SOFA software support for IAU 2000

Patrick Wallace
Rutherford Appleton Laboratory, UK
ptw@star.rl.ac.uk
Presentation outline

- Introduction to SOFA
- IAU 2000 and SOFA
- Software design choices
- Examples
SOFA

- SOFA (Standards of Fundamental Astronomy) is an IAU initiative to provide authoritative implementations of standard algorithms.
- Controlled by the international SOFA Reviewing Board.
- Currently offers 121 Fortran subroutines:
  - Vectors and matrices
  - Calendars and time scales
  - Precession, nutation
  - Reference frames
  - etc.
Standards Of Fundamental Astronomy

The IAU created the SOFA initiative to establish and maintain an accessible and authoritative set of constants, algorithms and procedures that implement standard models used in fundamental astronomy.

Background

SOFA operates under the auspices of the International Astronomical Union to provide a collection of approved algorithms for use in astronomical computing. The initiative is managed by an international panel, the SOFA Steering Board, appointed through IAU Division 1. The Board obtains the latest IAU-approved models and libraries from the fundamental astronomy community, implements them as computer code and checks them for accuracy. The product pages contain indices of the currently available software, instructions of how to obtain it, and instructions for its use. Some reports and papers from the IAU SOFA initiative can be found here.

In the future, as well as acquiring newer algorithms as they become available, the SOFA libraries will expand and diversify, and variants in other programming languages will appear.

SOFA Products

The SOFA products may be found on the index page:

**SOFA Product Index**

The SOFA product distribution service is co-ordinated from the Space Science and Technology Department, Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, United Kingdom.

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IAU 2000 inputs to SOFA software

- New, quasi-classical, model for precession-nutation of the “celestial intermediate pole”:
  - Starts with IAU 1976 precession and IAU 1980 obliquity
  - New nutation series + bias and precession corrections
  - Alternative abridged nutation model

- New way to express Earth rotation:
  - “Earth rotation angle” proportional to UT1
  - Zero point for matching RA is a “non-rotating origin”
Why there needed to be changes

Errors in 1976/1980 precession-nutation model (mas) from VLBI
**Numbers**

- 1 mas ~ aberration you get from walking pace
- 1 μas ~ 30 μm at Earth’s surface (~ 1 thou)
- Earth orientation known to 300 μas RMS, ~ 1 cm
- Smallest terms in nutation model ~ 1 μas
- Number of coefficients in nutation model = 2730
- Error in IAU 1976 precession ~ 1 mas/y
- Uncertainty in IAU 2000 precession ~ 30 μas/y
- Nutation-model noise floor ~ 500 μas (from FCN)
Free core nutation: the noise floor

X(2000A)-X(VLBI), FCN uncorrected

microarcseconds

t (Jcy)
Earth rotation, old and new

The old (classical) way:

\[
\text{GST}_{1982} (0h) = 24110.54841 + 8640184.812866 \ t + 0.093104 \ t^2 - 6.2e-6 \ t^3
+ \Delta \psi \cos \epsilon + \text{small correction terms}
\]

\( t \) is UT. The terms in red are “cross-talk” from precession-nutation.

The new (IAU 2000) way:

\[
\text{ERA} (0h) = 24110.54841 + 8639877.317376 \ t
\]

Calculating Greenwich hour angles:

For GHA = ERA-RA to work (cf. GHA = GST-RA), we simply use a different zero-point for RA, namely the CEO in place of the equinox.

Note the clean separation between Earth rotation and precession-nutation in the new system.
**IAU 2000 GST expression**

**Former IAU 1982/94 expression:**
\[
\text{GST}_{1982} (0h) = 24110.54841 + 8640184.812866 \ t + 0.093104 \ t^2 - 6.2\times10^{-6} \ t^3 + \Delta\psi \cos\varepsilon + \text{small correction terms (2)}
\]

**IAU 2000 compatible expression:**
\[
\text{GST}_{2000} (0h) = 24110.5493771 + 8639877.3173760 \ t_u + 307.4771600 \ t_e + 0.0931118 \ t_e^2 - 0.0000062 \ t_e^3 + 0.0000013 \ t_e^4 + \Delta\psi \cos\varepsilon + \text{small correction terms (34)}
\]

cf.
\[
\text{ERA} (0h) = 24110.54841 + 8639877.317376 \ t_u
\]
Zero points of right ascension

- **Classical**: zero point defined geometrically, by intersection with ecliptic. Messy (intersection of two moving planes) but familiar.

