Relativistic Time Scales: TCB and TCG

- \( t = TCB \) Barycentric Coordinate Time = coordinate time of the BCRS
- \( T = TCG \) Geocentric Coordinate Time = coordinate time of the GCRS

These are part of 4-dimensional coordinate systems so that the TCB-TCG transformations are 4-dimensional:

\[
T = t - \frac{1}{c^2} \left( A(t) + v_E' r_E'(t) \right) + \frac{1}{c^4} \left( B(t) + B'(t) r_E'(t) + B''(t) r_E'(t) r_E''(t) + C(t, x) \right) + O(c^{-6})
\]

- Therefore: \( TCG = TCG(TCB, \dot{x}_E) \)
- Only if space-time position is fixed in the BCRS \( \dot{x} = x_{\text{obs}}(t) \) TCG becomes a function of TCB:

\[
TCG = TCG(TCB, x_{\text{obs}}(TCB)) = TCG(TCB)
\]
Relativistic Time Scales: TCB and TCG

- Important special case \( x^i = x^i_c(t) \) gives the TCG-TCB relation at the geocenter:

\[
E_x(x,x,t) = \text{linear drift removed:}
\]

Relativistic Time Scales: proper time scales

- \( \tau \) proper time of each observer: what an ideal clock moving with the observer measures…

- Proper time can be related to either TCB or TCG (or both) provided that the trajectory of the observer is given:

\[
X_{\text{obs}}(t) \quad \text{and/or} \quad X_{\text{obs}}^a(T)
\]

The formulas are provided by the relativity theory:

\[
\frac{d\tau}{dt} = \left( -g_{\text{obs}}(t,x_{\text{obs}}(t)) - \frac{2}{c^2} g_{\text{obs}}(t,x_{\text{obs}}(t)) \dot{x}_{\text{obs}}^i(t) - \frac{1}{c^2} g_{\text{obs}}(t,x_{\text{obs}}(t)) \ddot{x}_{\text{obs}}^i(t) \dot{x}_{\text{obs}}^i(t) \right)^{1/2}
\]

\[
\frac{d\tau}{dT} = \left( -G_{\text{obs}}(T,X_{\text{obs}}(T)) - \frac{2}{c^2} G_{\text{obs}}(T,X_{\text{obs}}(T)) \dot{X}_{\text{obs}}^a(T) - \frac{1}{c^2} G_{\text{obs}}(T,X_{\text{obs}}(T)) \ddot{X}_{\text{obs}}^a(T) \dot{X}_{\text{obs}}^a(T) \right)^{1/2}
\]
Relativistic Time Scales: proper time scales

- Specially interesting case: an observer close to the Earth surface:

\[ \frac{d\tau}{dT} = 1 - \frac{1}{c^2} \left( \frac{1}{2} \dot{X}^2_{\text{obs}}(T) + W_x(T, X_{\text{obs}}) + \text{"tidal terms"} \right) + O\left(c^{-4}\right) \]

- Idea: let us define a time scale linearly related to \( T=TCG \), but which is numerically close to the proper time of an observer on the geoid:

\[ TT = (1 - L_w) TCG, \quad L_w = 6.969290134 \times 10^{-10} \]

\[ \frac{d\tau}{dT} = 1 - \frac{1}{c^2} \left( \text{"terms, h, \nu' + "tidal terms" + ...} \right) + \ldots \]

\( h \) is the height above the geoid
\( \nu' \) is the velocity relative to the rotating geoid

can be neglected in many cases

Relativistic Time Scales: TDB-1

- Idea: to scale TCB in such a way that the “scaled TCB” remains close to TT

- IAU 1976: TDB is a time scale for the use for dynamical modelling of the Solar system motion which differs from TT only by periodic terms.

- This definition taken literally is flawed:

  such a TDB cannot be a linear function of TCB!

But the relativistic dynamical model (EIH equations) used by e.g. JPL is valid only with TCB and linear functions of TCB…
Relativistic Time Scales: $T_{eph}$

- Since the original TDB definition has been recognized to be flawed, Myles Standish (1998) introduced one more time scale $T_{eph}$ differing from TCB only by a constant offset and a constant rate:

$$T_{eph} = K \times TCB + T_{eph0}$$

- The coefficients are different for different ephemerides.
- The user has NO information on those coefficients from the ephemeris.
- The coefficients could only be restored by some additional numerical procedure (Fukushima’s “Time ephemeris”)

- $T_{eph}$ is de facto defined by a fixed relation to TT:
  by the Fairhead-Bretagnon formula based on VSOP-87

