Implementation of the IAU 2000 precession-nutation and Universal Time using the new paradigm

Summary for the Division 1 "ICRS" Working Group and Commission 19 "Precession-Nutation" Working Group

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24 February 2003

1 The new paradigm

1.1 Definition

The conventional classical paradigm for the transformation from the GCRS to the frame of date is based on the use of the equinox. It uses a sequence of six consecutive rotations, for precession and nutation, followed by a further one for Earth rotation. The transformation is based on the developments as function of time of the precession angles ζ_A, θ_A, z_A (Lieske *et al.* 1977) and on the nutation angles $\Delta\psi$ in longitude and $\Delta\epsilon$ in obliquity referred to the ecliptic of date. It uses the relationship between Greenwich sidereal time and UT1, obtained from the GMST (Greenwich Mean Sidereal Time) at date t and the equation of the equinoxes. The frame bias (the fixed offset between the ICRS and mean J2000 frames) is included as part of the celestial pole offsets.

IAU 2000 Resolution B1.8 recommends the use of the CEO (Celestial Ephemeris Origin) and the TEO (Terrestrial Ephemeris Origin), as origins on the moving equator in the celestial and terrestrial reference systems respectively, and defines UT1 as linearly proportional to the Earth Rotation Angle (ERA) between the CEO and the TEO on the moving equator. This resolution recommends that the transformation between the International Terrestrial Reference System (ITRS) and the Geocentric Celestial Reference System (GCRS) be specified by the position of the Celestial Intermediate Pole, CIP, (defined by Resolution B1.7) in the GCRS, the position of the CIP in the ITRS and the ERA.

Two equivalent ways of implementing this resolution in the transformation from ITRS to GCRS can be used, namely (a) the new paradigm based on the direct use of the CEO and the ERA and (b) the classical paradigm based on the equinox and GST, but using the CEO and the ERA indirectly. They are called respectively "CEO-based" and "equinox-based" transformations in the following.

Common to both paradigms is the polar-motion matrix, which requires, in addition to the coordinates x_p, y_p of the CIP in the ITRS, the use of the quantity s' for providing the position of the TEO in the ITRS. This quantity, which was neglected in the classical form prior to 1 January 2003, corresponds to an exact realization of the "instantaneous prime meridian".

1.2 New paradigm versus classical paradigm

The CEO is the origin of right ascension[1] in the new system. Its counterpart in the classical system, namely the equinox, is defined geometrically. The two defining planes, the equator and ecliptic, are both in motion and not at right-angles, and consequently the motion of the equinox across the celestial sphere is complex (and comparatively rapid). The CEO, which is the "non-rotating origin" (Guinot 1979), in contrast is defined kinematically: from one moment to the next, the CEO moves only at right-angles to the instantaneous equator, and the ecliptic is not involved. This almost complete separation between the treatment of the precessing-nutating pole and the origin of right ascension leads to a much simpler relationship between stellar hour angles and Universal Time.

The concept of the CEO can be considered as extending that of a "uniform equinox" (Atkinson & Sadler 1951) by eliminating not only the equation of the equinoxes in sidereal time, as did the uniform equinox, but its precessional counterpart in the Greenwich Mean Sidereal Time (GMST) formula itself. This nearly eliminates the dependence of the origin of right ascension, which is just defined by a convention, from the precession-nutation motion. The kinematical definition of the CEO can also be considered as extending the concept of "departure point" (Newcomb 1895) currently used in celestial mechanics and satellite geodesy.

The CEO's evolution with time depends on s, the difference between the GCRS and instantaneous right ascensions of the ascending node of the instantaneous equator. The quantity s is defined by an integral that involves the path of the CIP, but the evolution of the CEO is both slow and insensitive to details of the CIP's motion. In the century 2000-2100, the CEO moves only 66 mas in GCRS right ascension. The CEO, realized by the quantity s, defines the origin of right ascension that matches the ERA, the time derivative of which is equal to the Earth's angular velocity.

