

*Effects of the VLBI procedure
on the estimated quantities for
precession, nutation and UT1*

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Implementation of IAU Resolution B1.8 (IERS Conventions 2003)
(transformation between ITRS and GCRS)

The IERS has implemented IAU Resolution B1.8 in parallel

- for the CEO-based transformation (new approach)*
- for the rigorous post-2003 classical (equinox-based) transformation*

The two options give results that agree to microarcsecond accuracy

The IERS implementation of B1.8 has been done so as to

- ensure consistency between post-2003 classical and new procedures*
- ensure continuity on **1st Jan 2003** between the pre-2003 classical and the post-2003 procedures (classical and new)*

Comparison between pre-2003 and post-2003 procedures

*There are 3 different procedures (see the 3 following slides)
for the TRS <-> CRS transformation*

- (1) Classical (equinox-based): **pre-2003** procedure
(**inaccuracies** at the level of a **few hundred microarcseconds/cy**)
- (2) New (CEO-based): **post-2003** rigorous procedure
(**microarcsecond accuracy**),
- (3) Classical (equinox-based): **post-2003** rigorous procedure
(**microarcsecond accuracy**)

TRS <-> CRS transformation

(1) Classical (equinox-based): *pre-2003 procedure*

$$R = P N T$$

Polar motion omitted

- P : Precession matrix
- N : Nutation matrix

IAU 1980 or IERS 1996

$$\text{celestial pole offsets} = \begin{pmatrix} \xi_0 / \sin \varepsilon_0 + d\psi \\ \eta_0 + d\varepsilon \end{pmatrix}$$

with: $d\psi, d\varepsilon$: corrections to the PN model ; ξ_0, η_0 : celestial pole offsets at J2000

- T: Earth's rotation

$$P = R_3(\zeta_A) R_2(-\theta_A) R_3(z_A)$$

$$N = R_1(-\varepsilon_A) R_3(\Delta\psi + \xi_0 / \sin \varepsilon_0 + d\psi) R_1(\varepsilon_A + \Delta\varepsilon + \eta_0 + d\varepsilon)$$

$$T = R_3(-(\text{GMST} + (\Delta\psi + d\psi) \cos \varepsilon_A + 2 \text{ complementary terms (from 1997)})$$

TRS <-> CRS transformation

(2) New (CEO-based): *post-2003 rigorous procedure*
(*microarcsecond level*)

$$R(t) = R_3(-E) R_2(-d) R_3(E + s - \theta) = Q(t) \cdot R_3(-\theta)$$

$$Q(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \bullet R_3(s)$$

$$a = 1/2 + (X^2 + Y^2)/8$$

$X = \sin d \cos E$, $Y = \sin d \sin E$: GCRS x , y -coordinates of the CIP unit vector: includes frame biases ($\xi_0, \eta_0, d\alpha_0$) + precession + nutation + cross terms precession x nutation)

$$\theta (UT1) = 2\pi (0.7790572732640 + 1.00273781191135448 \times (\text{Julian UT1date} - 2451545.0))$$

TRS <-> CRS transformation

(3) Classical (equinox-based): *post-2003 rigorous procedure (microarcsecond level)*

$$R = B P N T$$

Polar motion omitted

- B : Bias : CRS \rightarrow mean matrix at epoch $(\xi_0, \eta_0, d\alpha_0)$
- P : Precession matrix
- N : Nutation matrix *IAU 2000A*
- T: Earth's rotation

$$B = R_3(-d\alpha_0) R_2(-\xi_0) R_1(\eta_0)$$

$$P = R_1(-\varepsilon_0) R_3(\psi_A) R_1(\omega_A) R_3(-\chi_A)$$

$$N = R_1(-\varepsilon_A) R_3(\Delta\psi_A) R_1(\varepsilon_A + \Delta\varepsilon_A)$$

$$T = R_3(-\text{GMST} + (\Delta\psi + d\psi) \cos\varepsilon_A + \text{"complementary terms in the equation of the equinoxes"})$$

Implementation of IAU Resolution B1.8 (IERS Conventions 2003)

(IAU 2000 expressions for the transformation GCRS -> ITRS)

New (CEO-based) implementation

- $X(t)$, $Y(t)$, $s(t)$: IAU 2000A expressions for the GCRS positions of the CIP (X , Y) and the CEO (s)
- $\theta(UT1)$: IAU 2000 conventional relationship between the Earth Rotation angle (ERA) and UT1

Classical (equinox-based) implementation

- ψ_A , ω_A , ϵ_A , χ_A ((i):recommended way*), or z_A , ζ_A , θ_A ((ii): cf. pre-2003 procedure*): IAU 2000A precession

