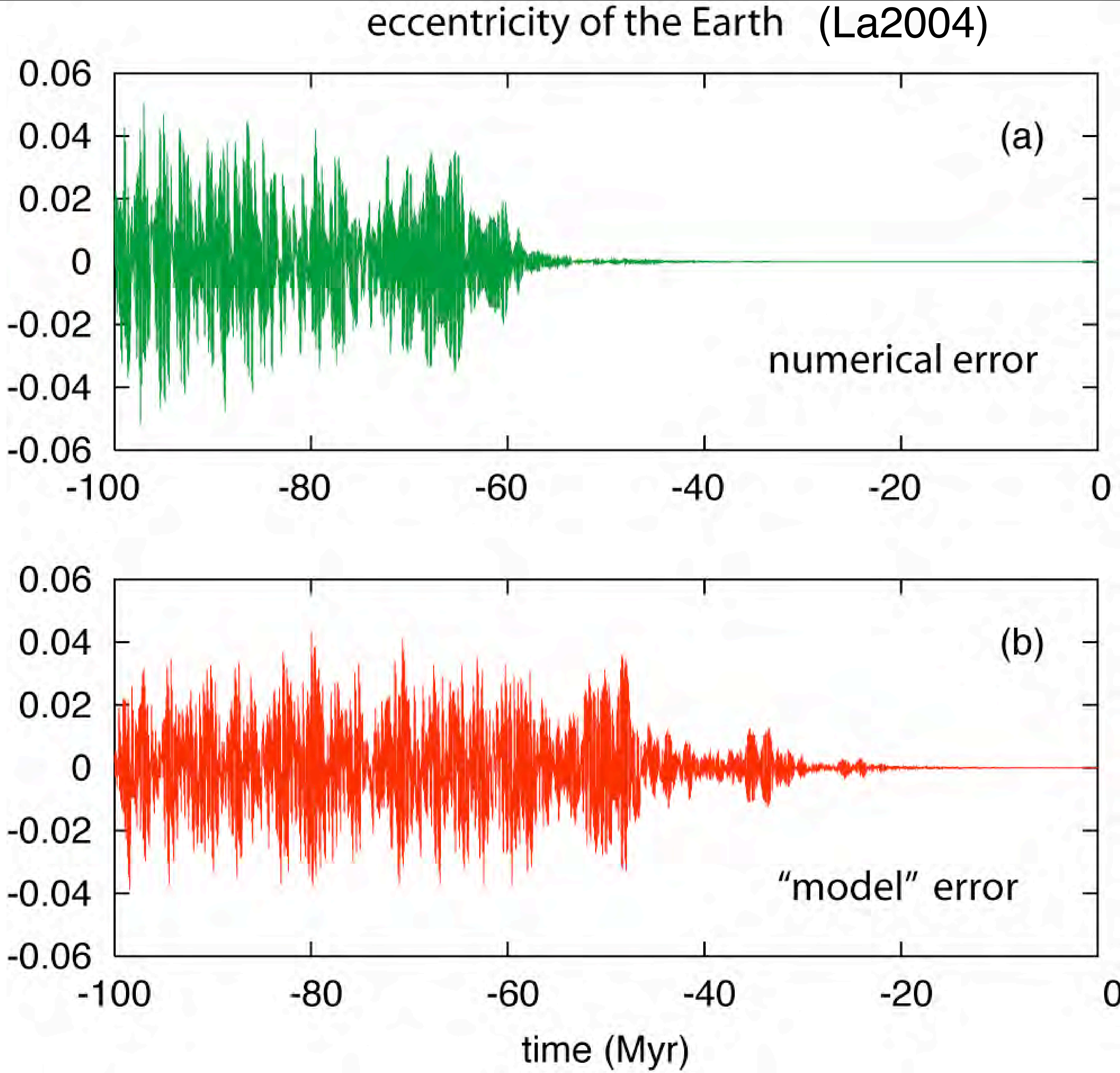
