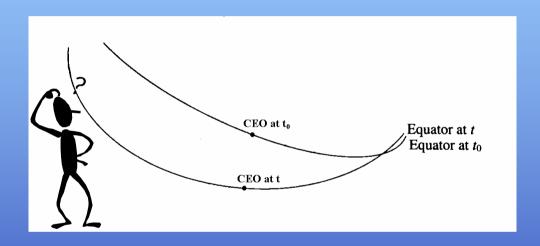
# SOFA software support for IAU 2000



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### Presentation outline

- Introduction to SOFA
- IAU 2000 and SOFA
- Software design choices
- Examples

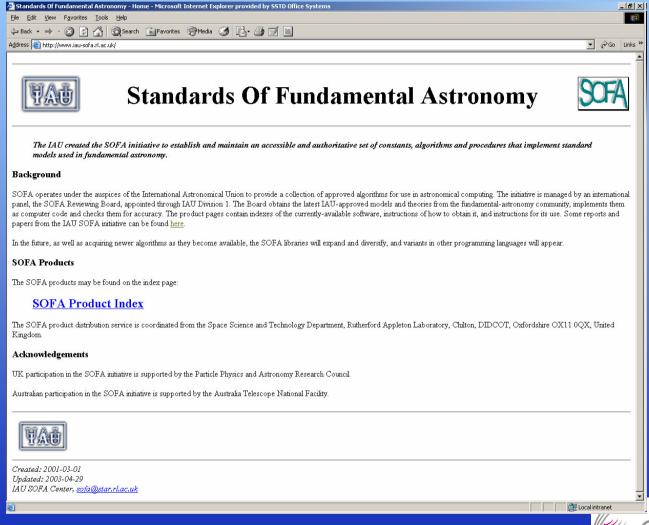


### SOFA

- SOFA (Standards of Fundamental Astronomy) is an IAU initiative to provide authoritative implementations of standard algorithms.
- Controlled by the international SOFA Reviewing Board.
- Currently offers 121 Fortran subroutines:
  - Vectors and matrices
  - Calendars and time scales
  - Precession, nutation
  - Reference frames
  - etc.



### www.iau-sofa.rl.ac.uk



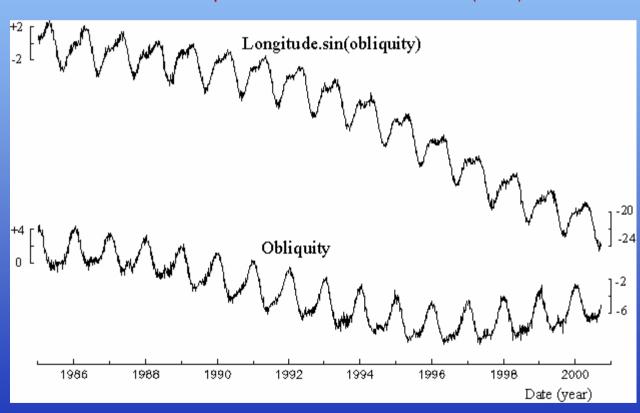
# IAU 2000 inputs to SOFA software

- New, quasi-classical, model for precessionnutation of the "celestial intermediate pole":
  - Starts with IAU 1976 precession and IAU 1980 obliquity
  - New nutation series + bias and precession corrections
  - Alternative abridged nutation model
- New way to express Earth rotation:
  - "Earth rotation angle" proportional to UT1
  - Zero point for matching RA is a "non-rotating origin"