* Note that IAU 2000 expressions as such that (i) and (ii) are equivalent, whereas they were not when using IAU 1980 expressions (see Fig. 1)

- $\Delta\psi$, $\Delta\epsilon$: IAU 2000A nutation
- GMST (IAU2000A expression for $\theta(UT1)$ + accumulated precession in RA)
- + «equation of the equinoxes» (IAU 2000A classical term + complementary terms in order to refer to the CEO)

New and classical implementations

- $s'(t)$: IAU 2000 expression for the ITRS position of the TEO
- $(\Delta x, \Delta y)_{\text{nutatation}}$ and $(\Delta x, \Delta y)_{\text{tidal}}$: numerical Tables for specific terms of the CIP motion in the ITR (polar motion)

Complementary terms in the equation of equinoxes

$$EE_{CT} = \sum_k (C'_{s,0})_k \sin \alpha_k + \sum_k (C'_{c,0})_k \sin \alpha_k - 0.87 \mu\text{as } t \sin \Omega$$

Pre-2003

Post-2003

Argument α_k	$(C'_{s,0})_k$	$(C'_{c,0})_k$
Ω	+2640.96	-0.39
2Ω	+63.52	-0.02
$2F - 2D + 3\Omega$	+11.75	+0.01
$2F - 2D + \Omega$	+11.21	+0.01
$2F - 2D + 2\Omega$	-4.55	+0.00
$2F + 3\Omega$	+2.02	0.00
$2F + \Omega$	+1.98	0.00
3Ω	-1.72	0.00
$l' + \Omega$	-1.41	-0.01
$l' - \Omega$	-1.26	-0.01
$l + \Omega$	-0.63	0.00
$l - \Omega$	-0.63	0.00
$l' + 2F - 2D + 3\Omega$	0.46	0.00
$l' + 2F - 2D + \Omega$	0.45	0.00
$4F - 4D + 4\Omega$	0.36	0.00
$F - D + \Omega - 8L_{V_e} + 12L_E$	-0.24	-0.12
$2F$	0.32	0.00
$2F + 2\Omega$	0.28	0.00
$l + 2F + 3\Omega$	0.27	0.00
$l + 2F + \Omega$	0.26	0.00
$2F - 2\Omega$	-0.21	0.00
$l' - 2F + 2D - 3\Omega$	0.19	0.00
$l' - 2F + 2D - \Omega$	0.18	0.00
$8L_{V_e} - 13L_E$	-0.10	0.05
$2D$	0.15	0.00
$2l - 2F - \Omega$	-0.14	0.00
$l - 2D + \Omega$	0.14	0.00
$l' + 2F - 2D + 2\Omega$	-0.14	0.00
$l - 2D - \Omega$	0.14	0.00
$4F - 2D + 4\Omega$	0.13	0.00
$2F - 2D + 4\Omega$	-0.11	0.00
$l - 2F - 3\Omega$	0.11	0.00
$l - 2F - \Omega$	0.11	0.00
Ω	$-0.87 \times t$	0.00

2 terms from 1997

Post-2003 additional terms

(Capitaine, Wallace, McCarthy 2003, A&A 406)

IERS Conventions 2003: The IERS routines

<http://maia.usno.navy.mil/ch5subs.html>

NEW (2) < equivalence > CLASSICAL RIGOROUS (3)

- SP2000: s'
- POM2000: polar motion matrix
- ERA2000: Earth Rotation Angle
- XY2000A: X, Y and s
- BPN2000: new N•P•B matrix
- SP2000: s'
- POM2000: polar motion matrix
- GST2000: GST
- GMST2000: GMST
- EE2000: equation of the equinoxes
- EECT2000: complementary terms
- NU2000A: nutation, IAU 2000A
- CBPN2000: classical N•P•B matrix

T2C2000: TRS-to-CRS matrix

Effects of the procedure

There is equivalence **to microarcsecond accuracy** between:

- New (CEO-based): **post-2003** rigorous procedure (2)
- and**
- Classical (equinox-based): **post-2003** rigorous procedure (3)

There are discrepancies of a **few hundred microarcseconds/cy** between:

- Classical (equinox-based): **pre-2003** procedure (1)
- and**
- Classical (equinox-based) and new (CEO-based) **post-2003** rigorous procedures (2) and (3)

Effects of the VLBI procedure in estimated X, Y, UT1 differences between (1) pre-2003 and (2) or (3) post-2003
(in μas , t in centuries)