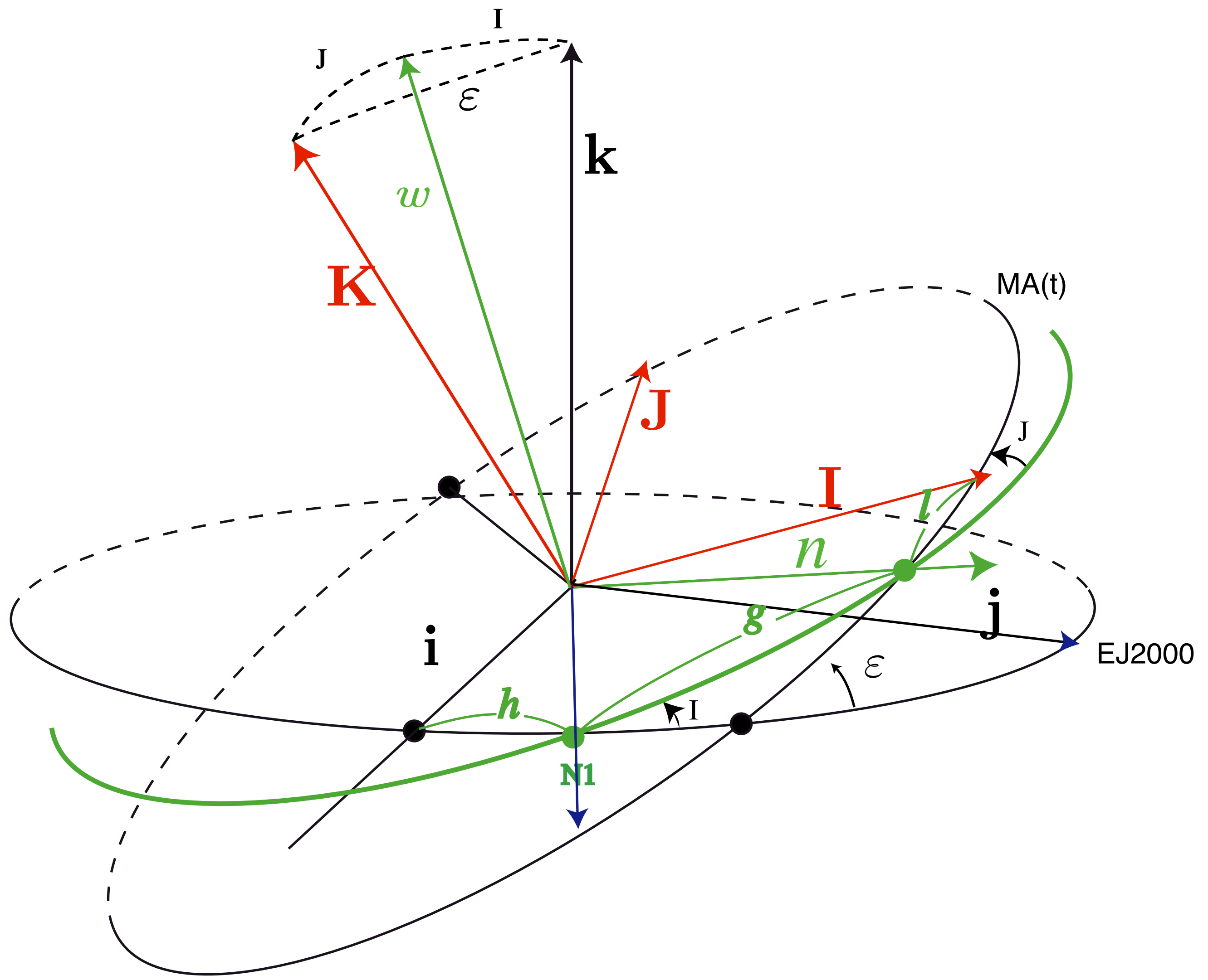

