Using the P03 Precession Model

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Improving the IAU 2000 precession

- The precession part of the IAU 2000A bias-precession-nutation model consists simply of rate adjustments to the former precession model, IAU 1976. This "IAU 2000 precession" fits the existing VLBI observations well, but is not consistent with dynamical theory beyond first order.

- To improve on IAU 2000, what was needed:
  - A new model for the precession of the ecliptic.
  - A model for the precession of the equator consistent with dynamical theories.

- The replacement precession model that has been chosen by the IAU WG *Precession and the Ecliptic* (James Hilton, chair) is the "P03" solution of Capitaine et al. (2003).

- In addition to the new precession models for ecliptic and equator, P03 includes:
  - Slight adjustments to the nutation model.
  - A parameterized solution to prepare for future revisions of precession and J_2 rates.

- Subject to IAU resolution, P03-based precession-nutation procedures will be needed for the IERS Conventions (and the SOFA software).

P03 precession

- The P03 precession of the ecliptic is based on a fit to DE406 over a 2000-year interval, after removal of periodic terms using VSOP87.
- The P03 precession of the equator is based on semi-analytical integration of the Earth's precession equations, and is consistent with non-rigid Earth models.
- P03 has been verified by comparing with VLBI observations 1985-2005, with FCN taken into account.

Capitaine N., Wallace P.T., Chapront J. (2003, A&A **412**, 567-586)
"Expressions for IAU 2000 precession quantities"
the P03 solution for the precession of the ecliptic and the precession of the equator

Capitaine N., Wallace P.T., Chapront J. (2004, A&A **421**, 365-379)
"Comparison between high precision precession models for the ecliptic and the equator"
comparison between the recent precession models P03/B03/F03/HF04/IAU 2000

Capitaine N., Wallace P.T., Chapront J. (2004, A&A **432**, 355-367)
"Improvement of the IAU 2000 precession model"
fit to VLBI, P03 Tables and the parameterized "P04_par" precession solution

Capitaine N., Wallace P.T., (2006, A&A **450**, 855-872)
"High precision methods for locating the celestial intermediate pole and origin"
comparison of the different ways of locating the (P03) CIP and CIO

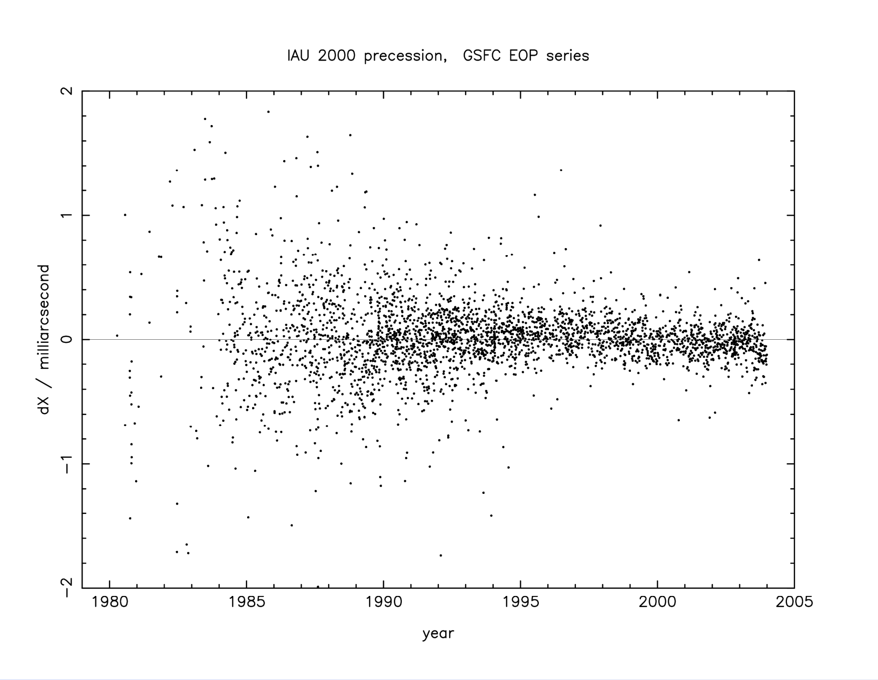
Wallace P.T., Capitaine N., (2006, submitted to A&A)
"Precession-nutation procedures consistent with IAU 2006 resolutions"
precession-nutation procedures suitable for (i) SOFA and (ii) IERS Conventions, and (iii) concise method