- *Current VLBI procedures (1) use correction for biases and precession as if they were nutation quantities
=> secular and cubic discrepancies in computing X, Y, UT1 w.r.t rigorous transformation (few 100 $\mu\text{as}/\text{cy}$)*
- Differences in X, Y due the celestial pole offsets at J2000 ($\xi_0 = -16.617 \text{ mas}$; $\eta_0 = -6.819 \text{ mas}$)
 - (post-2003 – pre-2003): $dX (\sim d\psi \sin\epsilon) = 153 t - 5 t^2$; $dY (\sim d\epsilon) = -372 t - 1.7 t^2$
- Differences in X, Y due to the equinox offset ($d\alpha_0 = -14.6 \text{ mas}$)
 - (post-2003 – pre-2003): $dX (\sim d\psi \sin\epsilon) = -1.6 t^2$; $dY (\sim d\epsilon) = -142 t \sim d\epsilon$
- Differences in X, Y due to correction to precession rates ($d\psi_A = -0''.29965/\text{c}$ $d\omega_A = -0''.02524/\text{c}$)
 - (post-2003 – pre-2003): $dX (\sim d\psi \sin\epsilon) = +64 t^2$; $dY (\sim d\epsilon) = -6 t^2$
- Differences in UT1 due the frame bias effects
 - (post-2003 – pre-2003): $GMST00_{mod} = GMST_{00} - 14600 + 274990 t$

Following slides

Plots of differences in the
computed parameters X, Y, UT1
due to effects of the VLBI procedure

post-2003 (rigorous) minus pre-2003 (non-rigorous)

Classical paradigm: IAU 1980 precession effects

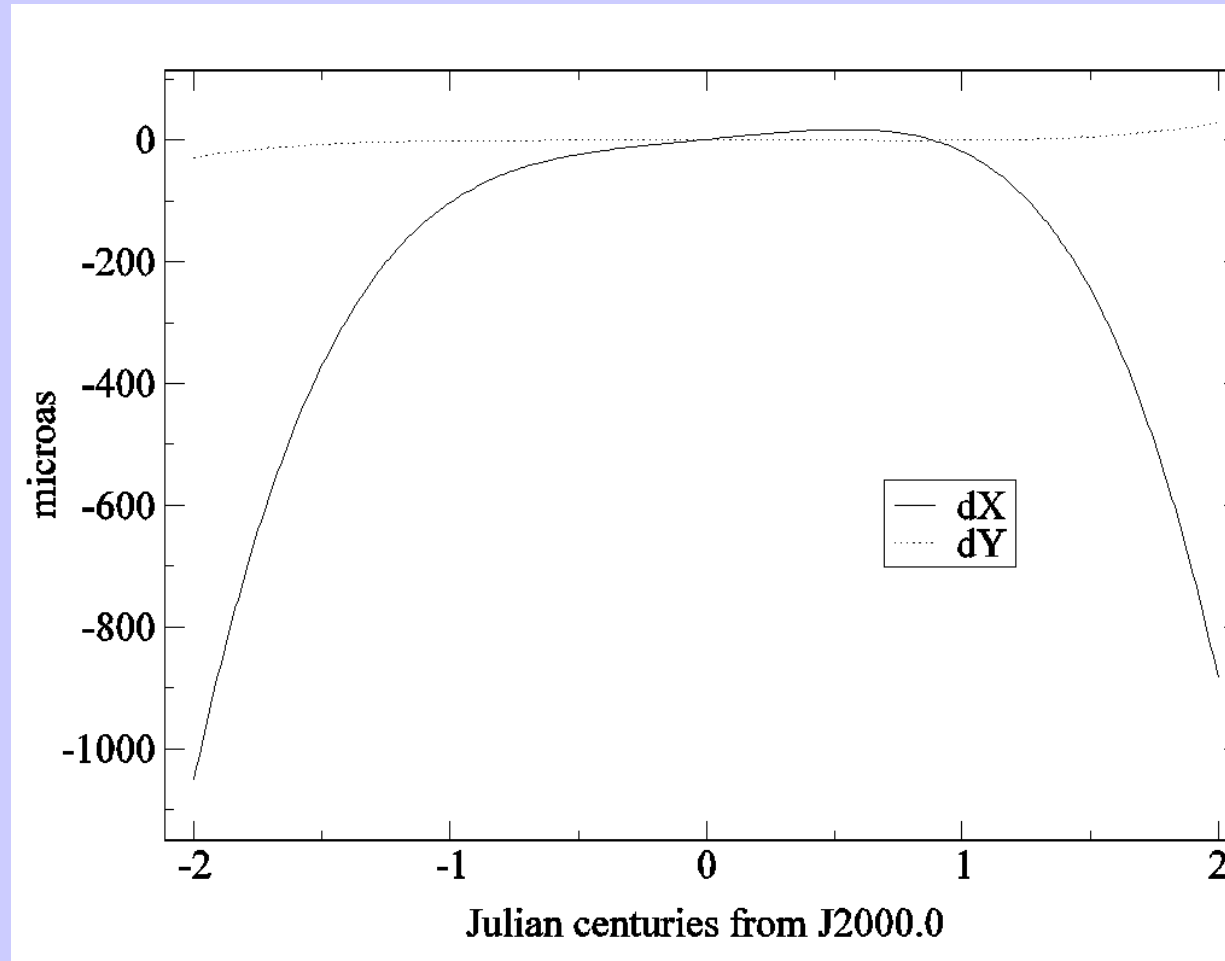
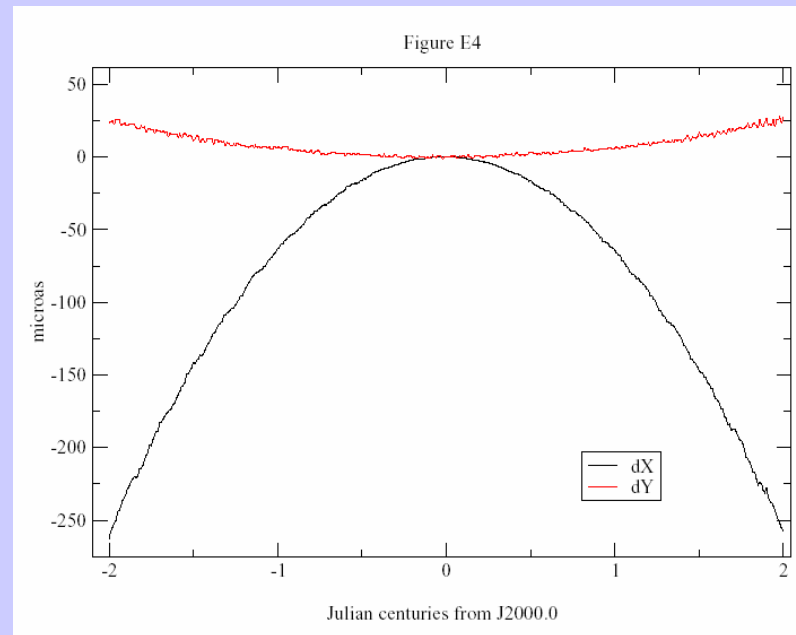