The classical and new paradigms share a common pole, the CIP, specified in terms of the X and Y components of its unit vector in the GCRS. In the new paradigm, these components are used as basic parameters which take rigorous account of frame bias and deal with precession-nutation. In the classical paradigm, these components are readily obtained simply by forming the frame bias and precession-nutation matrix and extracting elements (3, 1) and (3, 2).

In contrast to the classical paradigm, the new paradigm keeps the concepts of the pole position and Earth rotation separate, leading to a cleaner implementation that has particular advantages for high-accuracy Earth rotation studies. The complicated formulas for GMST and the equation of the equinoxes are eliminated and replaced by the linear relationship between the ERA and

[1] Note that, whereas existing definitions of RA are by their nature limited to past practice, any strict requirement that the RA origin be an equinox has already been abandoned with the use of the term "right ascension" in the context of ICRS coordinates (Ma *et al.* 1998). Note also that there are already several equinoxes in use, such as mean of some epoch, mean of date, true of date, and it is therefore necessary to be quite specific when quoting RA,Dec positions. We are here using the term RA in a generic sense, putting alpha$_{CEO}$ to distinguish such an RA from alpha$_{ICRS}$, which is similar to putting alpha$_{true}$ etc.

UT1. Of course, the intrinsic complexity of the dynamical interactions between precession-nutation and Earth rotation are still there in the new paradigm, in the form of the expression for $s + XY/2$. However, this is all taken care of in the transformation between GCRS and the intermediate frame and does not intrude into the Earth rotation calculation as it does using the classical paradigm.

The equivalence between the two paradigms requires that the classical one take account rigorously of the corrections to precession using the "canonical" four rotations (Wallace 2002), and frame bias, through appropriate rotation matrices. GST is then related to the ERA by considering the accumulated precession and nutation on the moving equator using the CEO as the origin.

2 Implementation of the IAU 2000 system

The implementation of the IAU 2000 resolutions we are discussing here, beginning on 1 January 2003, requires the adoption of (i) the IAU 2000 model for the motion of the Celestial Intermediate Pole (CIP) in the Geocentric Celestial Reference System and (ii) the new definition of UT1.

The IAU 2000 model for the motion of the CIP in the GCRS is the precession-nutation model recommended by Resolution B1.6 to replace the IAU 1976 precession (Lieske *et al.*, 1977) and the IAU 1980 theory of nutation (Wahr, 1981; Seidelmann, 1982), beginning on 1 January 2003. Specifically it is the MHB 2000 model provided in Mathews *et al.* (2002) designated IAU 2000A or, for applications not requiring the utmost accuracy, its shorter version IAU 2000B (McCarthy & Luzum 2003). The new defining relation of UT1 is the conventional relationship between ERA and UT1 which is provided in IAU Resolution B1.8 to ensure continuity with the previous conventional relationship between GMST and UT1 (Aoki *et al.* 1982).

The implementation of the IAU 2000 system requires computation of expressions compliant with the new precession-nutation model, to be used in the transformation between ITRS and GCRS, once the relationship ERA(UT1) is adopted. IERS Conventions 2000 (see McCarthy & Capitaine 2002 and Chapter 5 of the IERS Conventions 2000) provide equivalent expressions for implementing the IAU 2000 precession-nutation model and the IAU 2000 UT1 definition, using either the CEO-based or the equinox-based transformations.

2.1 The IAU 2000A precession-nutation model

The IAU 2000A model, developed by Mathews *et al.* (2002), is based on the solution of the dynamical equations for the wobble-nutation equations using basic Earth parameters and on the Souchay *et al.* (1999) rigid Earth nutation series. It is composed of: (i) the MHB 2000 nutation series which includes lunisolar and planetary terms, expressed as "in-phase" and "out-of-phase" components with their time variations, (ii) the IAU 1976 precession with corrected rates in longitude and obliquity, and (iii) the associated values for the CIP offsets at J2000 with respect to the GCRS pole. The basic Earth parameters, the corrections to precession rates and the celestial pole offsets at J2000 have been estimated from the most recent VLBI observations. The numerical coefficients of the model include the theoretical geodesic precession and nutation (Fukushima 1991) so that the BCRS and GCRS are without any time-dependent rotation.