eccentricity of the Earth (La2004)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-100
-80
-60
-40
-20
0
(a)
numerical error
(b)
"model" error
time (Myr)

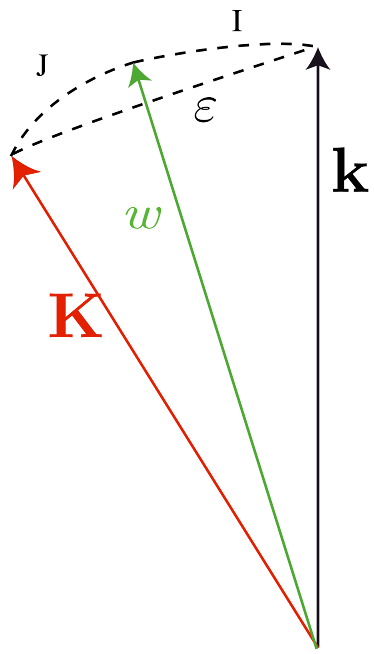
Table 2. Maximum difference between La2004 and DE406 over the whole time interval of DE406 (−5000 yr to +1000 yr with origin at J2000); Col. 1: −100 to +100 yr; Col. 2:−1000 to +1000 yr; Col. 3: −5000 to +1000 yr. EMB is the Earth–Moon barycenter.

	a (UA $\times 10^{10}$)		(15 m)
Mercury	4	43	58
Venus	13	23	27
EMB	29	62	62
Mars	27	76	76
Jupiter	268	470	565
Saturn	743	1073	1168
Uranus	1608	3379	3552
Neptune	3315	5786	6458
Pluto	4251	10 603	10 603
Moon	23	204	520
Earth	487	3918	10 238