P03 adjustments to IAU 2000A nutation

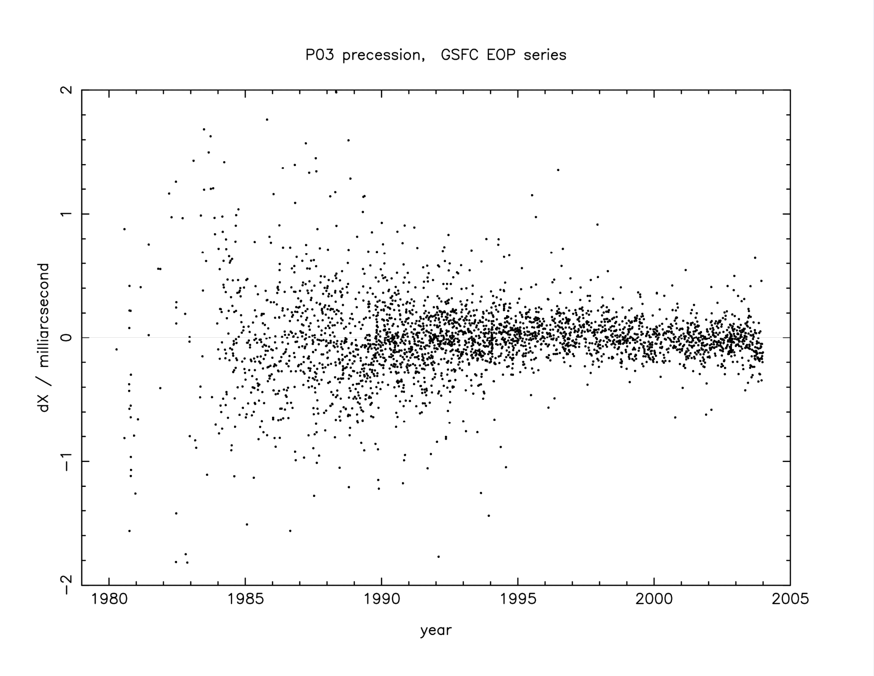
- For use with P03 precession, slight adjustments to the IAU 2000A nutation are needed:
  - Change of nutation amplitudes (in longitude only) to take account of the revised obliquity: a geometrical effect.
  - Change of nutation amplitudes (in both longitude and obliquity) due to the secular variation of the Earth's dynamical flattening: not considered in IAU 2000A.
- The adjustments are (at present) small: a few microarcseconds.