The implementation of this model, which is based on a semi-analytical development of nutation, requires the use of trigonometric series, valid over a limited interval of time.

2.2 Implementation of the new precession-nutation

Implementation of the IAU 2000A precession-nutation model using the new paradigm requires expressions for the positions of the CIP and the CEO in the GCRS computed to an accuracy of a few microarcseconds over a time span of a few hundred years, in order to meet the requirements of high-accuracy applications.

The coordinates X, Y of the CIP in the GCRS are functions of the combined precession and nutation, together with the coupling effects between precession and nutation, and include also the frame rotation between the precession-nutation model at J2000 and the GCRS. Developments for X, Y have been provided as polynomials in t plus a series of Poisson terms and the corresponding development for the quantity relative to the CEO has been derived (Capitaine *et al.* 2003a). The expression for the quantity $s + XY/2$ has been developed rather than an expression for s itself, because the former requires fewer terms to reach a given accuracy. This quantity has been derived from a semi-analytical integration, that uses the history of the CIP coordinates (X, Y), as derived from the IAU 2000A precession-nutation model.

2.3 Frame bias with respect to the GCRS

2.3.1 The celestial pole offsets at J2000

The IAU 2000 precession-nutation model is associated with the CIP offsets at J2000 with respect to the GCRS pole which have been derived from VLBI observations. This provides the two first components of the frame bias between the precession-nutation model and the GCRS. These offsets have been used in the computation of the X, Y coordinates of the CIP in the GCRS, either in the development of these quantities for use in the new paradigm, or indirectly through a specific bias rotation in the classical paradigm.

2.3.2 The equinox offset at J2000

The third component of the frame bias, $-d\alpha_0$, which is the GCRS right ascension of the mean equinox at J2000 (*i.e.* offset in right ascension) cannot be derived directly from VLBI observations, which are insensitive at the first order to the position of the ecliptic. Consequently, the current VLBI procedure in fact ignores the equinox offset, introducing in this way a small spurious distortion (*i.e.* second order terms) in the realization of the CIP of the order of $-d\alpha_0 Y$ and

$+d\alpha_0 X$ (Capitaine 1990) in its X and Y coordinates respectively, or equivalently precession-nutation in $\psi \sin \epsilon_0$ and ϵ.

The determination of this offset thus requires the use of observations which are dependent on the position of the ecliptic. It should be noted that to take into account the equinox offset in the proper way, it is necessary to use an equinox (i) which corresponds to an ecliptic dynamically consistent with the IAU's adopted precession-nutation model, (ii) which can be provided by high-accuracy observations.

The numerical value that has been used for the implementation of the IAU 2000 precession-nutation model is the GCRS right ascension of the mean dynamical equinox at J2000 (-14.6 ± 0.5 mas) as provided by Chapront *et al.* (2002) from a fit to LLR observations based jointly on the use of a dynamical theory for the Moon and of VLBI Earth Orientation parameters. The mean equinox of epoch derived in this way corresponds to the definition of the ecliptic in its "inertial" sense to which the recent analytical and numerical solutions for the Moon and planets refer. It differs by 93.66 mas from the "rotational dynamical mean equinox of J2000.0" (Standish, 1981) as used in the past when referring to the FK5 equinox or to the origin of the JPL ephemerides DE200 (Folkner *et al.* 1994), with an accuracy at least 20 times better.

The MHB precession and nutation, being derived from the theory of the Earth's rotation for a rigid Earth (Souchay *et al.* 1999) based on the analytical theories for the planets and the Moon, which refer to the inertial dynamical equinox (Simon *et al.* 1994), are more likely to refer to this equinox.