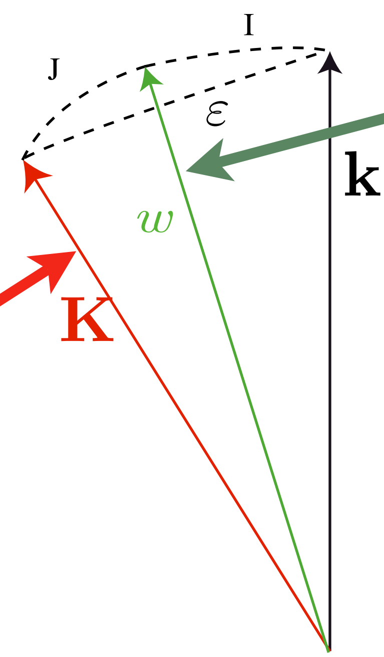
Remarks on Precession Equations



Remarks on Precession Equations



Remarks on Precession Equations



angular momentum

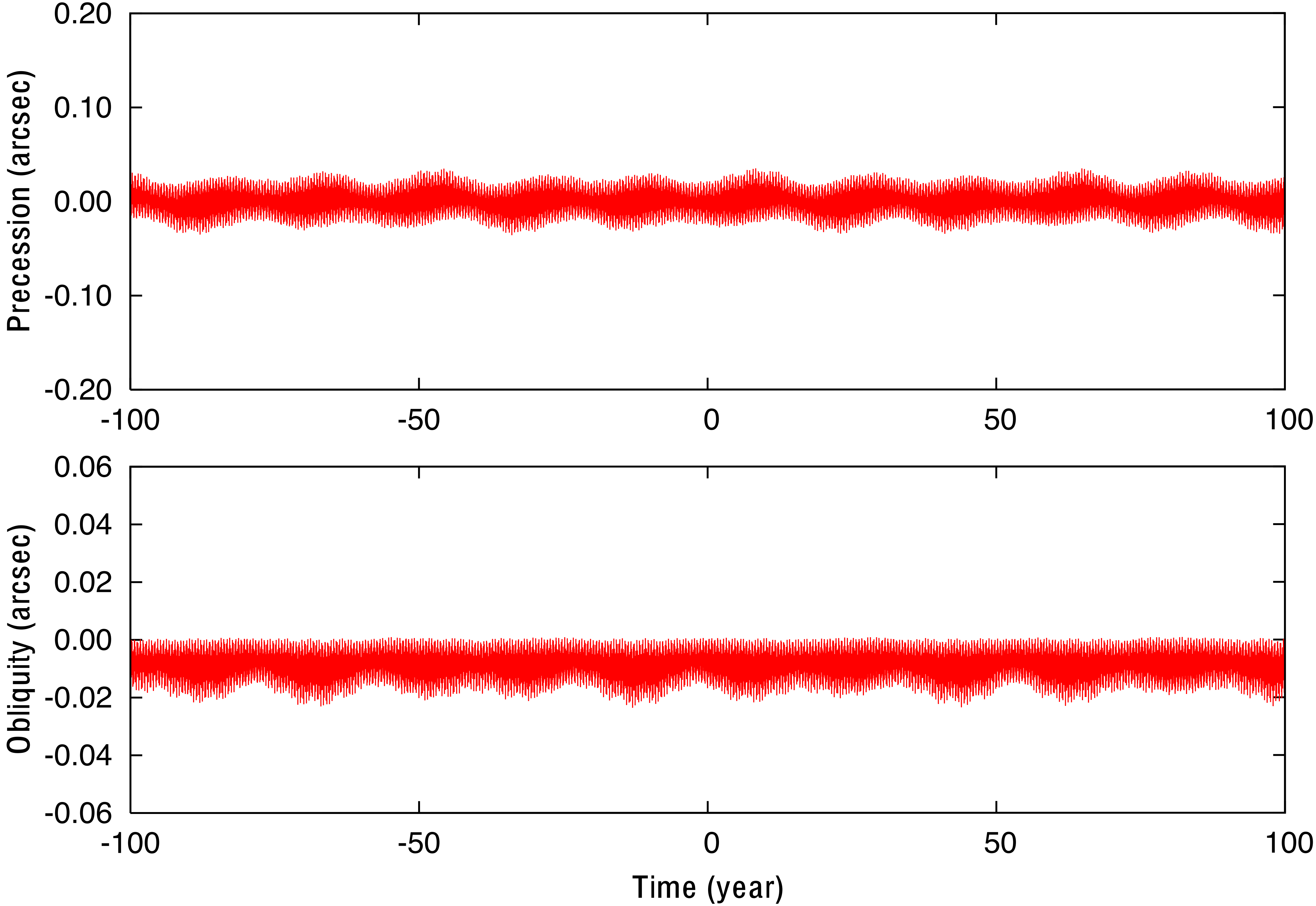
