# TDB or TCB: does it make a difference?

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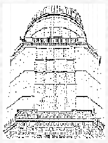

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## Relativistic Time Scales: TCB and TCG

- $t = TCB$ Barycentric Coordinate Time = coordinate time of the BCRS
- $T = TCG$ Geocentric Coordinate Time = coordinate time of the GCRS

These are part of 4-dimensional coordinate systems so that the TCB-TCG transformations are 4-dimensional: $( r_E^i = x^i - x_E^i(t) )$

$$T = t - \frac{1}{c^2}\left(A(t) + v_E^i r_E^i\right) + \frac{1}{c^4}\left(B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t,\boldsymbol{x})\right) + O\left(c^{-5}\right)$$

- Therefore: $TCG = TCG(TCB, x^i)$
- Only if space-time position is fixed in the BCRS $x^i = x_{obs}^i(t)$ TCG becomes a function of TCB:

$$TCG = TCG(TCB, x_{obs}^i(TCB)) = TCG(TCB)$$

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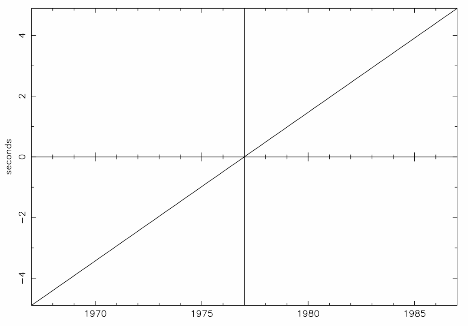
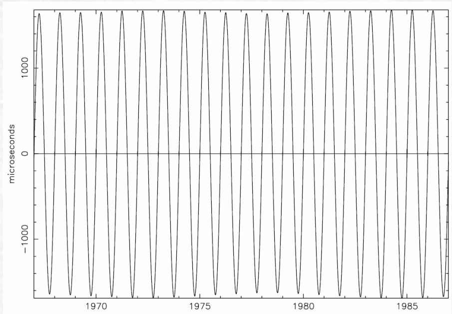
# Relativistic Time Scales: TCB and TCG

- Important special case $x^i = x^i_E(t)$ gives the TCG-TCB relation at the geocenter:




linear drift removed:

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# Relativistic Time Scales: proper time scales

- $\tau$ proper time of each observer: what an ideal clock moving with the observer measures…

- Proper time can be related to either TCB or TCG (or both) provided that the trajectory of the observer is given:

$$x^i_{obs}(t) \quad \text{and/or} \quad X^a_{obs}(T)$$

The formulas are provided by the relativity theory:

$$\frac{d\tau}{dt} = \left( -g_{00}\left(t, \mathbf{x}_{obs}(t)\right) - \frac{2}{c} g_{0i}\left(t, \mathbf{x}_{obs}(t)\right) \dot{x}^i_{obs}(t) - \frac{1}{c^2} g_{ij}\left(t, \mathbf{x}_{obs}(t)\right) \dot{x}^i_{obs}(t)\, \dot{x}^j_{obs}(t) \right)^{1/2}$$

$$\frac{d\tau}{dT} = \left( -G_{00}\left(T, \mathbf{X}_{obs}(T)\right) - \frac{2}{c} G_{0a}\left(T, \mathbf{X}_{obs}(T)\right) \dot{X}^a_{obs}(T) - \frac{1}{c^2} G_{ab}\left(T, \mathbf{X}_{obs}(T)\right) \dot{X}^a_{obs}(T)\, \dot{X}^b_{obs}(T) \right)^{1/2}$$

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## Relativistic Time Scales: proper time scales

- Specially interesting case: an observer close to the Earth surface:

$$\frac{d\tau}{dT} = 1 - \frac{1}{c^2}\left(\frac{1}{2}\dot{X}^2_{obs}(T) + W_E\left(T, \mathbf{X}_{obs}\right) + \text{"tidal terms"}\right) + O\left(c^{-4}\right)$$

$\approx 10^{-17}$

- Idea: let us define a time scale linearly related to T=TCG, but which is numerically close to the proper time of an observer on the geoid:

$$TT = (1 - L_G)\, TCG, \quad L_G \equiv 6.969290134 \times 10^{-10}$$

$$\frac{d\tau}{d\,TT} = 1 - \frac{1}{c^2}\left(\text{"terms } \sim h, v^i\text{"} + \text{"tidal terms"} + \dots\right) + \dots$$

can be neglected in many cases

$h$ is the height above the geoid

$v^i$ is the velocity relative to the rotating geoid

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## Relativistic Time Scales: TDB-1

- Idea: to scale TCB in such a way that the “scaled TCB” remains close to TT

- IAU 1976: TDB is a time scale for the use for dynamical modelling of the Solar system motion which differs from TT only by periodic terms.

- This definition taken literally is flawed:
  such a TDB cannot be a linear function of TCB!

But the relativistic dynamical model (EIH equations) used by e.g. JPL is valid only with TCB and linear functions of TCB…

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# Relativistic Time Scales: $T_{eph}$

- Since the original TDB definition has been recognized to be flawed Myles Standish (1998) introduced one more time scale $T_{eph}$ differing from TCB only by a constant offset and a constant rate:

$$T_{eph} = K \times TCB + T_{eph0}$$

- The coefficients are different for different ephemerides.
- The user has NO information on those coefficients from the ephemeris.
- The coefficients could only be restored by some additional numerical procedure (Fukushima's "Time ephemeris")

- $T_{eph}$ is de facto defined by a fixed relation to TT:

  by the Fairhead-Bretagnon formula based on VSOP-87

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# Relativistic Time Scales: TDB-2

The IAU Working Group on Nomenclature in Fundamental Astronomy suggested to re-define TDB to be a fixed linear function of TCB:

- TDB to be defined through a conventional relationship with TCB:

$$TDB = TCB - L_B \times (JD_{TCB} - T_0) \times 86400 + TDB_0$$

  - $T_0$ = 2443144.5003725 exactly,
  - $JD_{TCB} = T_0$ for the event 1977 Jan 1.0 TAI at the geocenter and increases by 1.0 for each 86400s of TCB,
  - $L_B \equiv 1.550519768 \times 10^{-8}$,
  - $TDB_0 \equiv -6.55 \times 10^{-5}$ s.