# Why there needed to be changes

#### Errors in 1976/1980 precession-nutation model (mas) from VLBI



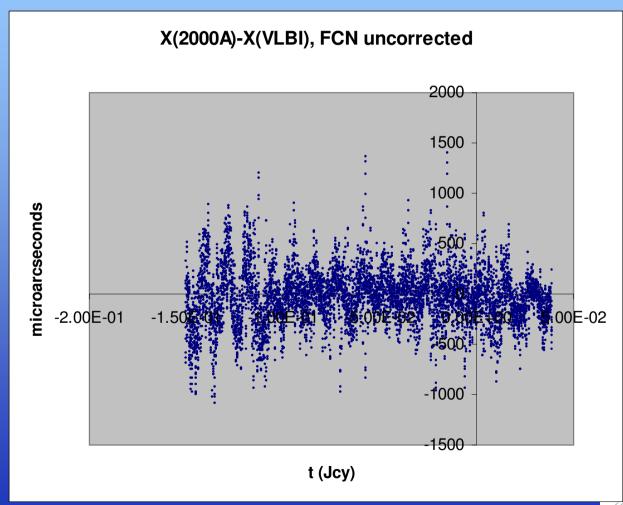


### **Numbers**

- 1 mas ~ aberration you get from walking pace
- 1 µas ~ 30 µm at Earth's surface (~ 1 thou)
- Earth orientation known to 300 µas RMS, ~ 1 cm
- Smallest terms in nutation model ~ 1 µas
- Number of coefficients in nutation model = 2730
- Error in IAU 1976 precession ~ 1 mas/y
- Uncertainty in IAU 2000 precession ~ 30 μas/y
- Nutation-model noise floor ~ 500 µas (from FCN)



### Free core nutation: the noise floor



## Earth rotation, old and new

#### The old (classical) way:

 $\overline{\text{GST}}_{1982}$  (0h) = 24110.54841 + 8640184.812866 t + 0.093104 t<sup>2</sup> - 6.2e-6 t<sup>3</sup> + Δψ cos ε + small correction terms

t is UT. The terms in **red** are "cross-talk" from precession-nutation.

#### The new (IAU 2000) way:

ERA (0h) = 24110.54841 + 8639877.317376 t

#### Calculating Greenwich hour angles:

For GHA = ERA-RA to work (cf. GHA = GST-RA), we simply use a different zero-point for RA, namely the CEO in place of the equinox.

Note the clean separation between Earth rotation and precession-nutation in the new system.



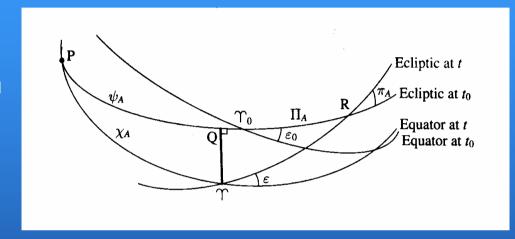
## IAU 2000 GST expression

```
Former IAU 1982/94 expression:
   GST_{1982} (0h) = 24110.54841 + 8640184.812866 t
IAU 2000 compatible expression:
   GST_{2000} (0h) = 24110.5493771 + 8639877.3173760 t<sub>11</sub>
                    + 307.4771600 t_0 + 0.0931118 t_0^2
                      -0.0000062 t_e^3 + 0.0000013 t_e^4
                          + \Delta \psi cos \varepsilon + small correction terms (34)
cf.
   ERA (0h) = 24110.54841 + 8639877.317376 t_{ij}
```

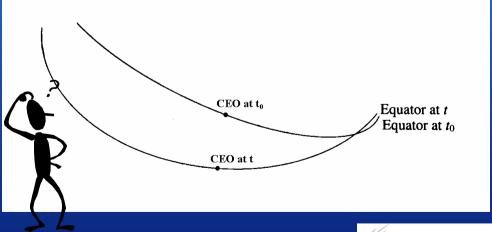


# Zero points of right ascension

 Classical: zero point defined geometrically, by intersection with ecliptic. Messy (intersection of two moving planes) but familiar.



 New: zero point defined kinematically. Tidy but unfamiliar.





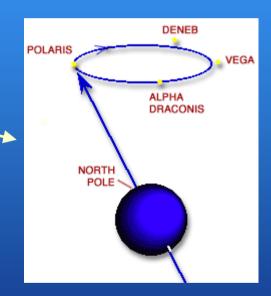
### Where is the CEO?