axis of figure

$$\dot{K} = \omega \frac{C}{A} w \wedge K$$

nearly diurnal motion around w

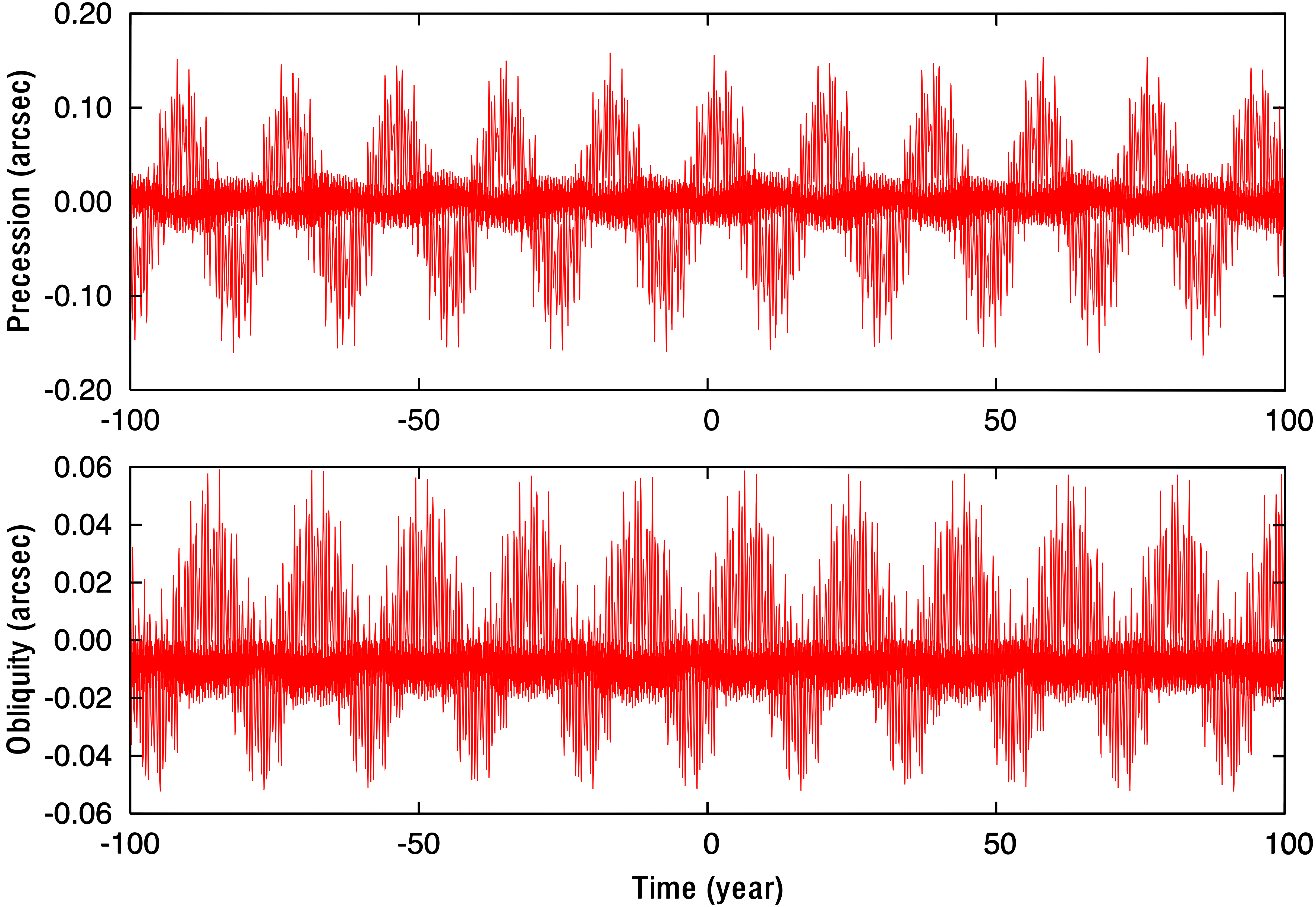
$$< K > = \cos J \; w = w + O(J^2) \qquad J < 0.2''$$