Fits of IAU2000 and P03 to VLBI: X

IAU 2000

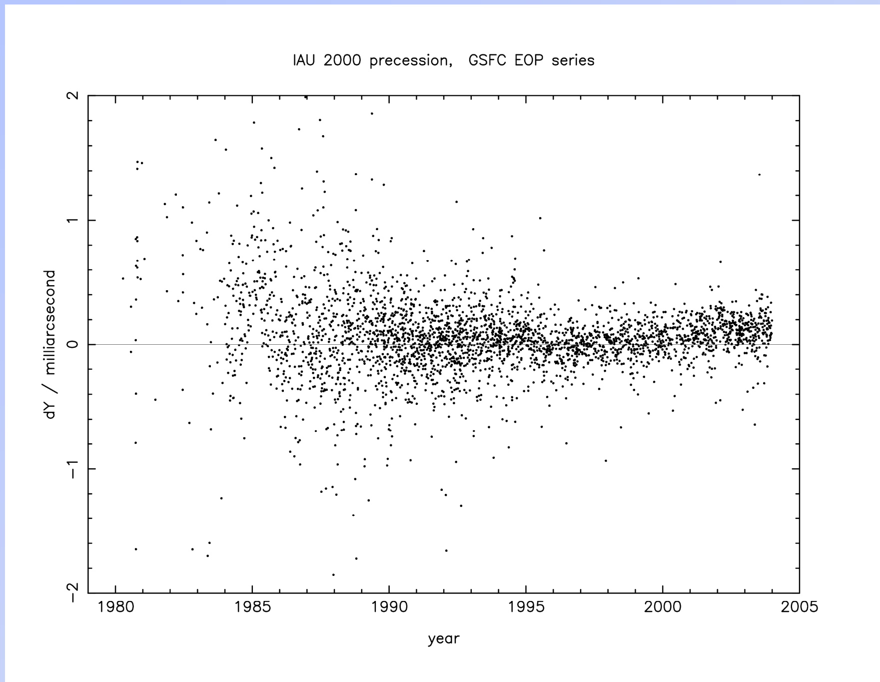


P03

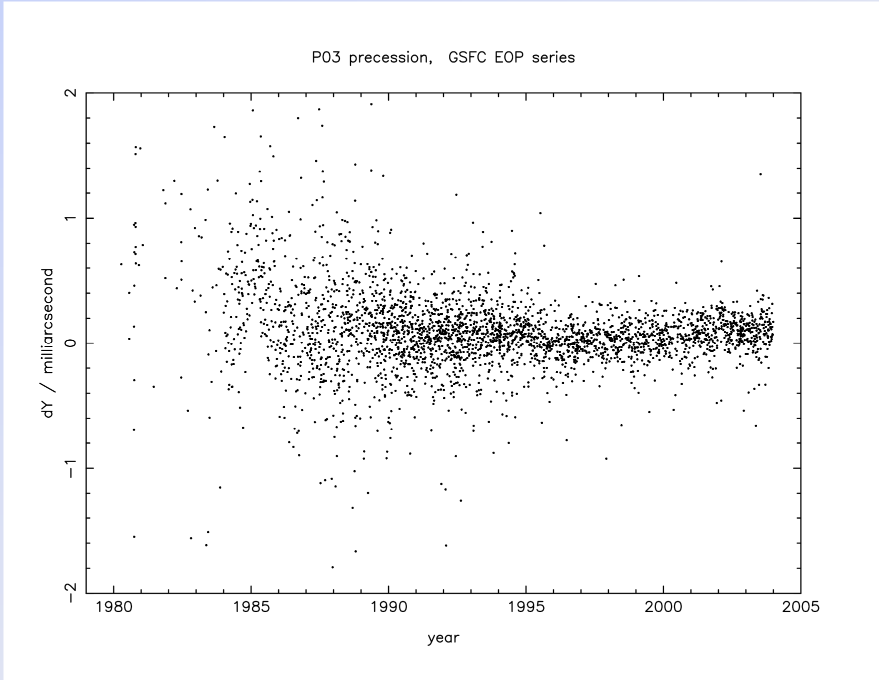


Fits of IAU2000 and P03 to VLBI: *Y*

IAU 2000



P03



Building-blocks

- The goal is to generate these matrices:
  - ${}_{GCRS}\mathbf{R}_{CIRS}$ (or ${}_{GCRS}\mathbf{R}_{true}$)
  - ${}_{GCRS}\mathbf{R}_{TIRS}$
- This can be done if we know these vectors:
  - $\mathbf{v}_{CIP}$
  - $\mathbf{v}_{CIO}$ (or $\mathbf{v}_{equinox}$)
- ...and this angle:
  - ERA (or GST)
- The *equation of the origins* links classical and new:
  - EO = ERA−GST = distance from CIO to equinox