- The CEO is merely a point on the moving celestial equator that stays as still as it can.
- Obviously, it has to move north-south in the sky as the equator precesses...
- ...but it doesn't move along the equator: from moment to moment it moves only at right-angles to the equator.
- In fact it does creep along a bit, but very slowly.
- The CEO's present  $\alpha_{ICRS}$  is about  $00^h$   $00^m$   $00^s$ .0001; by the end of the century it will have drifted only as far as  $\alpha_{ICRS} = 0^h$   $00^m$   $00^s$ .0046.
- The CEO can be thought of as "a kinematically defined place on the celestial equator close to where the ICRS prime meridian crosses".



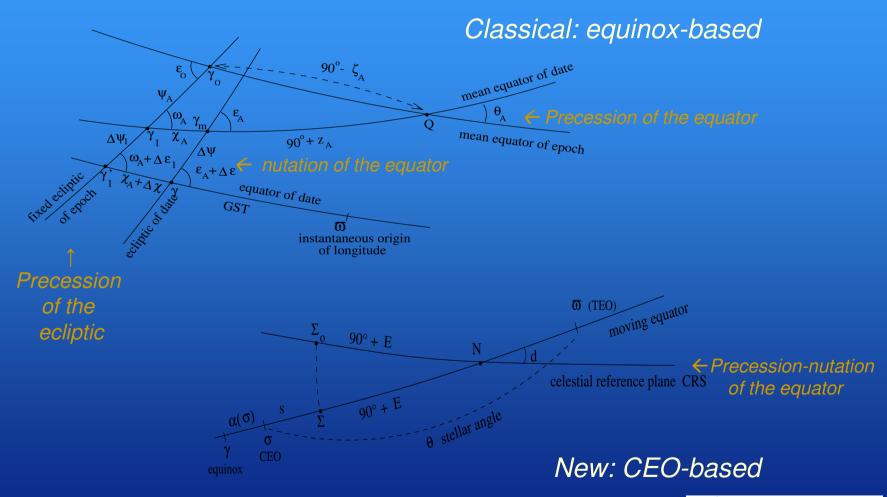
# What is precession?

- Astronomers have traditionally talked of "the precession of the equinoxes" and also have distinguished between luni-solar precession, planetary precession and general precession.
- Lay people just talk about what the pole does.
- Indeed, precession can simply be regarded as the slow component of the motion of the Earth's axis; this is the IAU 2000 picture.
- In IAU 2000, the clean separation between the pole's motion and Earth rotation makes things clearer and reduces "cross-talk" effects.





## Precession-nutation, old and new





# So what happened to the ecliptic?

- The ecliptic remains important in a qualitative and descriptive sense...
- ...and is part of constructing a precession model...
- ...but is no longer needed to define the zero point of right ascension.
- The ecliptic is in any case a rather slippery concept:
  - Is the ecliptic defined by the EMB's path, or the orbital angular momentum vector? n.b. Difference ~ 0.1 arcsec.
  - Does it go through the Sun? Solar system barycentre? Earth-Moon-Sun barycentre?
  - What about long-period nutation terms?
- There is no "IAU 2000 ecliptic" in the SOFA software.



## Precession-nutation matrix, new method

$$R(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E + S - \theta) = Q(t) \cdot R_3(-\theta)$$