Fig. 1: X, Y-differences between two ways (i) and (ii) (cf. slide 7) of forming the precession matrix based on the IAU 1980 expressions

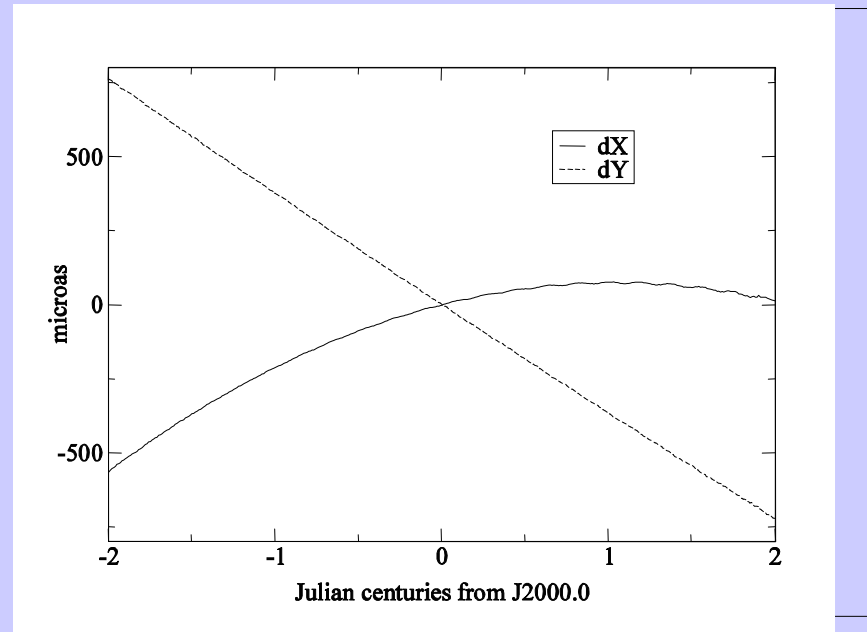
Classical paradigm at μas level: precession effects



(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

Fig. 2: *Effect in X and Y of considering precession corrections as nutation quantities (cf. pre-2003 VLBI codes)*