Note that this $d\alpha_0$ rotation does *not* cause shifts in the right ascensions of stars in either the GCRS or the intermediate frame and that the corresponding shifts in the "true" right ascensions are compensated by changes to the new sidereal time formulas (Capitaine *et al.* 2003b).

2.4 Implementation of the new definition of UT1

The implementation of the new definition of UT1 with the CEO-based transformation uses the conventional relationship between ERA and UT1 together with the quantity s that positions the CEO on the equator of the CIP. The secular and quadratic terms in the semi-analytical expression for the quantity $s + XY/2$, which nearly equals the GCRS right ascension of the CEO, has been fitted by a numerical integration.

The expression for GST to be used in the equinox-based transformation is derived from the relationship between ERA and UT1 and the expression for the accumulated precession and nutation (*i.e.* the equinox-based right ascension of the CEO) based on the IAU 2000A model. The polynomial part of GST is conventionally defined as GMST, whereas the non-polynomial part is the "complete equation of the equinoxes". This is the sum of the *classical* part and the *complementary terms.* The latter (*i.e.* the right ascension of the CEO in the mean frame at J2000) is very similar to the non-polynomial part of the expression for $s + XY/2$.

These expressions have been derived in conjunction with precession and nutation quantities consistent with the IAU 2000A precession-nutation model. The IAU 2000 numerical expressions (Capitaine *et al.* 20003b) linking GST and ERA and locating the CEO produce no discontinuity in UT1 on 1 January 2003 when changing from the current VLBI procedure to the new IAU 2000 system in which ERA(UT1) is a conventional relationship. This takes into account the systematic error of the order of 100 μas (see for more detail Gontier 2002) due to the incorrect use of UT (instead of TT) for computing precession in RA in the old expression for GST.

The expected discontinuity in UT1 rate, shown to be unavoidable due to the improved models and the fixed relationship between ERA and UT1, will have an effect on the determination of UT1 that is less than a few hundreds of microarcseconds over the next century.

The IERS Conventions provide the expressions to be used to implement the IAU 2000 definition of UT1 using the IAU 2000 precession-nutation model, either in the new (CEO-based) or classical (equinox-based) transformations between the ITRS and GCRS.

2.5 New and classical implementations

Various comparisons and numerical checks have been performed between the classical and the new transformations based on the IAU 2000A precession-nutation. These comparisons revealed the need for improvements to the classical form of the transformation in order to achieve the required level of accuracy. Once these improvements are applied, the consistency between (i) the positions of the CIP in the GCRS and (ii) the rotation about the CIP axis, in the CEO-based and equinox-based transformations, when using the IAU 2000 expressions, are at a level of a few microarcseconds after one century.

Such comparisons have demonstrated that the new method, in addition to providing an explicit separation between precession-nutation of the equator from Earth rotation, is more simple, compact and direct than the classical one, achieving accuracies at the level of a few microarcseconds with reduced scope for accidental misuse.

Fortran subroutines implementing the IAU 2000 celestial-to-terrestrial transformations are provided through the IERS Conventions website, and other software will in due course be released through the SOFA website (Wallace 2000). For applications where the classical transformations are to be used, for example where existing software is to be upgraded, the new routines follow a very similar pattern to those that they replace. Some differences are inevitable, because of the need to introduce the frame bias and because the new GMST formula requires TT in addition to UT1. However, the overall process of (for instance) predicting topocentric h, δ of stars is essentially the same as before. The routines offering the new (CEO-based) method follow a similar pattern but are a little more straightforward to use.

2.6 Limitation of the MHB 2000 precession model

It should be noted that, whereas more sophisticated developments of the precession quantities are available (Simon *et al.* 1994, Williams 1994), the MHB 2000 implementation makes use of a precession model which follows the straightforward approach of updating the secular terms only without interpreting them in a physical way. Although of practical utility for the next few years, this interpretation is in fact dynamically inconsistent and suffers, except of course for the improvements in the precession rates, from the same limitations as the IAU 1976 precession in the precision of the coefficients and compliance with up to date models for the ecliptic motion.