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# Scaled BCRS: not only time is scaled

- If one uses scaled version TCB – $T_{eph}$ or TDB – one effectively uses three scaling:

  - time $t^* = K \cdot TCB + t_0^*$
  - spatial coordinates $\mathbf{x}^* = K \cdot \mathbf{x}$
  - masses ($\mu$= GM) of each body $\mu^* = K \cdot \mu$

$$K = 1 - L_B$$

**WHY THREE SCALINGS?**

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# Scaled BCRS

- These three scalings together leave the dynamical equations unchanged:

  - for the motion of the solar system bodies:

(first published in 1917!)

$$\ddot{x}_A = -\sum_{B\neq A} \mu_B \frac{r_{AB}}{|r_{AB}|^3}$$
$$+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{r_{AB}}{|r_{AB}|^3}\left\{\sum_{C\neq B}\frac{\mu_C}{|r_{BC}|}+4\sum_{C\neq A}\frac{\mu_C}{|r_{AC}|}+\frac{3}{2}\frac{(r_{AB}\cdot\dot{x}_B)^2}{|r_{AB}|^2}\right.$$
$$-\frac{1}{2}\sum_{C\neq A,B}\mu_C\frac{r_{AB}\cdot r_{BC}}{|r_{BC}|^3}$$
$$\left.-2\,\dot{x}_B\cdot\dot{x}_B-\dot{x}_A\cdot\dot{x}_A+4\,\dot{x}_A\cdot\dot{x}_B\right\}$$
$$+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\dot{x}_A-\dot{x}_B}{|r_{AB}|^3}\left\{4\,\dot{x}_A\cdot r_{AB}-3\,\dot{x}_B\cdot r_{AB}\right\}$$
$$-\frac{1}{c^2}\frac{7}{2}\sum_{B\neq A}\frac{\mu_B}{|r_{AB}|}\sum_{C\neq A,B}\mu_C\frac{r_{BC}}{|r_{BC}|^3}+O(c^{-4}),$$

  - for light propagation:

$$c\,(t_2-t_1)=|x_2-x_1|$$
$$+\sum_A\frac{2\,\mu_A}{c^2}\ln\frac{|r_{1A}|+|r_{2A}|+|r_{21}|}{|r_{2A}|+|r_{1A}|-|r_{21}|}+O(c^{-4}),$$

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# TCB-based or TDB-based ephemeris?

- As soon as the redefined TDB is used there is no real difference between them!

- Once a TDB ephemeris is constructed, it is trivial to convert it to TCB and vice verse: just apply the three scalings given above!

- With fixed scaling constant $L_B$ (that is, with the re-defined TDB) it is impossible to have different post-fit residuals when using TDB and TCB.

  The fits must be absolutely equivalent!

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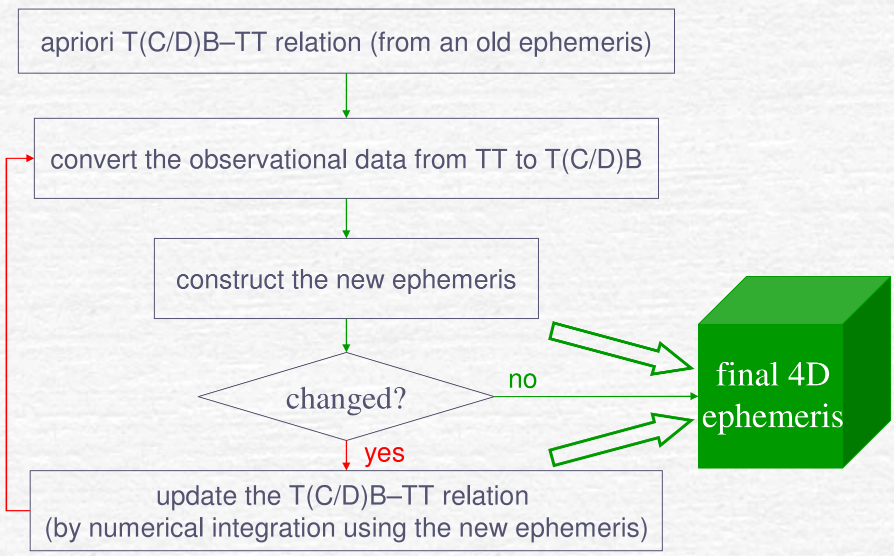
# Iterative procedure to construct ephemeris with TCB or TDB in a fully consistent way

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## Notes on the iterative procedure

- This scheme works even if the change of the ephemeris is (very) large
- The iterations are expected to converge very rapidly (after just 1 iteration)
- The time ephemeris (TCB-TCG relation) becomes a natural part of any new ephemeris of the Solar system:

  Self-consistent 4-dimensional ephemerides should be produced in the future

  Consequence of not doing it, e.g. TEMPO2 does it internally

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## Choices/questions which are still open

- "TDB-units" vs. SI units
- Astronomical units in the relativistic context
- Astronomical units in general:

  Do we need astronomical units in their current form?

  AU as a fixed value in SI meters?…

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