The solar quadrupole moment from planetary ephemerides: present state of the art

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Structure of this speech

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Part II. Present estimates of $J_2$
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   b. Possible variation with the solar cycle

Part III. Relevance of $J_n$
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   b. In other fields

Part IV. $J_2$ from planetary ephemerides?
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   b. Method
   c. Planetary ephemeris tests carried
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(Part V. Future space missions
   a. In solar astrophysics
   b. In fundamental physics (dedicated or not) )
Part I. Definition of $J_n$

- Outer solar gravitational potential at distance $r$ ($r > R_{\odot}$):

$$\phi_{grav} = -\frac{GM}{r} \left[ 1 - \sum \left( \frac{R_{\odot}}{r} \right)^{2n} J_{2n} \ P_{2n} (\cos \theta) \right]$$

Gravitational moments ($J_2 \sim 10^{-7}$)

- $J_n$ and solar shape are related:
  
  solar surface = equipotential wrt total potential (gravitational, rotational…)

Distorted shape analogy:

**Earth** = geoid    **Sun** = helioid
Part II. Present estimates of $J_2$

Ila. Methods yield estimates

- Stellar structure equations + Differential rotation model
  
ex: [Godier, Rozelot, 1999] $\sim 1.6 \times 10^{-7}$

- Theory of the Solar Figure
  
ex: [Rozelot, Lefebvre, 2003] $\sim 6.5 \times 10^{-7}$

- Helioseismology
  
ex: [Pijpers, 1998] $\sim 2.2 \times 10^{-7}$

Ilb. Possible variation of $J_2$ with the solar cycle

... related to solar shape variations
Part III. Relevance of $J_2$

Illa. In solar astrophys.: reflects the physics of solar models

- Influence of solar core properties on $J_n$, $\odot$ shape and $\vec{J_\odot}$?

- Influence of latitudinal or differential rotation on $J_n$ and $\odot$ shape?

- Precise determination of the $J_n$ and $\odot$ shape, still poorly known?

- Are the $\odot$ shape and $J_n$ time dependant?

- Constraining solar - models (differential rotation, density inhomogeneities) - evolution?
IIIb. In other fields:

**A/ Dynamical conseq. of $J_2$: relativistic astrometry**

![Diagram of space-time curved by the solar body with light deflection measured by angle $\hat{\alpha}$]

**Relativistic light deflection**

**\[ \hat{\alpha}_{\text{grazing}} = +2(1 + \gamma) \frac{GM}{R_\odot c^2} \]**

need precise knowledge of $(J_2, \overrightarrow{J_\odot})$ for precise astrometry in solar neighborhood:

\[ + 2(1 + \gamma)J_2 \frac{GM}{R_\odot c^2} \]

~1.75 arcsec

\[ \pm 2(1 + \gamma)\left| \frac{G\overrightarrow{J_\odot}}{R_\odot^2 c^3} \right| \]

~0.4-0.3 μarcsec

~0.7 μarcsec

Post-Newtonian parameter encoding the amount of curvature of space-time per unit rest mass

\[ \Theta\left( \frac{1}{c^4} \right) \]

[Pireaux, 2002]
B/ Dynamical conseq. of $J_2$: relativistic celestial meca.

- **Indirect influences of $J_2$:**

Through solar system bodies spin-orbit couplings, $J_2$ and $\overrightarrow{J_\odot}$ will influence the motion $(a, e, i)$ of solar system bodies.

\[ J_2 \odot \rightarrow \Phi \rightarrow \zeta \]

**dynamical constraints on** $J_2 \odot$ and $\overrightarrow{J_\odot}$

**ex:** by lunar librations: $J_2 \odot \leq 3 \cdot 10^{-6}$

[Rozelot, Rösch, 1997]
[Bois, Girard, 1999]
**A direct influence of** \( J_2 \): 

Purely relativistic contribution (PPN) → \( J_2 \) in planetary perihelion advances

\[
\Delta \omega = \Delta \omega_{0,\text{GR}} \bullet \left[ \frac{1}{3} \left( 2 + 2 \gamma - \beta \right) + \frac{R_{\odot}^2}{R a (1-e^2)} J_{2,\odot} \left( 3 \sin^2 i - 1 \right) \right]
\]

42,98 arcsec/century for \( \odot \)

Post-Newtonian parameter encoding the amount of non-linearity in the superposition law of gravitation

<table>
<thead>
<tr>
<th>Mercury perihelion advance (arcsec/century)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PPN</strong></td>
</tr>
<tr>
<td>( J_{2,\odot} )</td>
</tr>
<tr>
<td>~ 43</td>
</tr>
<tr>
<td><strong>Equinoxes</strong></td>
</tr>
<tr>
<td>~ 5000</td>
</tr>
<tr>
<td><strong>planets:</strong></td>
</tr>
<tr>
<td>Venus ( \sim 280 )</td>
</tr>
<tr>
<td>Jupiter ( \sim 150 )</td>
</tr>
<tr>
<td>Others ( \sim 100 )</td>
</tr>
</tbody>
</table>
Precise $J_2$ will be needed for precise ephemeris.

