Chaos and relativistic effects in the rotational dynamics of minor planetary satellites //

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Abstract

The chaotic regime of rotation of minor planetary satellites taking place during tidal evolution is considered. The possibility of the formation of strange attractors in the rotational dynamics of all known minor planetary satellites of the Solar system is estimated. A detailed analysis of the presence of anisotropy in the orientation of the satellite figure during its chaotic rotation was carried out. The relativistic effect (the geodetic precession, which is the part of the effect of the geodetic rotation) in the rotation of some minor satellites for the first time is investigated. As a result, in Euler angles, the most significant systematic terms of the geodetic rotation are calculated.

Currently, the total number of known planetary moons is approaching two hundred [6]. Wisdom et al. [13] showed that a satellite of a strongly non-spherical shape (a typical shape of the small satellites) in an elliptical orbit can rotate chaotically. It was found [4, 5, 13] that the satellite of Saturn Hyperion (S7) is in chaotic rotation mode; Prometheus (S16) and Pandora (S17) have a high probability [8, 10] of transition from synchronous to chaotic rotation. Studies [2, 7, 9] of the planar rotational motion of a satellite in the presence of tidal dissipation have shown that, in the phase space of rotational motion, a strange attractor can exist. In [10, 12] indicated that chaotic rotation of a satellite may result in a preferred orientation of the largest axis of the satellites figure toward the planet. If we consider the rotational motion of the satellite in the gravitational field of the planet in the post-Newtonian approximation, it is necessary to take into account the effects of geodetic precession and nutation, which together make up the geodetic rotation [3].

Consider the plane (in the orbit plane) rotational motion of the satellite relative to the center of mass. The equation of motion taking into account tidal interaction (within the framework of the MacDonald model) has the form [2, 7, 9]:

$$(1+e\cos f)\frac{\mathrm{d}^2\theta}{\mathrm{d}f^2}+\left[\beta(1+e\cos f)^5-2e\sin f\right]\frac{\mathrm{d}\theta}{\mathrm{d}f}+\omega_0^2\sin\theta\cos\theta=2e\sin f,$$

where e — the eccentricity, f — the true anomaly, $\omega_0 \simeq \sqrt{3(a^2-b^2)/(a^2+b^2)}$, $a>b>c$ — the semiaxes of a triaxial ellipsoid approximating the satellite figure; θ — the angle between the axis of the smallest principal central moment of inertia of the satellite (the largest axis of the triaxial ellipsoid approximating the figure of the satellite) and the radius-vector "planet – center of mass of the satellite"; the dimensionless parameter $\beta \geq 0$ characterizes the value of the tidal interaction. By calculating the Lyapunov exponents (LE), we found that for certain values of e, ω_0 and β in the phase space of rotational motion there is a strange attractor for which the maximum LE is greater than zero, that is, the motion is chaotic. This conclusion is confirmed by the analysis of representative phase space sections constructed for the selected values of e, ω_0 and β. At the sections (see the example for Hyperion in Fig. 1a, b), there is a structure characteristic of a strange attractor. For most planet satellites whose shape parameters are determined, the values of $\beta \in [10^{-6}, 10^{-4}]$ [9]. If we place these satellites on the plane (ω_0, e), where the existence regions of the strange attractor are highlighted, then can be verify (see Fig. 1c, d) that Hyperion (S7) and Phoebe (S9) fall into these regions. Phoebe is in fast non-synchronous rotation and it is located in the phase space far from the strange attractor that exists in the vicinity of synchronous resonance. Hyperion, for which $\beta \sim 10^{-6}$, is currently [4, 5] in the chaotic rotation mode and, most likely, the rotation occurs on a strange attractor. Consequently, the probability that Hyperion can leave the chaotic rotation regime is very small.