Forming the GCRS-to-TIRS matrix

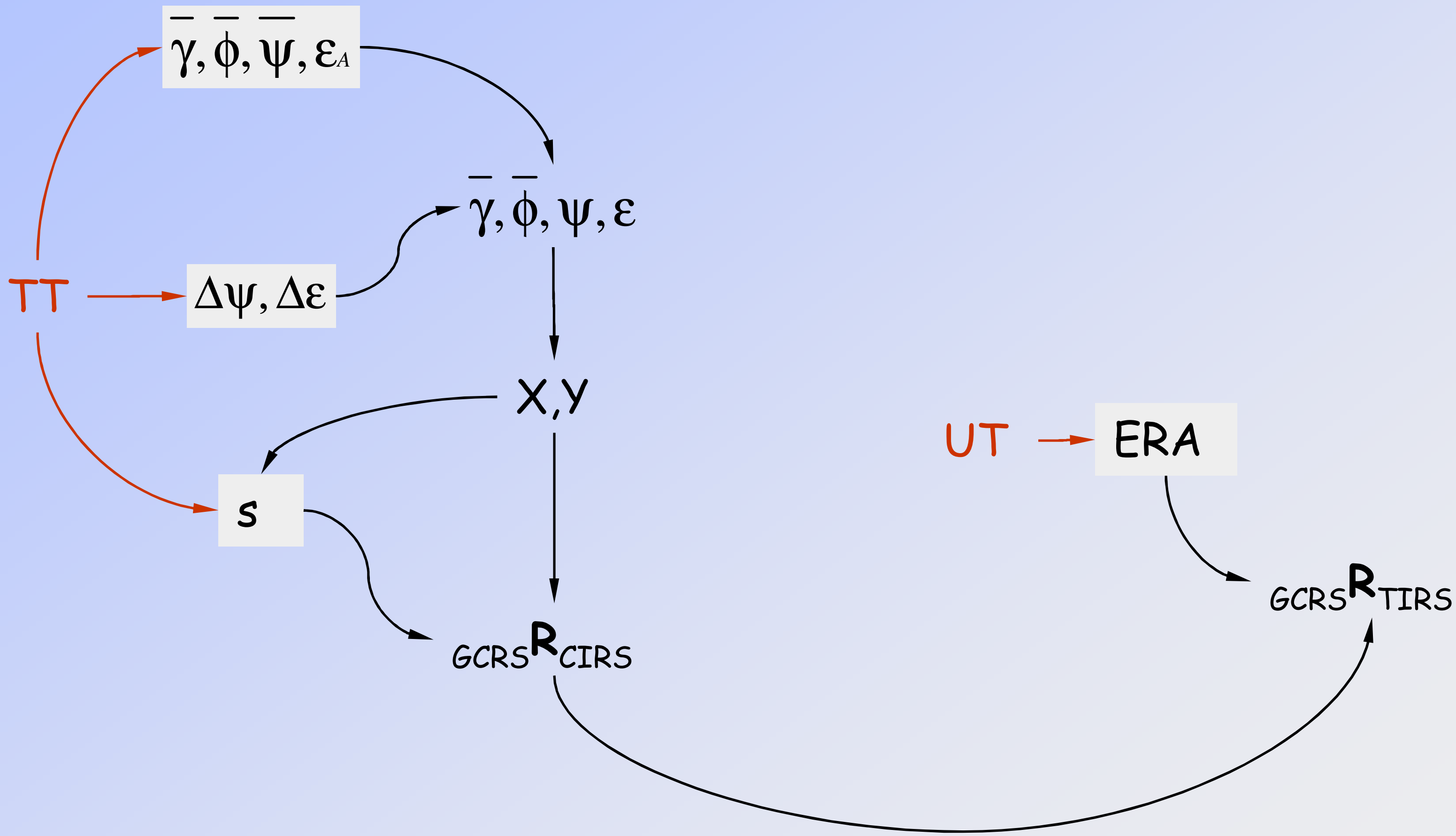
$$ {}_{GCRS}\mathbf{R}_{TIRS} = \mathrm{R}_3(\mathrm{ERA}) \times \begin{pmatrix} \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{CIO} \\ \mathbf{v}_{CIP} \end{pmatrix} = \mathrm{R}_3(\mathrm{GST}) \times \begin{pmatrix} \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \times \mathbf{v}_{equinox} \\ \mathbf{v}_{CIP} \end{pmatrix} $$