Presently, strong correlation $(J_2, \beta)$ or $(J_2, \gamma)$ in planetary ephemerides [to be detailed]

- Other direct influences of $J_2$:
  - planetary spin
  - variation of the ecliptic plane

Precise $J_2$ needed for long-term solar system models [Laskar, 1999, 2004]

C/ Testing alternative theories of gravitation

$J_2$, $\beta$, $\gamma$, $G$ in PPN formalism and in IAU2000 BCRS standard metric
Part IV.  $J_2$ from planetary ephemerides

IVa. Aim: dynamical constraints on solar $J_2$

\[ J_2, \gamma - 1, \beta - 1, \frac{\dot{G}}{G} = ? \]

IVb. Method:

- **Simultaneous** least square fit of parameters (planetary, $J_2$, PPN) in a planetary motion model to observations of planets
- Planetary ephemerides considered:
  - JPL (E.M. Standish, NASA, USA): DE413, DE415
  - EPM (E. Pitjeva, IAARAS, Russia): EPM2003, EPM2004
  - joint JPL-EPM 2006 solution
IVc. Planetary ephemeris tests carried

A/ Observational data sets available (inner planets)

[PIreaux, Standish, Pitjeva, Rozelot, Celest. Mec. in prepa]

B/ Different tests... for a given epoch (JD2440400.5) and ± epochs

Testing relevance of data sets?

Tests #1 → 8 : GR assumed ( \( \tilde{G}/G = \beta - 1 = \gamma - 1 = 0 \) )

with #0 = base solution = all data sets → \( J_2 \) estimate in the setting of

\( \text{GR} = J_2^{GR} \)

\((J_2, \beta, \gamma, G)\) parameters correlation?

Tests #9 → 12 : all data sets, GR relaxed

with #10: \( \tilde{G}/G, \beta, \gamma \) relaxed → \( J_2 \) estimate in the setting of

fully conservative theories

\( = J_2^{\text{fully-cons.}} \)

Discriminate between \( J_2 \) prior assumptions?

Tests #13 → 20 : all data sets, assumption on \( J_2 = 2.0 \times 10^{-7} \) or \( 6.5 \times 10^{-7} \)

[Rozelot, Lefebvre, 2003] Theory of Solar Figure

[Lydon, Sofia, 1996] Solar oblateness measurements
IVd. PRELIMINARY results

Testing relevance of data sets?
- Mars data is crucial (largest data sets, more precise)
- Mercury data is important (strongest relativistic perihelion advance)
- Among Mars data, ranging is the most important (precise, numerous)
- Post-1985 Marsian data is important (more precise)

Testing \((J_2, \beta, \gamma G)\) parameters correlation?
- For fully-cons. theories of gravitation: strong \((J_2, \beta), (J_2, \gamma)\) correlations !!
  ➡️ improved significantly with the recent ephemerides used here
  ➡️ might improve in the future:
    - more data
    - space missions to decorrelate \(\beta-J_2\) (GAIA, Bepi Colombo…)
- High sensitivity of fully conservative solution to any perturbation
  ⬅️ least-square minimum = very flat surface
  ⬅️ \(\beta-1, \gamma-1\) are very small (solar system tests), \(J_2 \sim 10^{-7}\)
- \(\beta\) and \(\gamma\) estimates from ephemerides agree with those of solar system tests

Testing \(J_2\) assumptions?
- constraining \(J_2\)-value tends to distort other parameter values
All tests:

- $\sigma = 1$ formal errors, are too optimistic (by a factor 10 or more).

- For fully-cons. theories of gravitation: $\beta, \gamma$ estimates sensitive to data-set weight.

- $J_2$-values close to helioseismic estimates are favored by ephemeris tests

  \( \leftrightarrow J_2 \text{Theory of Solar Figure} \)

- $J_2^{GR}$ central value estimate $< J_2^{fully-cons.}$ central value estimate

- Should not (yet?) trust exact central-value estimate of $J_2$ from ephemerides:
  - epoch dependency problem to be solved
  - sensitivity to data weight
    - … but order of magnitude ($10^{-7}$) is NOW correct.

- Comparisons between JPL and EPM planetary ephemerides
  - improved both ephemerides
  - some errors were found and corrected
  - converging towards an estimate of $J_2$ in the setting of GR
Other transparencies...
Part IV. Future space missions

1. In solar astrophysics

- Better knowledge of solar diameter, rotation, core dynamics: space missions
- Solar cycle dependency
2. In fundamental physics (dedicated or not)

A/ Adopted missions

- **Stronger constraints on PPN:**
  \(~10^{-7} \text{ (GAIA)} \) or \(~10^{-7} \text{ (BeppiC)}\) on \(\gamma\) from light deflection
  \(~(2-9) \times 10^{-4} \text{ (GAIA)} \) or \(~2 \times 10^{-6} \text{ (BeppiC)}\) on \(\beta\) from \(\Delta\omega\).

- **Possible decorrelation \(J_2\) in \(\Delta\omega\):**
  \(~10^{-8}\) on \(J_2\) from \((a,e,i)\)-dependancy of \(J_2\)-term in \(\Delta\omega_{\text{minor planets}}\)

- **Possible measurement of the precession of the orbital plane around the polar axis of the Sun:** \(J_2\)-effect only,
  \(~2 \times 10^{-9}\) on \(J_2\)

  - Improved planetary ephemerides (topo) and \(\Delta\omega\)

B/ Proposed missions

- **ASTROD**
  \(~10^{-9}\) on \(\gamma\) from light deflection, \(~10^{-7}\) on \(\beta\), \(~1 \times 10^{-8}-5 \times 10^{-9}\) on \(J_2\)

- **LATOR**
  \(~10^{-7}\) on \(\gamma\) from light deflection, \(~10^{-4}\) on \(\beta\), \(~10^{-7}\) on \(J_2\)
References

Lask, http://hal.ccsd.cnrs.fr/ccsd-00001603
Pitjeva, Astron. Letter, 31, 5, 340-349, 2005
Standish, private communication 2004