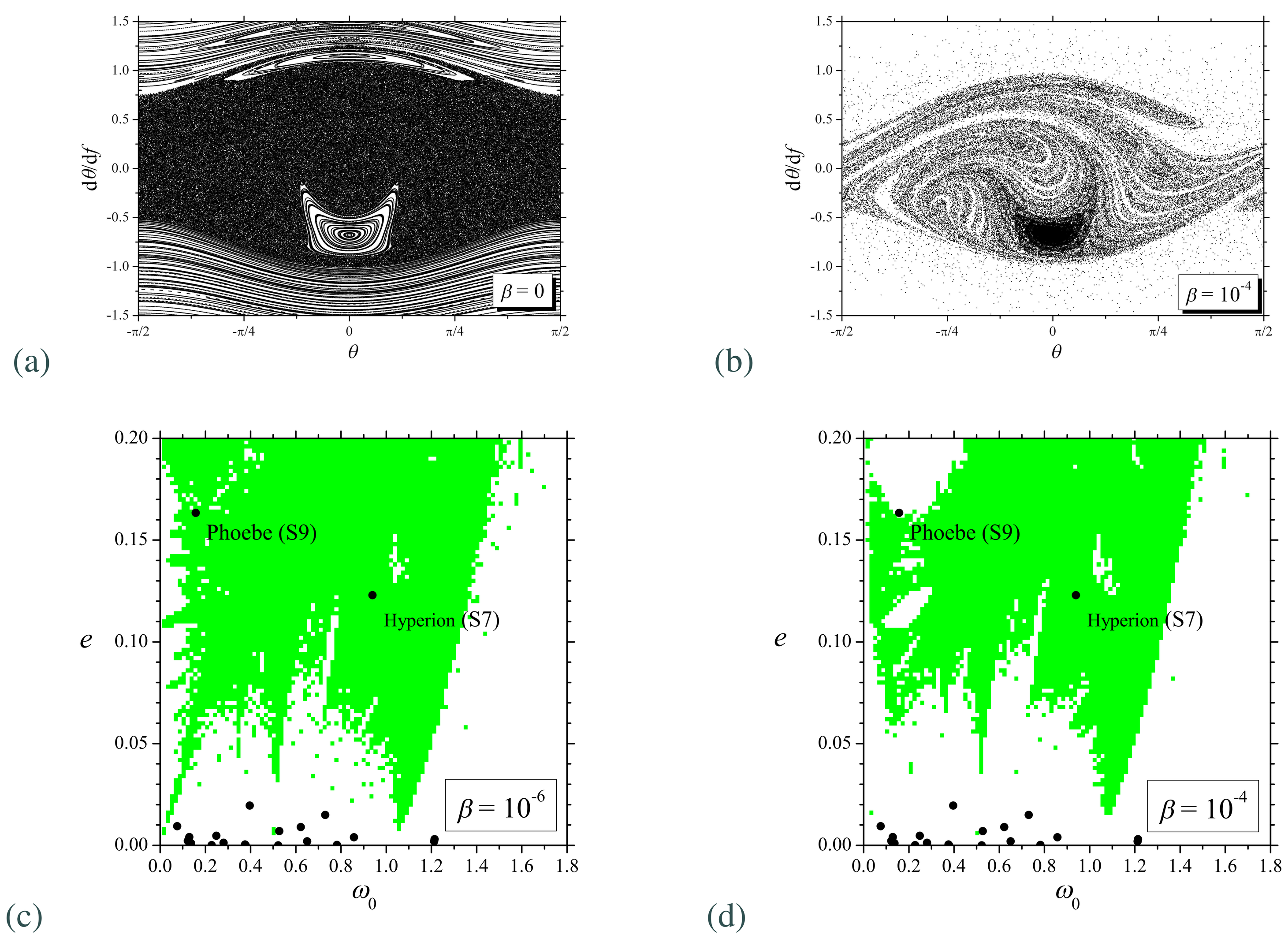

Figure 1: The phase space section of the planar rotational motion for $e = 0.123$, $\omega_0 = 0.936$ (Hyperion (S7)): (a) $\beta = 0$ — there is no tidal interaction, (b) an example of a strange attractor for $\beta = 10^{-4}$. (c) and (d) — the regions of existence of the strange attractor (green) for different values of β. Dots mark the locations of a some of known planetary satellites.

In [8, 10] it was shown that for two satellites of Saturn — Prometheus (S16) and Pandora (S17) there is a high probability of a transition from a synchronous rotation mode to a chaotic one. Our massive numerical experiments on modeling the spatial chaotic rotation of Prometheus and Pandora showed that the chaotic rotation of these satellites is similar to ordinary synchronous rotation in the sense that the satellites retain their preferred orientation over long time intervals (100–1000 orbital periods) — the largest axis of the satellites figure is directed mainly to Saturn (see the example for the case of Prometheus in Fig. 2). In Fig. 3, on the integration time interval $t = 10^5$ of orbital periods, the behavior of σ — the relative amount of time during which the largest axis of the satellite figure is oriented to Saturn on a segment of 1000 of orbital periods is shown. It can be seen that the value $\sigma \simeq 9\%$, which in the case of Prometheus corresponds to "isotropic" chaotic rotation, is several times exceeded. The chaotic rotational dynamics of Pandora has a similar character. It should be expected that in the chaotic rotational dynamics of other small planetary satellites in the vicinity of synchronous resonance, the effect of the predominant orientation of their largest axis of the figures on the planet should be manifested. The discovered phenomenon can make it difficult to detect chaotic rotation of small satellites, for which there are indications of a possible chaotic rotation, by analyzing observational data if the observation interval is not large enough.