 $X = \sin d \cos E$ ,  $Y = \sin d \sin E$ , and  $Z = \cos d$ 

$$Q(t) = \begin{pmatrix} 1 - aX^{2} & -aXY & X \\ -aXY & 1 - aY^{2} & Y \\ -X & -Y & 1 - a(X^{2} + Y^{2}) \end{pmatrix} \bullet R_{3}(s)$$

where  $a = \frac{1}{2} + (X^2 + Y^2)/8$ 



# Direct models for CIP X, Y

(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

```
X = -0. "016617 + 2004."191743 t - 0."4272190 t^2
     -0."1986205 t^3 - 0."0000460 t^4 + 0."0000060 t^5
     + \sum_{i} [(a_{s,0})_{i} \sin(ARGUMENT) + (a_{c,0})_{i} \cos(ARGUMENT)]
     + \sum_{i} [(a_{s,1})_i t \sin(ARGUMENT) + (a_{c,1})_i t \cos(ARGUMENT)]
     + \sum_{i} [(a_{s,2})_i t^2 \sin(ARGUMENT) + (a_{c,2})_i t^2 \cos(ARGUMENT)]
     + ...
Y = -0.006951 - 0.025382 t - 22.4072510 t^{2}
     +0.0018423 t^3 + 0.0011131 t^4 + 0.0000099 t^5
     + \sum_{i} [(b_{c,0})_i \cos(ARGUMENT) + (b_{s,0})_i \sin(ARGUMENT)]
     + \sum_{i} [(b_{c,1})_i t \cos(ARGUMENT) + (b_{s,1})_i t \sin(ARGUMENT)]
     + \sum_{i} [(b_{c,2})_i t^2 \cos(ARGUMENT) + (b_{s,2})_i t^2 \sin(ARGUMENT)]
      + ...
      precession; bias effect; nutation; cross terms precession × nutation
```

# X,Y from classical precession-nutation matrix

$$\mathbf{v}_{\mathsf{GCRS}} = \mathbf{B} \cdot \mathbf{P} \cdot \mathbf{N} \cdot \mathbf{v}_{\mathsf{TRUE}} = \mathbf{R} \cdot \mathbf{v}_{\mathsf{TRUE}}$$

**B** = frame bias matrix (GCRS → mean J2000)

$$= R_3(-\Delta\alpha_0) \cdot R_2(-\Delta\xi_0) \cdot R_1(\Delta\eta_0)$$

**P** = precession matrix

$$= R_1(-\varepsilon_0) \cdot R_3(\omega_A) \cdot R_1(\psi_A) \cdot R_3(-\chi_A)$$

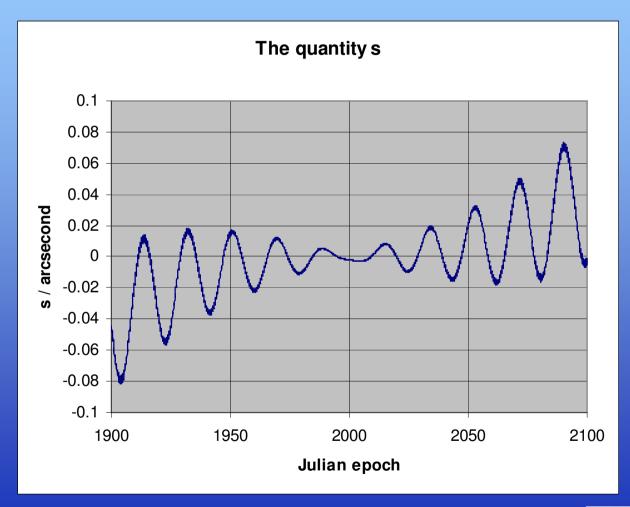
**N** = nutation matrix

$$= R_1(-\varepsilon_A) \cdot R_2(\Delta \psi) \cdot R_1(\varepsilon_A + \Delta \varepsilon)$$

 $\mathbf{R}$  = classical precession-nutation matrix =  $\mathbf{B} \cdot \mathbf{P} \cdot \mathbf{N}$ (X, Y)<sub>CIP</sub> = matrix elements  $\mathbf{R}(1,3)$  and  $\mathbf{R}(2,3)$ 