(GST) $= \mathrm{ERA} - \mathrm{EO}$

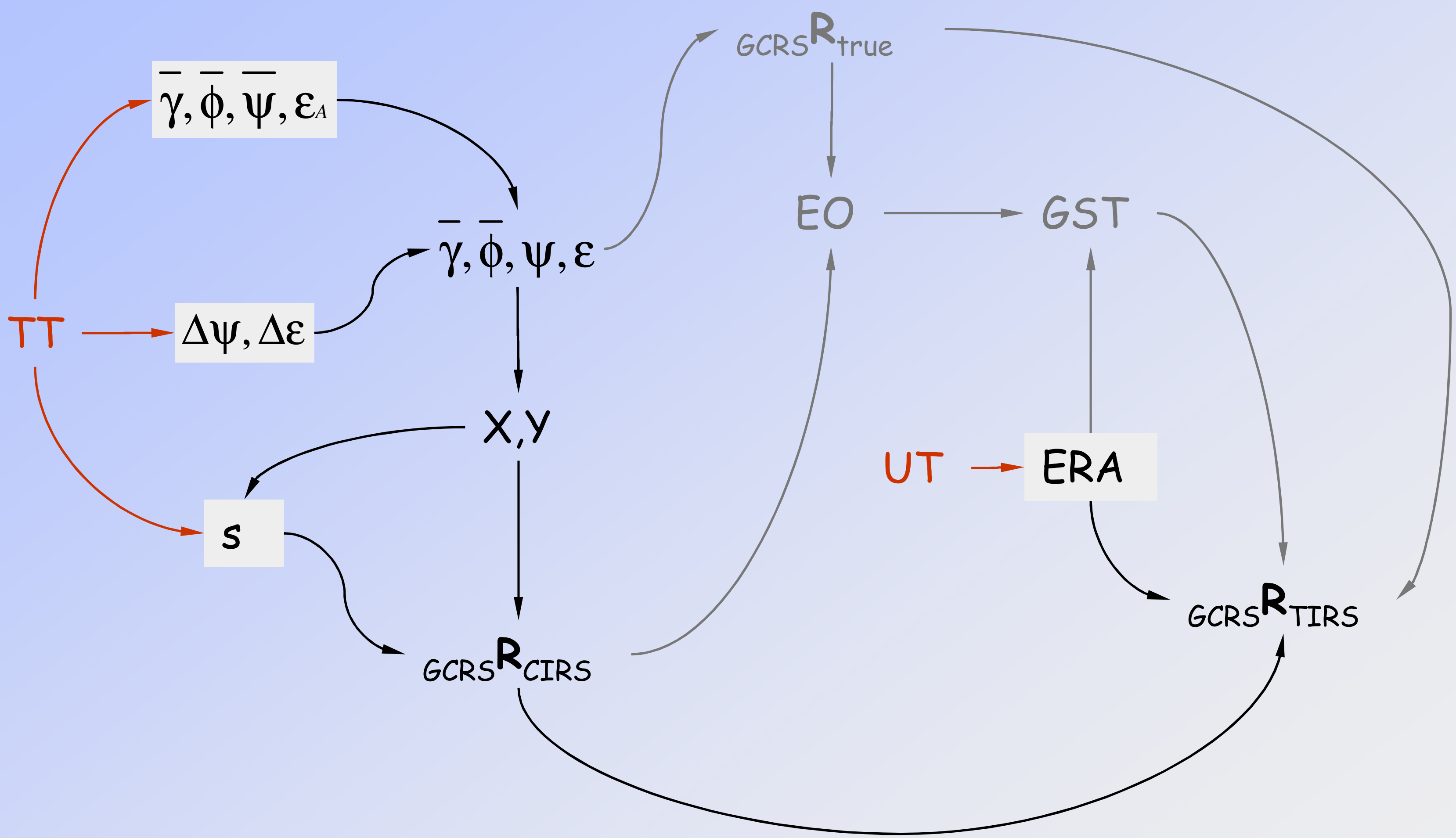
Algorithm choices

- The *Precession and the Ecliptic* WG report leaves open the choice of parameterization.
- Capitaine and Wallace (2006) describes:
  - 6 ways of forming ${}_{GCRS}\mathbf{R}_{CIRS}$
  - 3 ways of obtaining $\mathbf{v}_{CIP}$
  - 8 ways of obtaining $\mathbf{v}_{CIO}$
- Different applications have different priorities. For example:
  - A general-purpose toolkit such as SOFA must
    - have a clear provenance
    - be versatile, concise and efficient
    - be highly self-consistent
  - IERS Conventions addresses a demanding but highly focused application:
    - it must be straightforward to use
    - it must perform efficiently and accurately
  - In many applications accuracy can be traded off against size and speed.
- Wallace and Capitaine (2006, submitted):
  - Concentrates on two specific choices.
  - Provides detailed numerical examples.