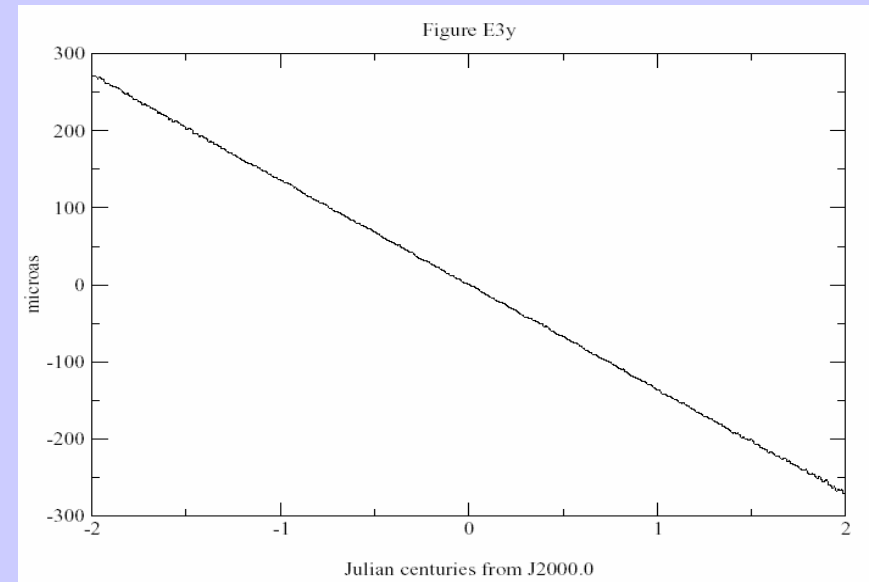
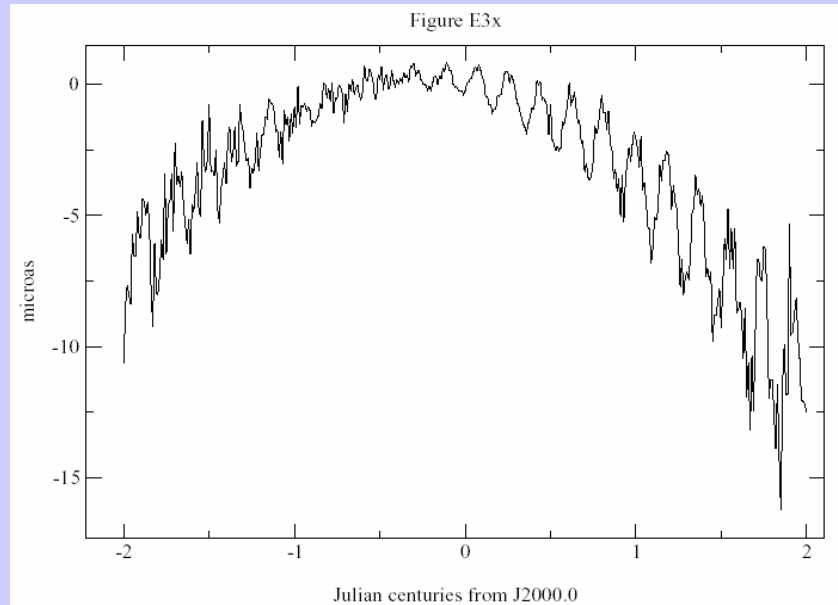
Classical paradigm at μas level: frame bias effects



(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

Fig. 3: Effect in X and Y of considering frame biases corrections as nutation quantities (cf. pre-2003 VLBI codes)

Classical paradigm at μas level: frame bias effects



(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

Fig. 4: Effect in X and Y of omitting the frame bias $d\alpha_0$
(cf. pre-2003 VLBI codes)