- **New**: zero point defined kinematically. Tidy but unfamiliar.
Where is the CEO?

- The CEO is merely a point on the moving celestial equator that stays as still as it can.
- Obviously, it has to move north-south in the sky as the equator precesses…
- …but it doesn’t move *along* the equator: from moment to moment it moves only *at right-angles to the equator*.
- In fact it does creep along a bit, but very slowly.
- The CEO’s present $\alpha_{ICRS}$ is about $00^h 00^m 00^s.0001$; by the end of the century it will have drifted only as far as $\alpha_{ICRS} = 0^h 00^m 00^s.0046$.
- The CEO can be thought of as “a kinematically defined place on the celestial equator close to where the ICRS prime meridian crosses”.
What is precession?

- Astronomers have traditionally talked of “the precession of the equinoxes” and also have distinguished between luni-solar precession, planetary precession and general precession.

- Lay people just talk about what the pole does.

- Indeed, precession can simply be regarded as the slow component of the motion of the Earth’s axis; this is the IAU 2000 picture.

- In IAU 2000, the clean separation between the pole’s motion and Earth rotation makes things clearer and reduces “cross-talk” effects.
Precession-nutation, old and new

Classical: equinox-based

New: CEO-based
So what happened to the ecliptic?

- The ecliptic remains important in a qualitative and descriptive sense…
- …and is part of constructing a precession model…
- …but is no longer needed to define the zero point of right ascension.
- The ecliptic is in any case a rather slippery concept:
  - Is the ecliptic defined by the EMB’s path, or the orbital angular momentum vector? *n.b.* Difference ~ 0.1 arcsec.
  - Does it go through the Sun? Solar system barycentre? Earth-Moon-Sun barycentre?
  - What about long-period nutation terms?
- There is no “IAU 2000 ecliptic” in the SOFA software.
Precession-nutation matrix, new method

\[ R(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E + s - \theta) = Q(t) \cdot R_3(-\theta) \]

\[ X = \sin d \cos E, \quad Y = \sin d \sin E, \quad \text{and} \quad Z = \cos d \]

\[ Q(t) = \begin{pmatrix}
1 - aX^2 & -aXY & X \\
-aXY & 1 - aY^2 & Y \\
-X & -Y & 1 - a(X^2 + Y^2)
\end{pmatrix} \cdot R_3(s) \]

where \( a = \frac{1}{2} + \frac{(X^2 + Y^2)}{8} \)
Direct models for CIP X, Y

(Capitaine, Chapront, Lambert, Wallace 2003, A&A 400)

\[
X = -0.016617 + 2004.191743\ t - 0.4272190\ t^2 \\
- 0.1986205\ t^3 - 0.0000460\ t^4 + 0.0000060\ t^5 \\
+ \sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,1})_i t \sin(\text{ARGUMENT}) + (a_{c,1})_i t \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,2})_i t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i t^2 \cos(\text{ARGUMENT})] \\
+ ... \\
\]

\[
Y = -0.006951 - 0.025382\ t - 22.4072510\ t^2 \\
+ 0.0018423\ t^3 + 0.0011131\ t^4 + 0.000099\ t^5 \\
+ \sum_i [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,1})_i t \cos(\text{ARGUMENT}) + (b_{s,1})_i t \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,2})_i t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i t^2 \sin(\text{ARGUMENT})] \\
+ ... \\
\]

precession; bias effect; nutation; cross terms precession × nutation
X, Y from classical precession-nutation matrix

\[ \mathbf{v}_{\text{GCRS}} = \mathbf{B} \cdot \mathbf{P} \cdot \mathbf{N} \cdot \mathbf{v}_{\text{TRUE}} = \mathbf{R} \cdot \mathbf{v}_{\text{TRUE}} \]