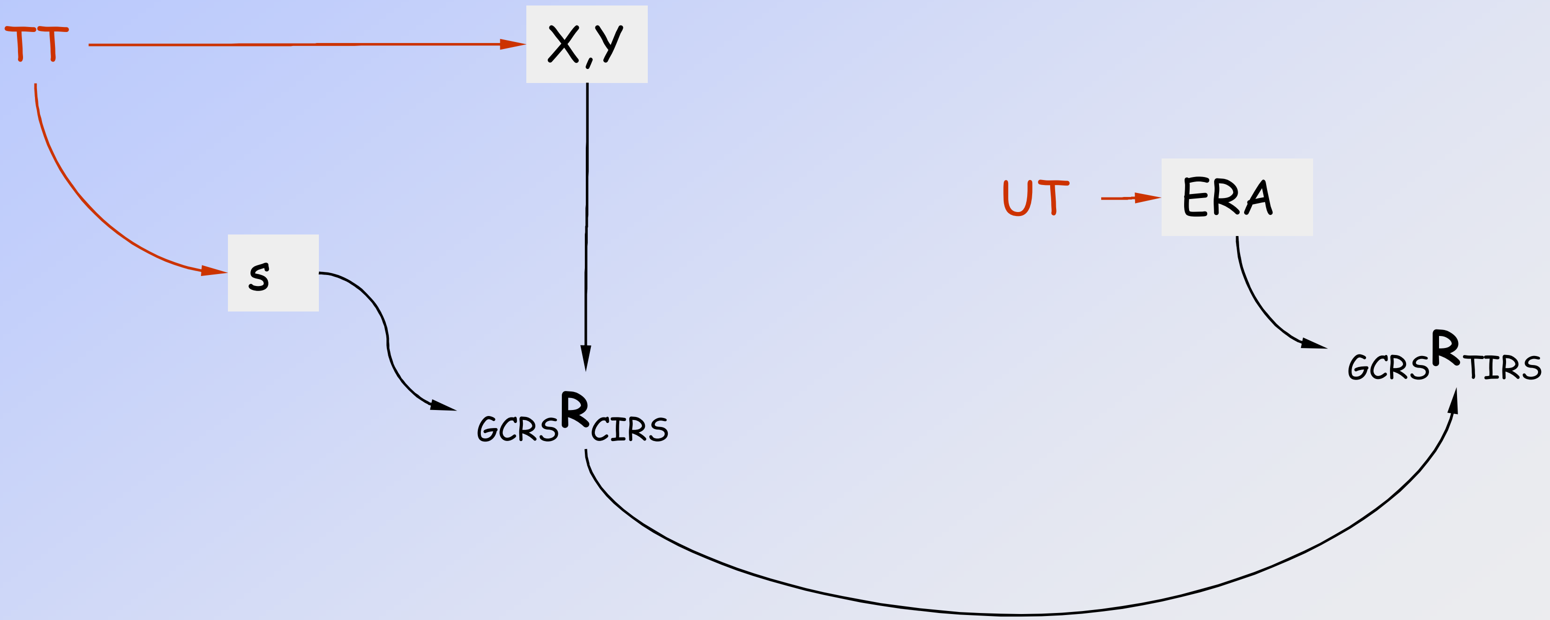
“Angles” method



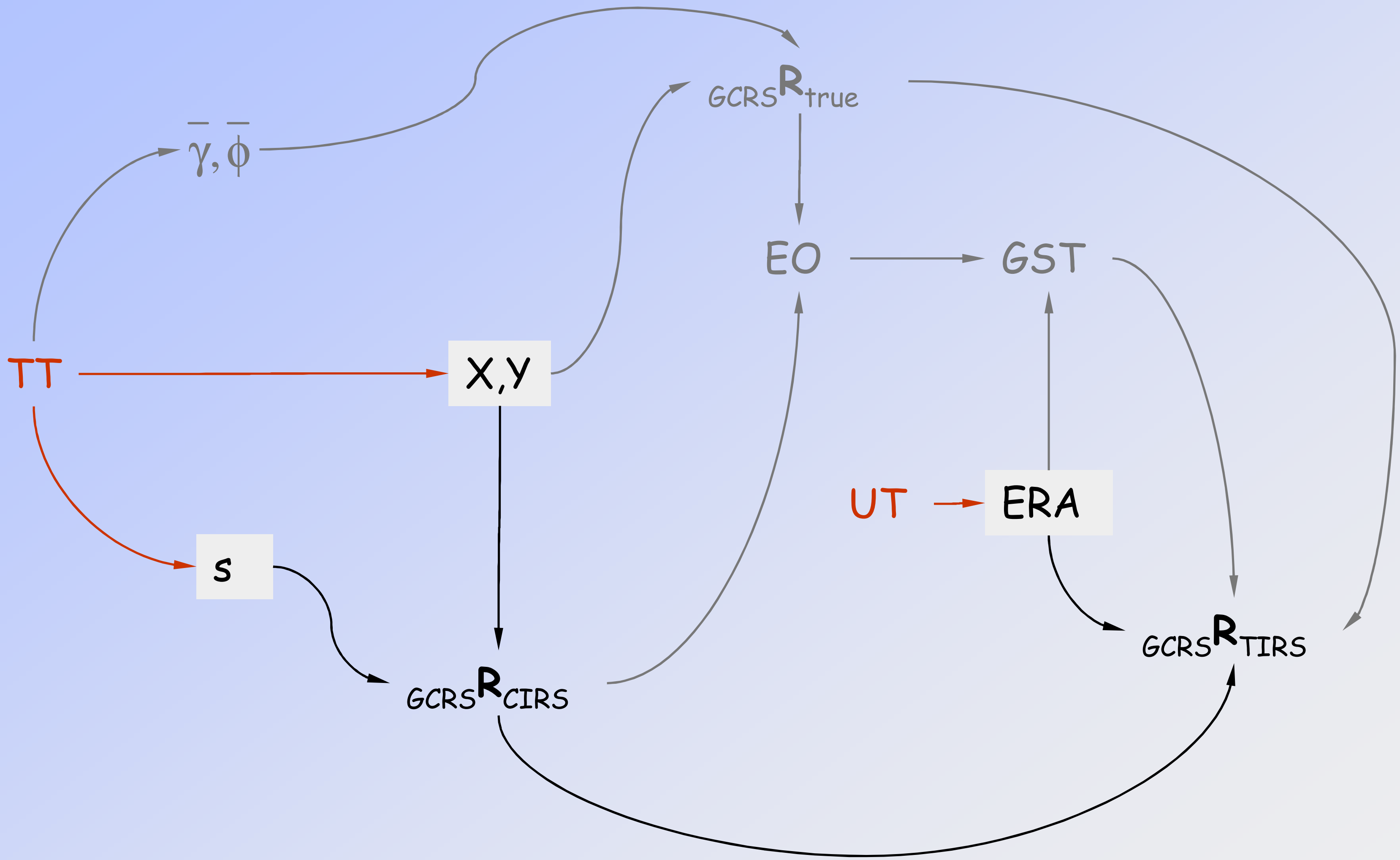
"Angles" method – equinox option



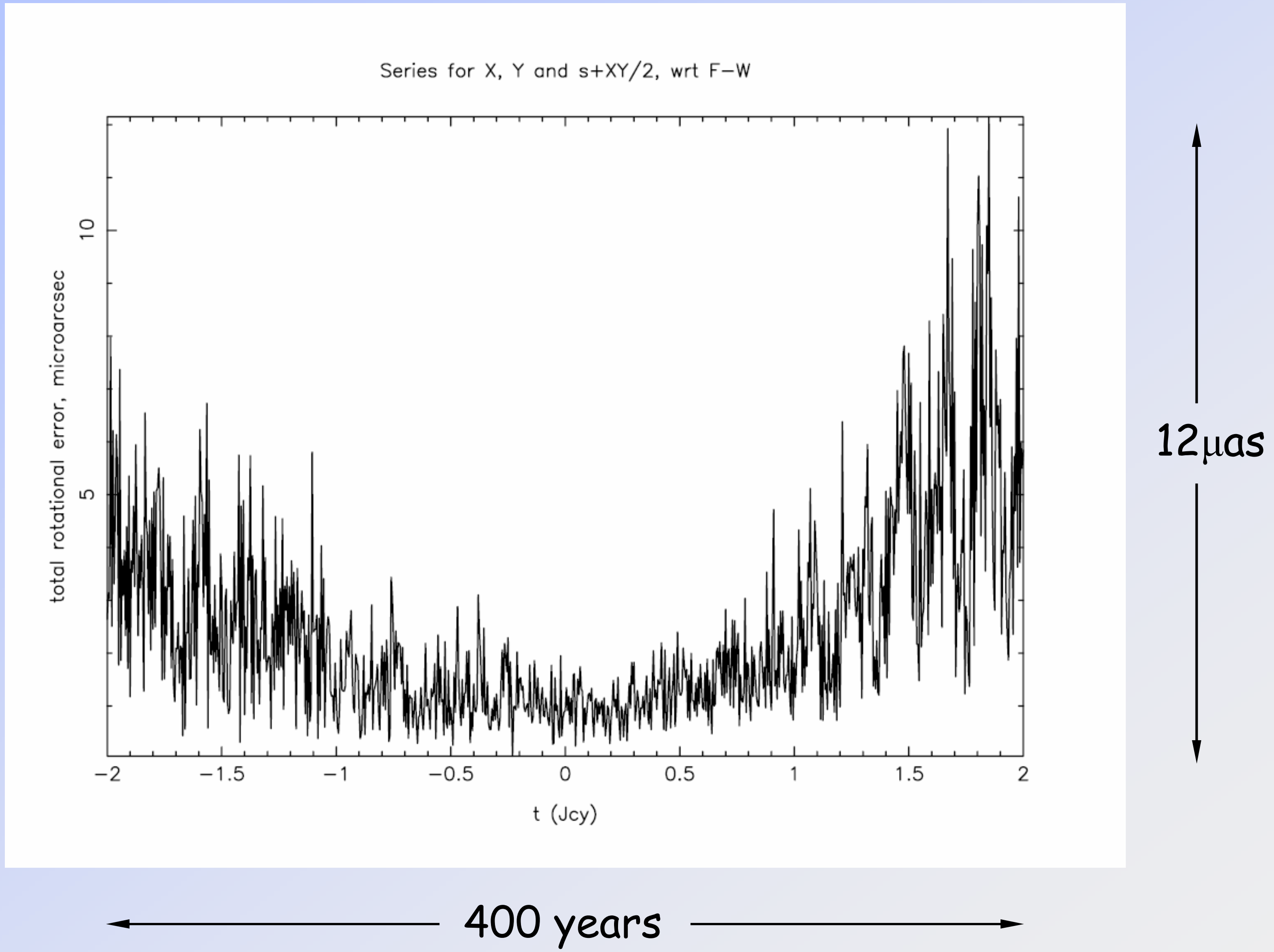
"X,Y series" method



"X,Y series" method - equinox option



Agreement between the two methods



The series compared

		number of terms					
		1	*t*	t^2	t^3	t^4	t^5
angles method	$\bar{\gamma}$	1	1	1	1	1	1
	$\bar{\phi}$	1	1	1	1	1	1
	ψ	1321	38	1	1	1	1
	ε_A	1038	20	1	1	1	1
X,Y method	X	1307	254	37	5	2	1
	Y	963	278	31	6	2	1
both	$s + XY/2$	34	4	26	5	2	1

Approximate $_{GCSR}\mathbf{R}_{TIRS}$ matrices