INPOP K - w

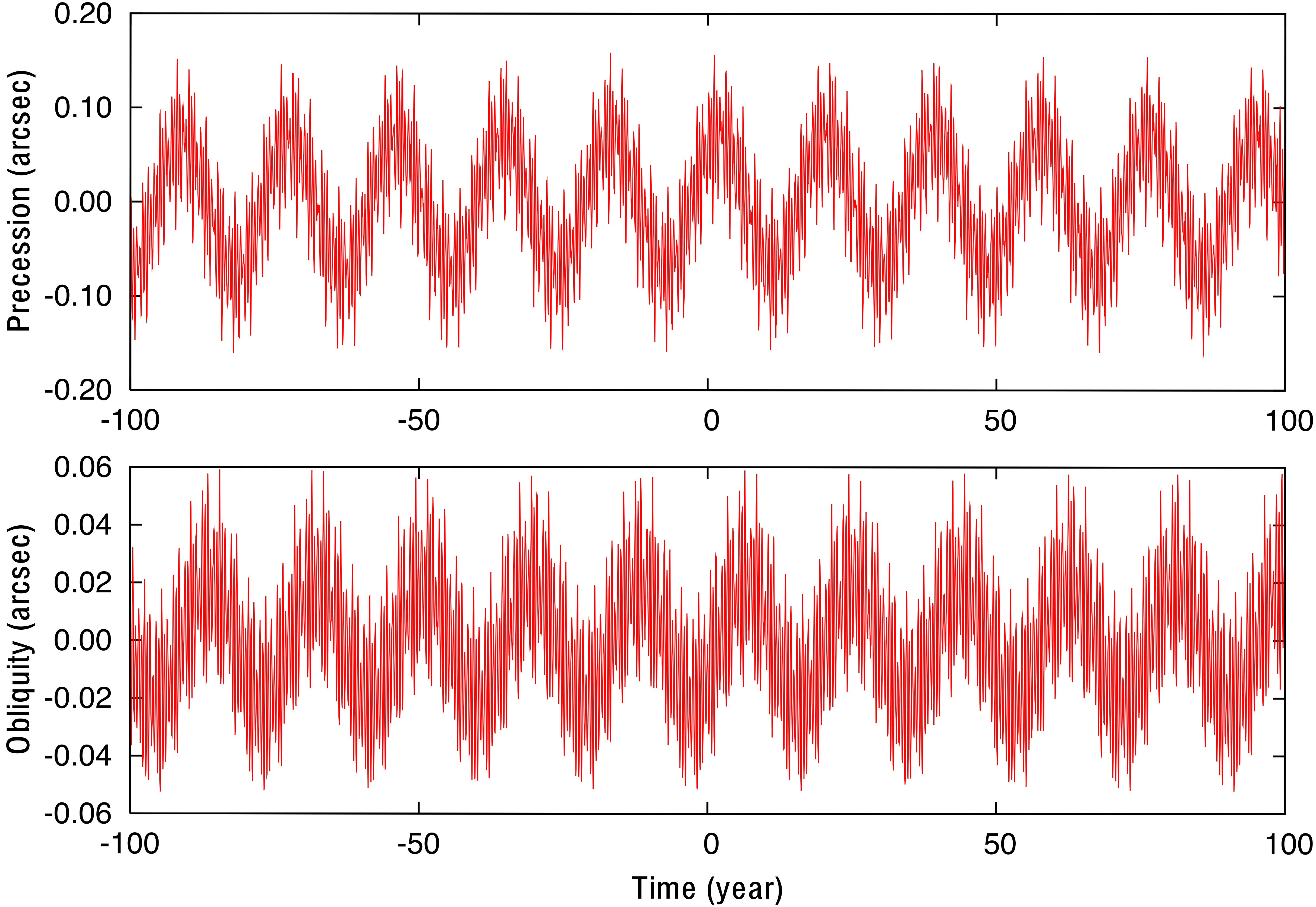


Precession (arcsec)
0.20
0.10
0.00
-0.10
-0.20
-100
-50
0
50
100
Obliquity (arcsec)
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-100
-50
0
50
100
Time (year)

