The VLBI contribution to precession (present and future)

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Precession-nutation parameters

Classical: equinox-based

New: CEO based

Precession of the equator

Nutation of the equator

Precession of the ecliptic

Third IVS Meeting, Ottawa, February 2004
VLBI estimation of celestial pole offsets

w.r.t. 1976/1980 precession-nutation model

VLBI provides the actual position of the pole in the GCRS

-> corrections to the model + frame biases
Position of the CIP in GCRS and ITRS

position of the CIP in the GCRS \((OX_0Y_0Z_0)\): \(E, d\)

position of the CIP in the ITRS \((Ox_0y_0z_0)\): \(F, g\)

\(X = \sin \theta \cos \phi, \ Y = \sin \theta \sin \phi\)

are the \(x, y\)-coordinates of the CIP unit vector

provides directly where the pole is in the sky

 Celestial Intermediate Pole
IAU 2000 precession-nutation

- **IAU 2000 Nutation (FCN not included)**
  IAU 2000A: for high precision level
  Iuni-solar terms (678) + planetary terms (687) (Mathews et al. 2002) based on rigid Earth nutation (Souchay et al. 1999), MHB transfer function and VLBI data
  → 7 basic Earth parameters (BEP) including $H$ (dynamical flattening)
  IAU 2000B (its shorter version): for 1 mas level (McCarthy & Luzum 2002)

- **Precession component of the IAU 2000 model**
  Lieske et al. 1977 + improved rates of precession
  
  \[
  d\psi_A (IAU 2000) = (-0.299650 \pm 0.000400)''/c \\
  d\omega_A (IAU 2000) = (-0.025240 \pm 0.000100)''/c 
  \]

- **Celestial pole offsets at J2000 in the GCRS (VLBI estimates)**
  \[
  \xi_0 (IAU 2000) = (-16.6170 \pm 0.0100) \text{ mas} \\
  \eta_0 (IAU 2000) = (-6.8192 \pm 0.0100) \text{ mas} 
  \]
IAU 2000 expressions for X and Y

(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

\[ X = - 0.\ "016617 + 2004."191743 \ t - 0."4272190 \ t^2 \\
  - 0."1986205 \ t^3 - 0."0000460 \ t^4 + 0."0000060 \ t^5 \\
  + \sum \[(a_{s,0})_i \ \sin(\text{ARGUMENT}) + (a_{c,0})_i \ \cos(\text{ARGUMENT})] \\
  + \sum \[(a_{s,1})_i \ \ t \ \sin(\text{ARGUMENT}) + (a_{c,1})_i \ \ t \ \cos(\text{ARGUMENT})] \\
  + \sum \[(a_{s,2})_i \ t^2 \ \sin(\text{ARGUMENT}) + (a_{c,2})_i \ t^2 \ \cos(\text{ARGUMENT})] \\
  + ... \]

\[ Y = - 0."006951 - 0."025382 \ t - 22."4072510 \ t^2 \\
  + 0."0018423 \ t^3 + 0."0011131 \ t^4 + 0."0000099 \ t^5 \\
  + \sum \[(b_{c,0})_i \ \cos(\text{ARGUMENT}) + (b_{s,0})_i \ \sin(\text{ARGUMENT})] \\
  + \sum \[(b_{c,1})_i \ \ t \ \cos(\text{ARGUMENT}) + (b_{s,1})_i \ \ t \ \sin(\text{ARGUMENT})] \\
  + \sum \[(b_{c,2})_i \ t^2 \ \cos(\text{ARGUMENT}) + (b_{s,2})_i \ t^2 \ \sin(\text{ARGUMENT})] \\
  + ... \]

\textit{precession ; bias effect ; nutation ; cross terms precession\times nutation}
IAU 2000 expression for GST


\[
\text{GST} = \theta + \int_{t_0}^{t_0} (\psi_\lambda + \Delta \psi) \cos(\omega_\lambda + \Delta \varepsilon) \, dt - \chi_\lambda + \Delta \psi \cos \varepsilon_\lambda + \Delta \psi_1 \cos \omega_\lambda
\]

\[
\text{GMST}_{2000} = \theta \text{ (UT1)} + 0''.014506 + 4612''.15739966 \, t + 1''.39667721 \, t^2 - 0''.00009344 \, t^3 + 0''.0000188 \, t^4
\]

accumulated precession in RA (in TT)

\[
\text{EE}_{2000} = + \Delta \psi \cos \varepsilon_\lambda - \sum_k C_k \sin \alpha_k - 0.87 \, \mu \text{as} \, t \, \sin \Omega
\]

kinematical equation of equinoxes
Further improvement in Precession

Expressions for the precession of the equator should be (i) solutions of the dynamical equations for the Earth’s rotation and (ii) referred to an improved ecliptic precession w.r.t Lieske et al. 1977

-> New models compatible with IAU 2000 with improved dynamical consistency

  VSOP87 ecliptic + SMART 97 nutation + MHB precession rate in longitude

- Fukushima (2003, A.J. 126): F03
  ecliptic fitted to DE405 (1600-2200) + SF01 nutation + quadratic fit to VLBI

  ecliptic based on both VSOP87 and fit to DE406 (2000-yr interval) + integration of dynamical equations for precession + MHB precession rates in longitude and obliquity

-> IAU Division I WG on « Precession and ecliptic » established at the 2003 GA (Chair: J. Hilton) to recommend a new model