Improved quantities for precession, both compliant with the IAU 2000 Resolution B1.6 and dynamically consistent, are being developed (Hilton 2002, Fukushima 2003, Bretagnon *et al.* 2003, Capitaine *et al.* 2003c).

3 References

Aoki, S., Guinot, B., Kaplan, G. H., Kinoshita, H., McCarthy, D. D., and Seidelmann, P. K., 1982, A&A, 105, 359

Atkinson, R.d'E. and Sadler, D.H., 1951, Mon. Not. R. Astr.Soc. 111, 619

Bretagnon, P., Fienga, A., Simon, J.-L., 2003, A&A, in press

Capitaine, N., 1990, Celest. Mech. Dyn. Astr. 48, 127

Capitaine, N., Guinot, B., McCarthy, D. D., 2000, A&A 355, 398

Capitaine, N., Chapront J., Lambert, S., Wallace, P.T, 2003a, A&A, in press

Capitaine, N., Wallace, P.T, McCarthy, D.D., 2003b, submitted to A&A

Capitaine, N., Wallace, P.T, Chapront, J., 2003c, to be submitted to A&A

Chapront, J., Chapront-Touzé M., and Francou, G., 2002, A&A 387, 700

Folkner, W.M., Charlot, P., Finger, et al., 1994, A&A 287, 279

Fukushima, T., 1991, A&A 244, L11

Fukushima, T., 2003, AJ, in press

Gontier, A.M., 2002, in IERS Technical Note No. 29, N. Capitaine *et al.* eds, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am main, 71

Guinot, B., 1979, in *Time and the Earth's Rotation*, D. D. McCarthy and J. D. Pilkington (eds), D. Reidel Publishing Company, pp. 7–18

IERS Conventions 2000, http://www.usno.mil/Conventions 2000, draft

Herring, T., Mathews P.M., Buffett, B.A., 2002, J. Geophys.Res., 107, B4, 10.1029/2001JB000390

Hilton, J., 2002, IERS Technical Note 29, N. Capitaine *et al.* eds, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am main, 101

Lieske, J. H., Lederle, T., Fricke, W., and Morando, B., 1977, A&A 58, 1

Ma, C., et al. 1998, A.J. 116, 516

Mathews, P.M., Herring, T.A., Buffett B.A., 2002, J. Geophys.Res. 107, B4, 10.1029/2001JB000165

McCarthy, D.D. & Capitaine, N., 2002, in IERS Technical Note No. 29, N. Capitaine *et al.* eds, Verlag des Bundesamts für Kartographie und Geodäsie,

Frankfurt am main, 85
McCarthy & Luzum 2003, Celest. Mech. Dyn. Astr. 85(1), 37
Newcomb, S., 1895, in *Astronomical Papers for the American Ephemeris and Nautical Almanac, AP, Vol. 5, Part IV-3*
Seidelmann, P. K., 1982, Celest. Mech. 27, 79
Simon, J. L., Bretagnon, P., Chapront, J. et al., 1994, A&A 282, 663
Souchay, J., Loysel B., Kinoshita, H., Folgueira, M., 1999, A&A Supp. Ser. 135, 111
Standish, E. M., 1981, A&A , 101, L17.
Wahr, J. M., 1981, Geoph. J. R. astr. Soc., 64, 705.
Wallace, P. T., 2000, in *Towards Models and Constants for Sub-Microarsecond Astrometry*, K. Johnston, D. D. McCarthy, B. Luzum and G. Kaplan (eds), US Naval Observatory, Washington, D.C., pp. 353–362
Wallace, P., 2002, IERS Technical Note 29, N. Capitaine *et al.* eds, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am main, 65
Williams, 1994, Astron. J. 108 (2), 711