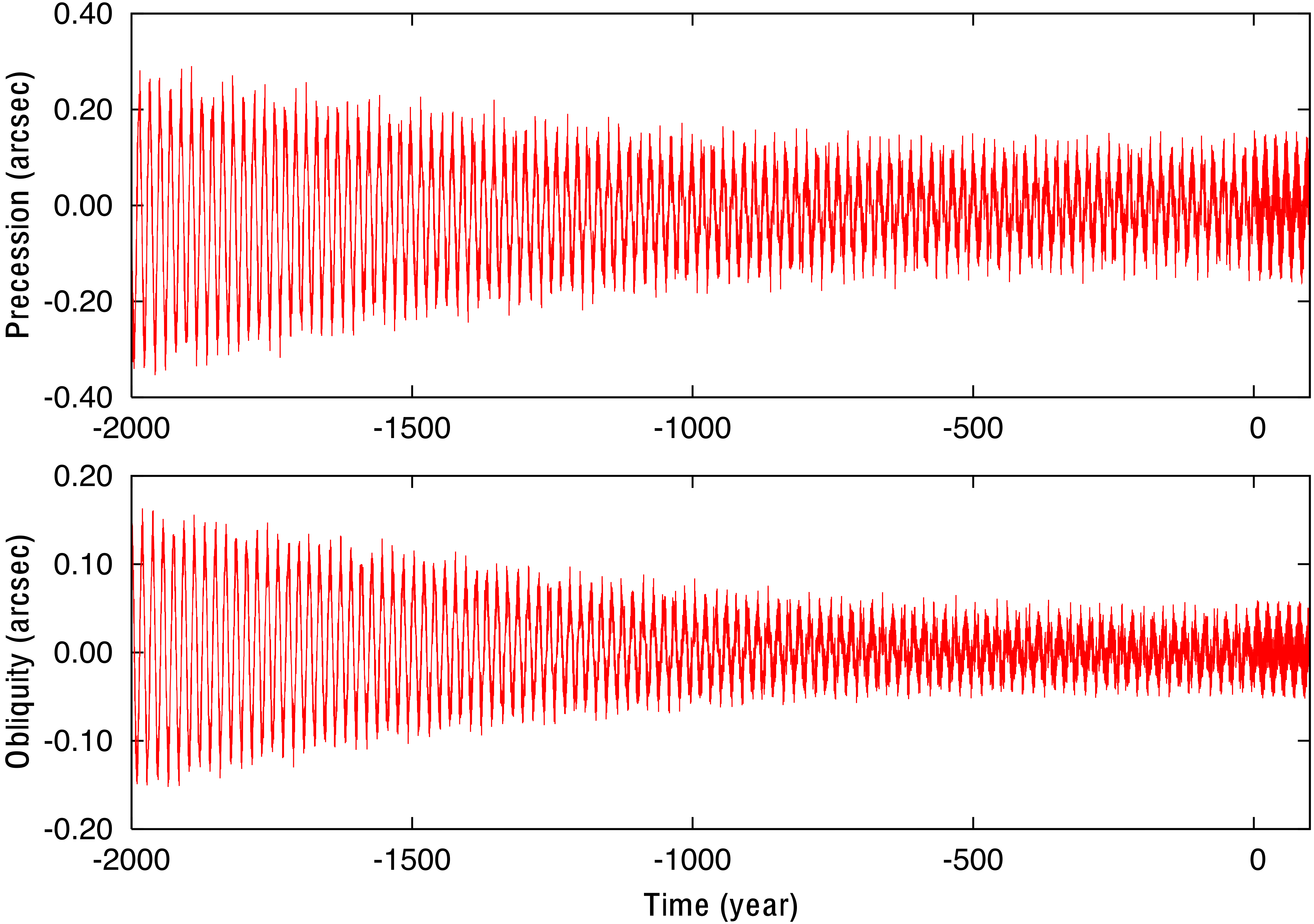
CIP (P03) - INPOP (w)