  Comparison between precession models
P_A : Differences with respect to IAU2000

Year

mas

mas

-5

0

2.5

5

1800 1900 2000 2100 2200

Year

1800 1900 2000 2100 2200

Third IVS Meeting, Ottawa, February 2004
Q.A: Differences with respect to IAU2000

Third IVS Meeting, Ottawa, February 2004
P03 precession


• Improvement of the ecliptic precession
  (based on VSOP/DE406)

• Choice of the integration constants
  (non-rigid Earth model based on MHB2000, Williams 1994 and corrections for perturbing effects on the observed quantities)

• Improvement of the precession of the equator
  (based on: dynamical precession equations, improved ecliptic, updated MHB integration constants)

• Discussion on the precession parameters
  (rotation vector approach, X, Y approach)
The derived precession in longitude is in fact the projection of the observed quantity on the conventional ecliptic with a conventional obliquity used in the current VLBI software.
### Comparison: equator precession solutions

*unit: mas*

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<th>Source</th>
<th>( t )</th>
<th>( t^2 )</th>
<th>( t^3 )</th>
<th>( t^4 )</th>
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<td>5038778.4</td>
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</tbody>
</table>
\[ \Psi_A : \text{Differences with respect to P03} \]
$\omega_A$: Differences with respect to P03

- IAU 2000
- B03
- F03
P03 expressions for X and Y

\[
X = -0.016617 + 2004.191898\ t - 0.4297829 \ t^2 \\
- 0.19861834 \ t^3 + 0.000007578 \ t^4 + 0.0000059285 \ t^5 \\
+ \sum_i [(a_{s,0})_i \ \sin(\text{ARGUMENT}) + (a_{c,0})_i \ \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,1})_i \ t \ \sin(\text{ARGUMENT}) + (a_{c,1})_i \ t \ \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,2})_i \ t^2 \ \sin(\text{ARGUMENT}) + (a_{c,2})_i \ t^2 \ \cos(\text{ARGUMENT})] \\
+ ... 
\]

\[
Y = -0.006951 - 0.025896 \ t - 22.4072747 \ t^2 \\
+ 0.00190059 \ t^3 + 0.001112526 \ t^4 + 0.000001358 \ t^5 \\
+ \sum_i [(b_{c,0})_i \ \cos(\text{ARGUMENT}) + (b_{s,0})_i \ \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,1})_i \ t \ \cos(\text{ARGUMENT}) + (b_{s,1})_i \ t \ \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,2})_i \ t^2 \ \cos(\text{ARGUMENT}) + (b_{s,2})_i \ t^2 \ \sin(\text{ARGUMENT})] \\
+ ... 
\]
(i) the IAU 2000 model gives a considerably better fit to the data

(ii) the data before 1990 are very noisy

(iii) The FCN must be corrected
$$C_0 + C_1 t + C_2 t^2$$

\begin{align*}
\text{RMS} &= 374 \mu\text{as} \\
C_0 &= (1 \pm 10) \mu\text{as} \\
C_1 &= (-1368 \pm 304) \mu\text{as} \\
C_2 &= (-11414 \pm 2579) \mu\text{as} \\
\text{Corr}(C_1, C_2) &= 0.9
\end{align*}

\begin{align*}
\text{RMS} &= 374 \mu\text{as} \\
C_0 &= (2 \pm 10) \mu\text{as} \\
C_1 &= (-1119 \pm 304) \mu\text{as} \\
C_2 &= (-13296 \pm 2580) \mu\text{as} \\
\text{Corr}(C_1, C_2) &= 0.9
\end{align*}
\[ C_0 + C_1 t + C_2 t^2 \]

\[ \text{RMS} = 416 \, \mu\text{as} \]

\[ C_0 = (33 \pm 11) \, \mu\text{as} \]

\[ C_1 = (2134 \pm 338) \, \mu\text{as} \]

\[ C_2 = (24024 \pm 2870) \, \mu\text{as} \]

\[ \text{Corr}(C_1, C_2) = 0.9 \]
Summary

• **VLBI** is potentially the most powerful technique to estimate the motion of the celestial equator w.r.t. GCRS (and consequently the precession of the equator) but not the ecliptic precession.

• **The IAU 2000A precession-nutation** which consists only on corrections to the precession rates in longitude and obliquity of the IAU 1976 precession is not ‘dynamically consistent’ but gives a very good fit to VLBI.

• **Improvement in the precession model for the ecliptic is necessary** (few mas in the t term): the resulting effect on the precession of the equator is at a level of 100 μ as for one century (on the t^2 term in ψ^2).

• **Improvement in the precession model for the equator is necessary.** However, the possible improvement in the model is dependent on the model for some parameters of the non-rigid Earth as the J2 rate.

• **Observations covering a longer period of time are necessary to separate**
  - the precession rates and amplitudes of long-period nutations
  - the precession rates and the t^2 terms in the developments (indirect access to the motion of the ecliptic).
Conclusion

- What is required from VLBI in the future in order it can be powerful at the present level of accuracy for discriminating between various precession models:
  - Refined ICRF (without any global rotation + the largest number of sources)
  - Longest interval of celestial pole offsets with low noise level and no systematic error (FCN)
  - Adopt IAU 2000 precession-nutation model in order than the celestial pole offsets can be considered as being first order quantities
  - Adopt the X, Y representation that is close to the parameters to which VLBI is sensitive
  - Adopt the CEO instead of the equinox to avoid possible risks of inconsistencies in the terrestrial to celestial transformation (for microarcsecond accuracy)

- In parallel:
  Models for contribution ($t$ term and $t^2$ term) to precession of a non-rigid Earth have to be improved (‘improved’ MHB)