# The quantity s





# SOFA provision for IAU 2000

- SOFA supports both the new (CEO-based) and the classical (equinox-based) procedures.
  - The CEO-based algorithms are definitive, but the classical equivalents deliver equal accuracy for practical purposes.
- Both IAU 2000A and IAU 2000B are supported.
  - The IAU 2000B nutation series is almost as accurate (1 mas) as the full IAU 2000A series, despite being *shorter* than the IAU 1976 series.
- All these choices mean that IAU 2000 support accounts for nearly a third of the 121 SOFA routines.
  - The routines range from building blocks (e.g. compute nutation  $\Delta\psi$ ,  $\Delta\epsilon$ ) to high-level ensembles (e.g. compute celestial-to-terrestrial matrix), enabling different simplicity/efficiency trade-offs.



## Using the SOFA software

- The improved classical procedures are, as far as possible, "plug-compatible" with existing ones.
  - However, frame bias is new, and GST requires both UT and TT, so applications will need to be changed accordingly.
- You don't have to understand the new CEO-based methods in order to benefit from the improved accuracy.
  - But for newcomers the CEO/ERA method is simpler.
- SOFA is Fortran-only at present.
  - Programmers in C++, Java etc. can use the SOFA routines as blueprints and benchmarks.



# Other software supporting IAU 2000

### IERS routines: http://maia.usno.navy.mil/ch5subs.html

- IAU2000A: MHB 2000 nutation
- IAU2000B: abbreviated nutation
- XYS2000A: X, Y and s
- GMST2000: GMST
- GST2000: GST
- EE2000: equation of the equinoxes
- EECT2000: complementary terms

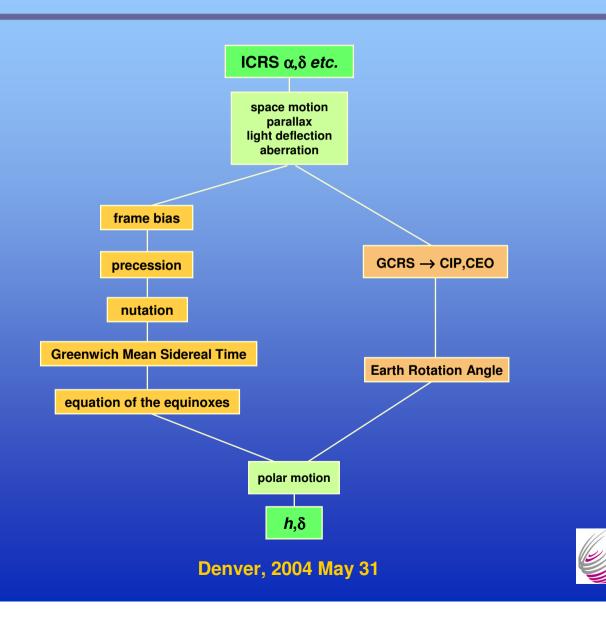
- ERA2000: Earth Rotation Angle
- SP2000: *s*′
- T2C2000: TRS-to-CRS matrix
- POM2000: polar motion matrix
- CBPN2000: classical bpn matrix
- BPN2000: new N•P•B matrix
- NU2000A: nutation, IAU 2000A
- NU2000B: nutation, IAU 2000B

#### In preparation:

- NOVAS
- SLALIB



## Greenwich Hour Angles, old and new: methods



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**AAS 2004** 

# Greenwich hour angles, old and new: example

```
2004 May 31, 22h UTC \lambda = -104.9950, \phi = +39.7427 Fictitious star
```

#### Old-style prediction using Sidereal Time

ICRS	23 32 55.171 +52 16 38.29
Apparent RA,Dec	23 33 06.176 +52 17 43.50
Local HA,Dec	+ 8 05 50.276 +52 17 43.66

### New-style prediction using Earth Rotation Angle

ICRS	23 32 55.171 +52 16 38.29
Intermediate RA,Dec	23 32 53.329 +52 17 43.50
Local HA,Dec	+ 8 05 50.276 +52 17 43.66