$$\begin{pmatrix} \cos s + X(Y\sin s - X\cos s)/(1+Z) & -\sin s + Y(Y\sin s - X\cos s)/(1+Z) & -(X\cos s - Y\sin s) \\ \sin s - X(Y\cos s + X\sin s)/(1+Z) & \cos s - Y(Y\cos s + X\sin s)/(1+Z) & -(Y\cos s + X\sin s) \\ X & Y & Z \end{pmatrix}$$

rigorous

$$\begin{pmatrix} 1-\frac{X^2}{2} & -s-\frac{XY}{2} & -X \\ s-\frac{XY}{2} & 1-\frac{Y^2}{2} & -Y-sX \\ X & Y & 1-\frac{(X^2+Y^2)}{2} \end{pmatrix}$$

8 μas 2000-2100

165 μas 1800-2200

$$\begin{pmatrix} 1-\frac{X^2}{2} & 0 & -X \\ 0 & 1 & -Y \\ X & Y & 1-\frac{X^2}{2} \end{pmatrix}$$

0.08″ 2000-2100

0.38″ 1800-2200

$$\begin{pmatrix} 1 & 0 & -X \\ 0 & 1 & -Y \\ X & Y & 1 \end{pmatrix}$$

0.12″ 2000-2100

0.85″ 1800-2200

BUT these figures assume that full-accuracy *X*, *Y*, *s* are used. In order to achieve significant computational savings, these quantities must themselves be approximated.

A very approximate method

The expression:

$$ {}_{GCRS}\mathbf{R}_{CIRS} \cong \begin{pmatrix} 1 & 0 & -X \\ 0 & 1 & -Y \\ X & Y & 1 \end{pmatrix} $$

where:

$$ X = 2.6603\times10^{-7} - 32.2\times10^{-6}\sin\Omega $$

$$ Y = -8.14\times10^{-14}\tau^2 + 44.6\times10^{-6}\cos\Omega $$

with τ days since J2000.0 and:

$$ \Omega = 2.182 - 9.242\times10^{-4}\tau \quad \text{radians} $$

delivers 0.9 arcsecond accuracy throughout 2000-2100, which is adequate for some real-world applications.

END