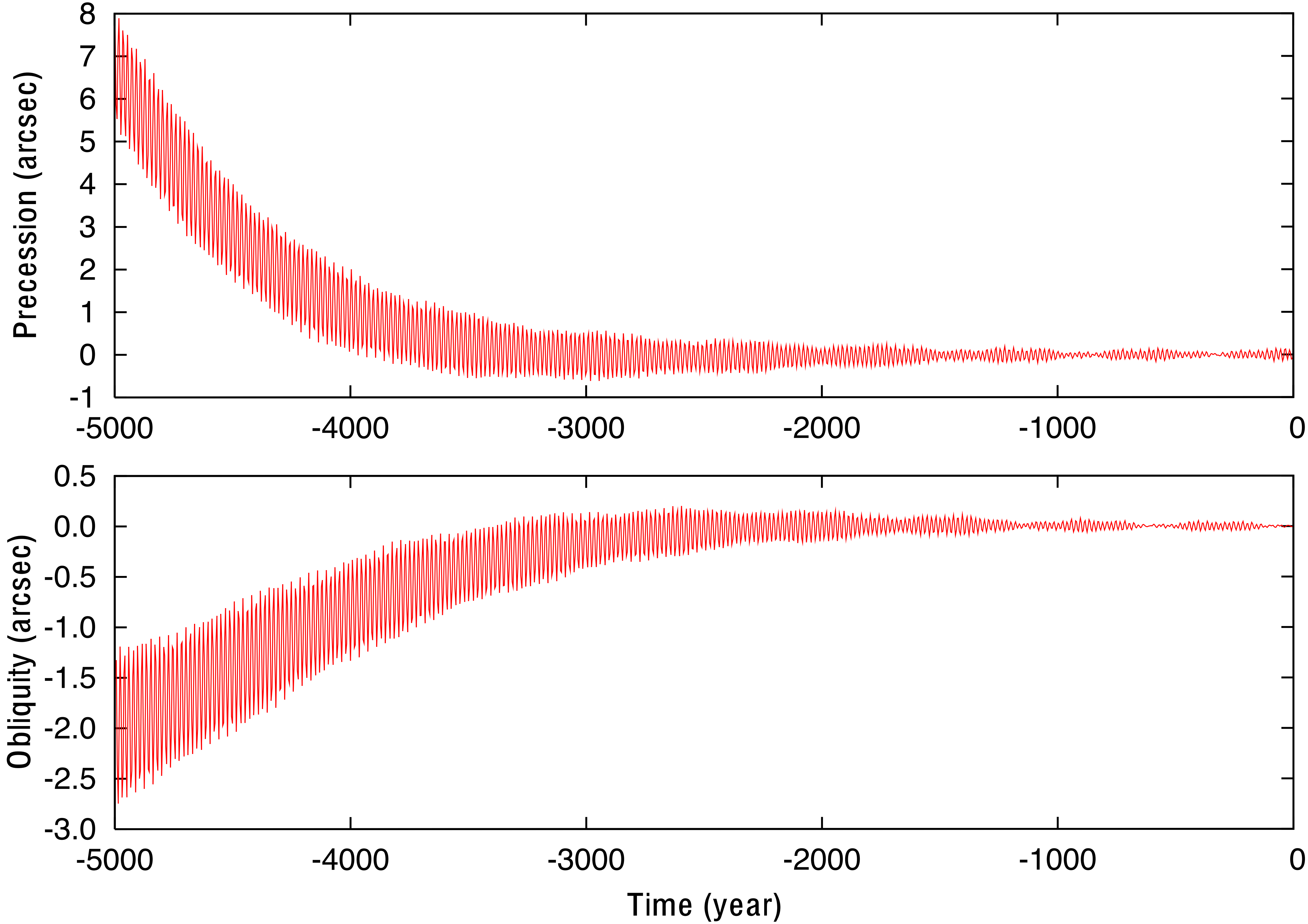
CIP (P03) - INPOP (w)



CIP (P03) - INPOP06 (same $\dot{J}_2$)



CIP (P03) - INPOP06 (same $\dot{J}_2$)



Integration of the angular momentum

- Averaging over the diurnal motion (Boué and Laskar, 2006)

- Takes into account the tidal deformation of the Earth, or post glacial deformation

- Do not depend on the internal model of the Earth

- The precession motions of K and w are the same

Suggestion : Any rotational solution for the Earth should provide also its angular momentum solution !