\( \mathbf{B} \) = frame bias matrix (GCRS \( \rightarrow \) mean J2000)
\[ = R_3(-\Delta\alpha_0) \cdot R_2(-\Delta\xi_0) \cdot R_1(\Delta\eta_0) \]

\( \mathbf{P} \) = precession matrix
\[ = R_1(-\varepsilon_0) \cdot R_3(\omega_A) \cdot R_1(\psi_A) \cdot R_3(-\chi_A) \]

\( \mathbf{N} \) = nutation matrix
\[ = R_1(-\varepsilon_A) \cdot R_2(\Delta\psi) \cdot R_1(\varepsilon_A + \Delta\varepsilon) \]

\( \mathbf{R} \) = classical precession-nutation matrix = \( \mathbf{B} \cdot \mathbf{P} \cdot \mathbf{N} \)
(\( X, Y \))\(_\text{CIP} \) = matrix elements \( \mathbf{R}(1,3) \) and \( \mathbf{R}(2,3) \)
The quantity $s$
SOFA provision for IAU 2000

- SOFA supports both the new (CEO-based) and the classical (equinox-based) procedures.
  - The CEO-based algorithms are definitive, but the classical equivalents deliver equal accuracy for practical purposes.

- Both IAU 2000A and IAU 2000B are supported.
  - The IAU 2000B nutation series is almost as accurate (1 mas) as the full IAU 2000A series, despite being shorter than the IAU 1976 series.

- All these choices mean that IAU 2000 support accounts for nearly a third of the 121 SOFA routines.
  - The routines range from building blocks (e.g. compute nutation $\Delta \psi, \Delta \epsilon$) to high-level ensembles (e.g. compute celestial-to-terrestrial matrix), enabling different simplicity/efficiency trade-offs.
Using the SOFA software

- The improved classical procedures are, as far as possible, “plug-compatible” with existing ones.
  - However, frame bias is new, and GST requires both UT and TT, so applications will need to be changed accordingly.
- You don’t have to understand the new CEO-based methods in order to benefit from the improved accuracy.
  - But for newcomers the CEO/ERA method is simpler.
- SOFA is Fortran-only at present.
  - Programmers in C++, Java etc. can use the SOFA routines as blueprints and benchmarks.
Other software supporting IAU 2000


- IAU2000A: MHB_2000 nutation
- IAU2000B: abbreviated nutation
- XYS2000A: X, Y and s
- GMST2000: GMST
- GST2000: GST
- EE2000: equation of the equinoxes
- EECT2000: complementary terms
- ERA2000: Earth Rotation Angle
- SP2000: $s'$
- T2C2000: TRS-to-CRS matrix
- POM2000: polar motion matrix
- CBPN2000: classical bpn matrix
- BPN2000: new N•P•B matrix
- NU2000A: nutation, IAU 2000A
- NU2000B: nutation, IAU 2000B

In preparation:

- NOVAS
- SLALIB
Greenwich Hour Angles, old and new: methods

ICRS $\alpha, \delta$ etc.

- space motion
- parallax
- light deflection
- aberration

- frame bias
- precession
- nutation

Greenwich Mean Sidereal Time

- equation of the equinoxes

GCRS $\rightarrow$ CIP, CEO

- Earth Rotation Angle

- polar motion

$h, \delta$
Greenwich hour angles, old and new: example

2004 May 31, 22h UTC
\( \lambda = -104.9950, \ \varphi = +39.7427 \)
Fictitious star

Old-style prediction using Sidereal Time

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<th>ICRS</th>
<th>Apparent RA,Dec</th>
<th>Local HA,Dec</th>
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New-style prediction using Earth Rotation Angle

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