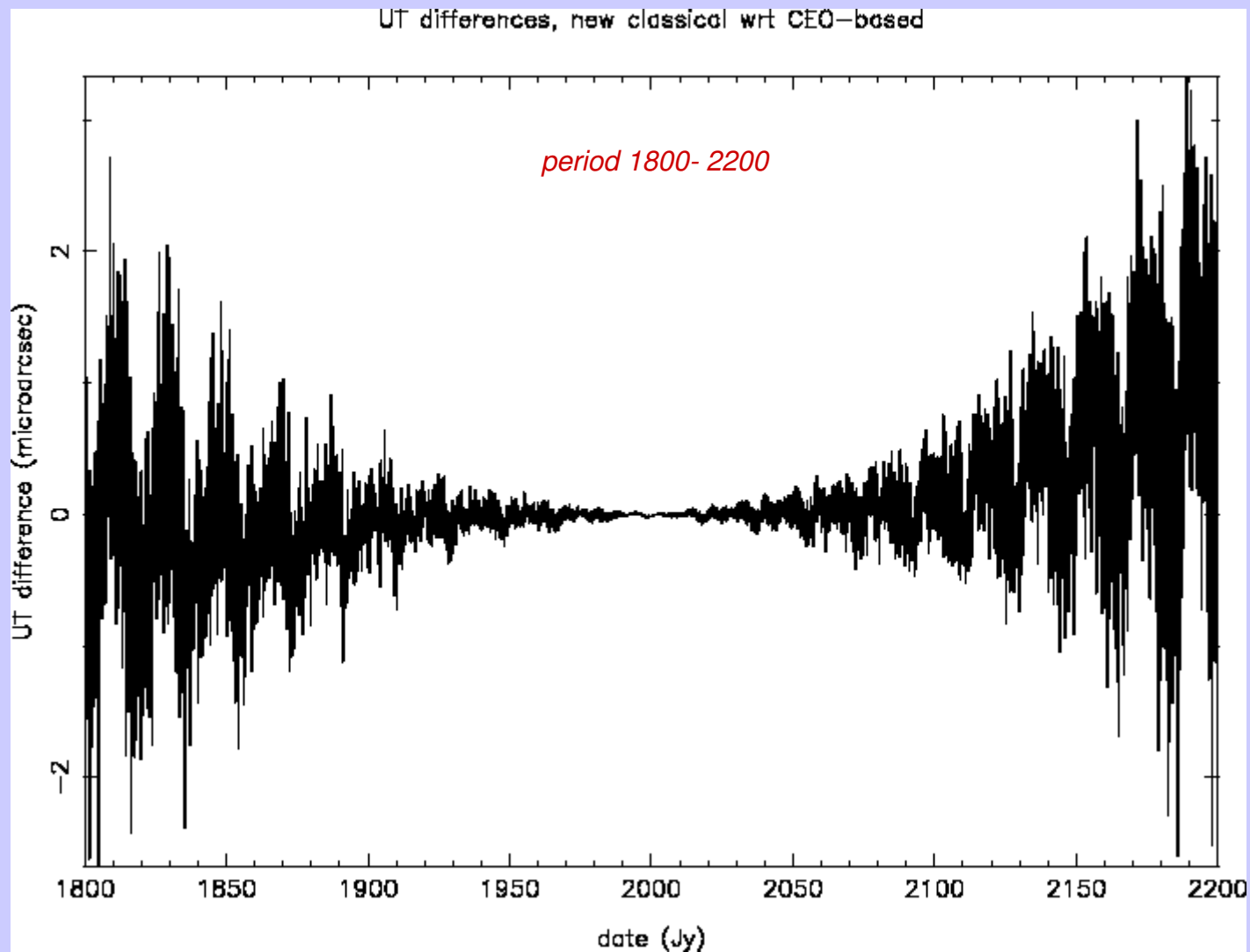