Relativistic Time Scales: TDB-2

The IAU Working Group on Nomenclature in Fundamental Astronomy suggested to re-define TDB to be a fixed linear function of TCB:

- TDB to be defined through a conventional relationship with TCB:

$$TDB = TCB - L_B \times (JD_{TCB} - T_0) \times 86400 + TDB_0$$

- $T_0 = 2443144.5003725$ exactly,
- $JD_{TCB} = T_0$ for the event 1977 Jan 1.0 TAI at the geocenter and increases by 1.0 for each 86400s of TCB,
- $L_B = 1.550519768 \times 10^{-8}$,
- $TDB_0 = -6.55 \times 10^{-5}$ s.
Scaled BCRS: not only time is scaled

- If one uses scaled version TCB – T_{eph}, or TDB – one effectively uses three scaling:
  - time
    \[ t^* = K \cdot TCB + t_0^* \]
  - spatial coordinates
    \[ x^* = K \cdot x \]
  - masses (\mu = GM) of each body
    \[ \mu^* = K \cdot \mu \]
    \[ K = 1 - \frac{L_B}{10} \]

**WHY THREE SCALINGS?**

Scaled BCRS

- These three scalings together leave the dynamical equations unchanged:
  - for the motion of the solar system bodies:
    (first published in 1917!)
  - for light propagation:

\[
\ddot{x}_A = - \sum_{B \neq A} \frac{\mu_B}{|r_{AB}|^3} \left[ \frac{r_{AB}}{|r_{AB}|^3} \right] + \frac{1}{c^2} \sum_{B \neq A} \frac{\mu_B}{|r_{AB}|} \left[ \frac{r_{AB}}{|r_{AB}|^3} \right] + \sum_{C \neq A} \frac{\mu_C}{|r_{AC}|} + \frac{3}{2} \left( \frac{r_{AB} \cdot \dot{x}_B}{|r_{AB}|^2} \right) \]
\[
- \frac{1}{2} \sum_{C \neq A, B} \frac{\mu_C}{|r_{BC}|^2} \left( \frac{r_{AB} \cdot \dot{x}_B}{|r_{AB}|^2} \right) - 2 \dot{x}_B \cdot \dot{x}_A - \dot{x}_A \cdot \dot{x}_A + 4 \dot{x}_A \cdot \dot{x}_B \right)
\]
\[
+ \frac{1}{c^2} \sum_{B \neq A} \frac{\mu_B}{|r_{AB}|^3} \left[ \frac{\dot{r}_{AB}}{|r_{AB}|^3} \right] + 4 \dot{x}_A \cdot \dot{r}_{AB} - 3 \dot{x}_B \cdot \dot{r}_{AB} \right)
\]
\[
- \frac{1}{c^2} \frac{7}{2} \sum_{B \neq A} \frac{\mu_B}{|r_{AB}|} \left[ \frac{r_{BC}}{|r_{BC}|^3} \right] + O(c^{-4}),
\]
\[
c(t_2 - t_1) = |x_2 - x_1|
\]
\[
+ \sum_A \frac{2 \mu_A}{c^2} \ln \left[ \frac{|r_{1A}| + |r_{2A}| + |r_{21}|}{|r_{1A}|} \right] + O(c^{-4}).
\]
TCB-based or TDB-based ephemeris?

- As soon as the redefined TDB is used there is no real difference between them!

- Once a TDB ephemeris is constructed, it is trivial to convert it to TCB and vice versa: just apply the three scalings given above!

- With fixed scaling constant $L_B$ (that is, with the re-defined TDB) it is impossible to have different post-fit residuals when using TDB and TCB. The fits must be absolutely equivalent!

Iterative procedure to construct ephemeris with TCB or TDB in a fully consistent way

1. *apriori* T(C/D)B–TT relation (from an old ephemeris)
2. convert the observational data from TT to T(C/D)B
3. construct the new ephemeris
4. changed?
   - yes: update the T(C/D)B–TT relation (by numerical integration using the new ephemeris)
   - no: final 4D ephemeris
Notes on the iterative procedure

• This scheme works even if the change of the ephemeris is (very) large

• The iterations are expected to converge very rapidly (after just 1 iteration)

• The time ephemeris (TCB-TCG relation) becomes a natural part of any new ephemeris of the Solar system:

  Self-consistent 4-dimensional ephemerides should be produced in the future

  Consequence of not doing it, e.g. TEMPO2 does it internally

Choices/questions which are still open

• “TDB-units” vs. SI units

• Astronomical units in the relativistic context

• Astronomical units in general:

  Do we need astronomical units in their current form?

  AU as a fixed value in SI meters?…