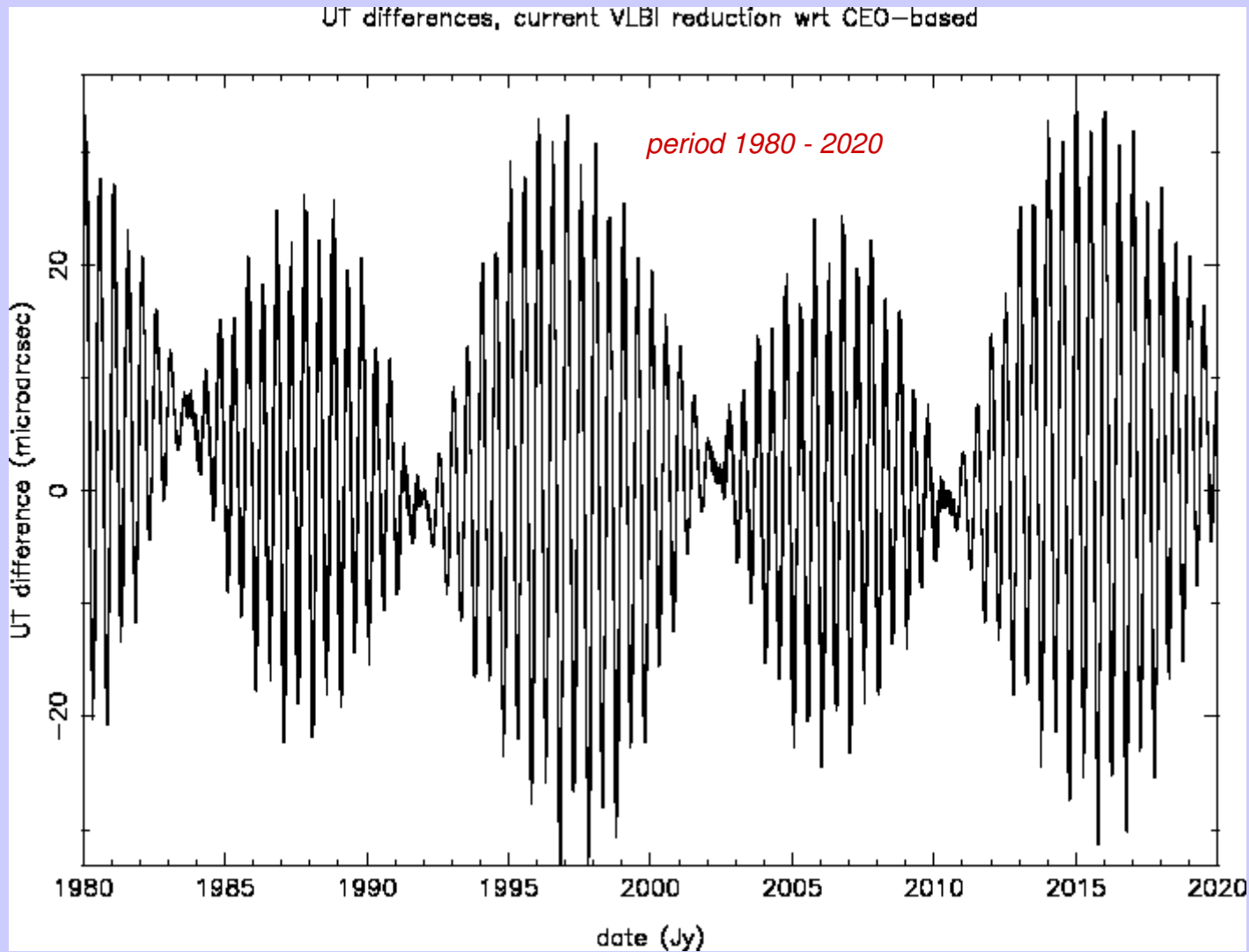


Fig. 5: UT1 differences between CEO-based and equinox-based rigorous procedures

(Capitaine, Wallace, McCarthy 2003, A&A 406)



*Fig. 6: Differences in UT1 (computed) between post-2003 and pre-2003 procedures
(Capitaine, Wallace, McCarthy 2003, A&A 406)*

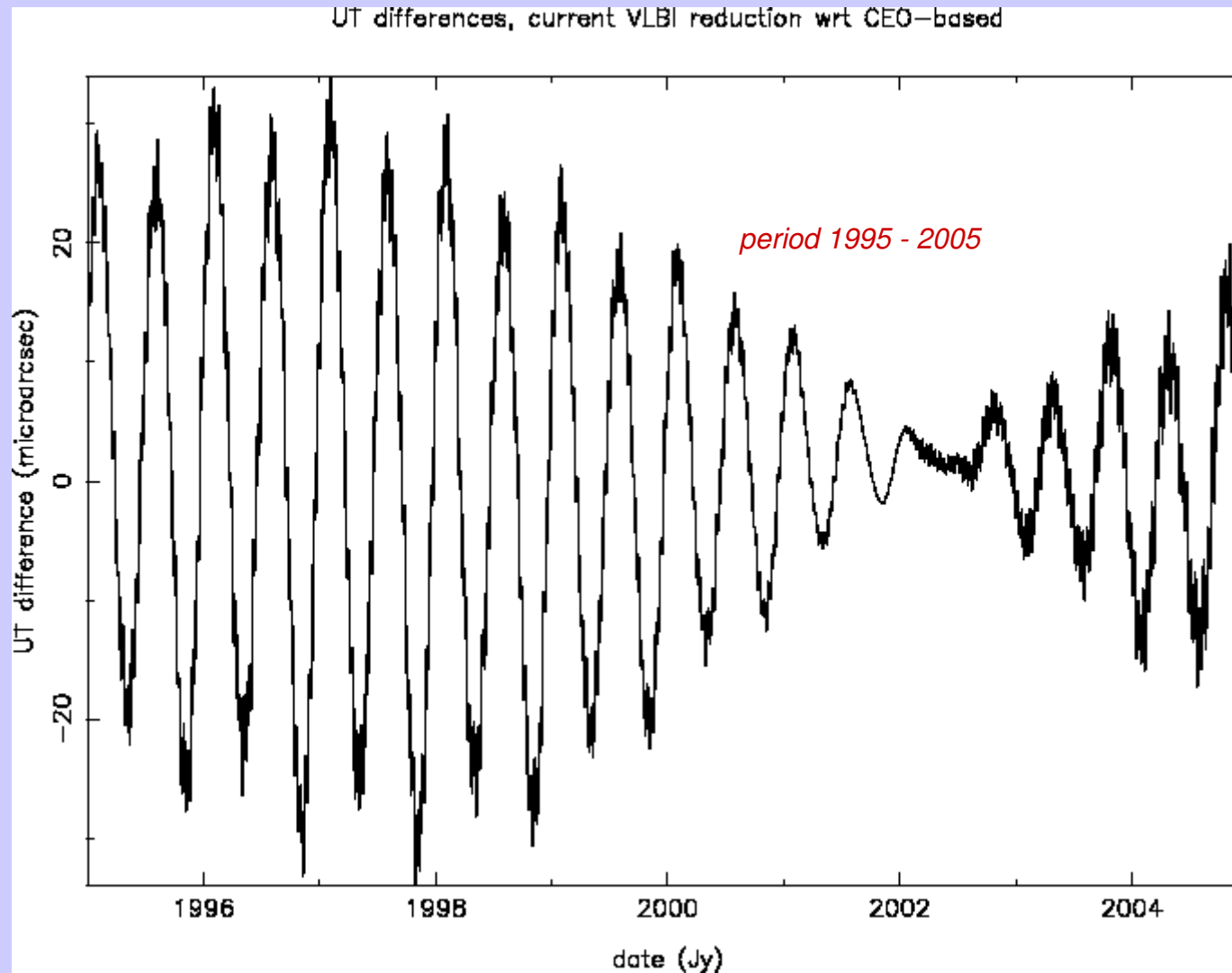


Fig. 7: Differences in UT1 (computed) between post-2003 and pre-2003 procedures

(Capitaine, Wallace, McCarthy 2003, A&A 406)

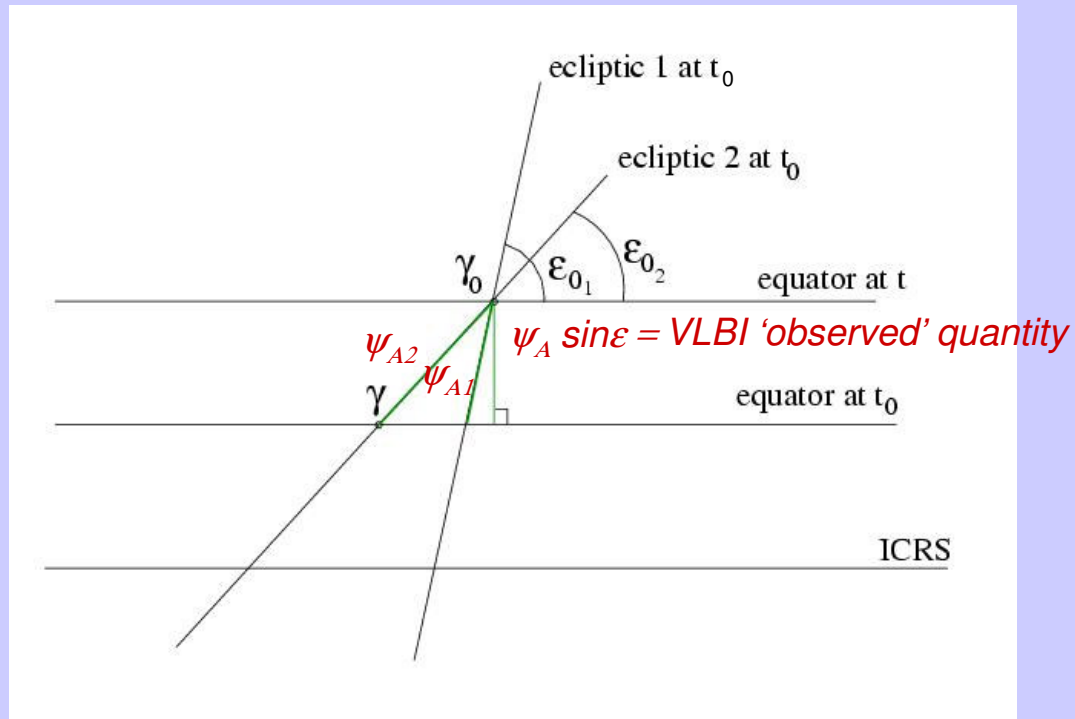
Following slide: Effect of the VLBI procedure on the estimated precession rate in longitude ψ_A

This effect is due to the fact that VLBI is not sensitive to an ecliptic but is sensitive to the GCRS position of the CIP (i.e. of the equator)

P03 solution for precession

Effect of the VLBI procedure on the estimated precession rate in longitude

Ecliptic 1 used in pre-2003 VLBI procedures $\Rightarrow \psi_{A1}$
Ecliptic 2 used for new precession model (cf P03) $\Rightarrow \psi_{A2}$



(Capitaine, Wallace, Chapront 2004, A&A in press)