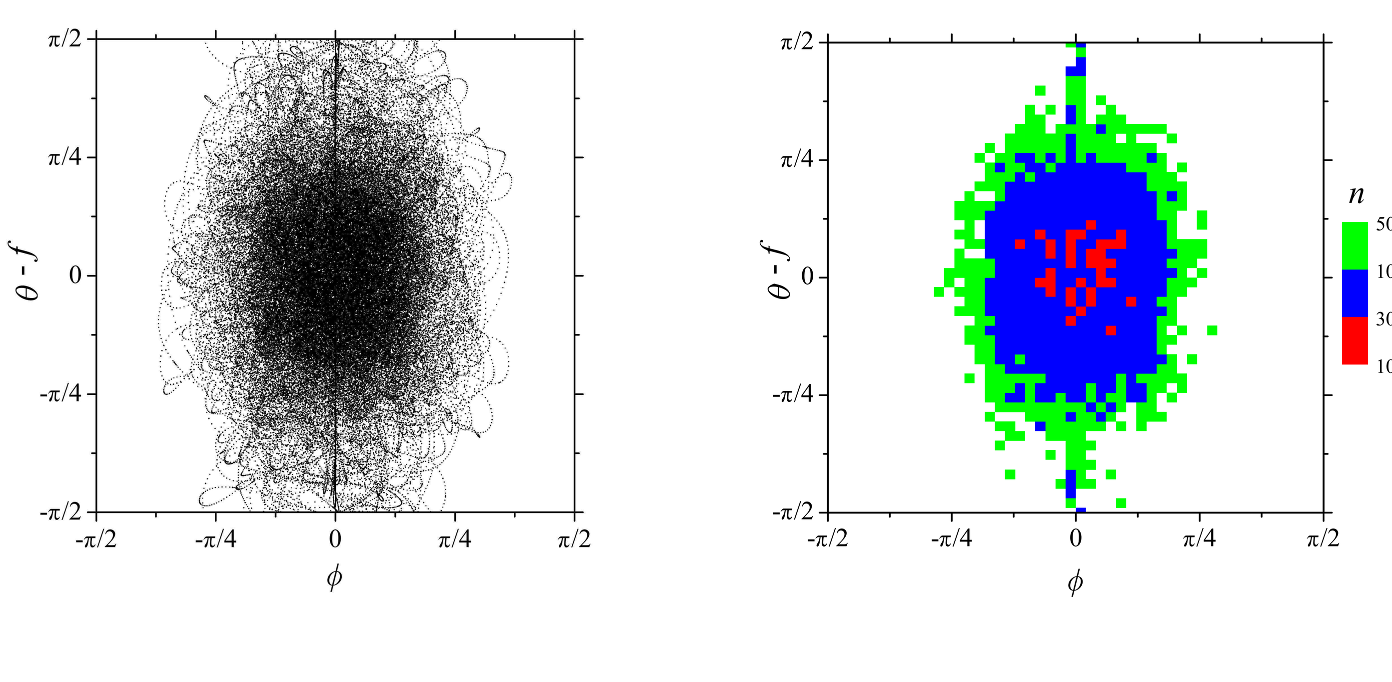

Figure 2: Orientation of Prometheus (S16) in chaotic rotation. (a) The projection of the chaotic trajectory to $(\phi, \theta - f)$ plane, where ϕ is the angle between the axis of rotation and the normal to the plane of the orbit, θ is the angle between the largest axis of the satellites figure and the direction to Saturn, f is the true anomaly. The integration time is 1 000 orbital periods. (b) Density graph of discrete projections of a chaotic trajectory to $(\phi, \theta - f)$ plane. The integration time is 10 000 orbital periods. The white area corresponds to $n < 50$. The direction to Saturn corresponds to the point $(0, 0)$.

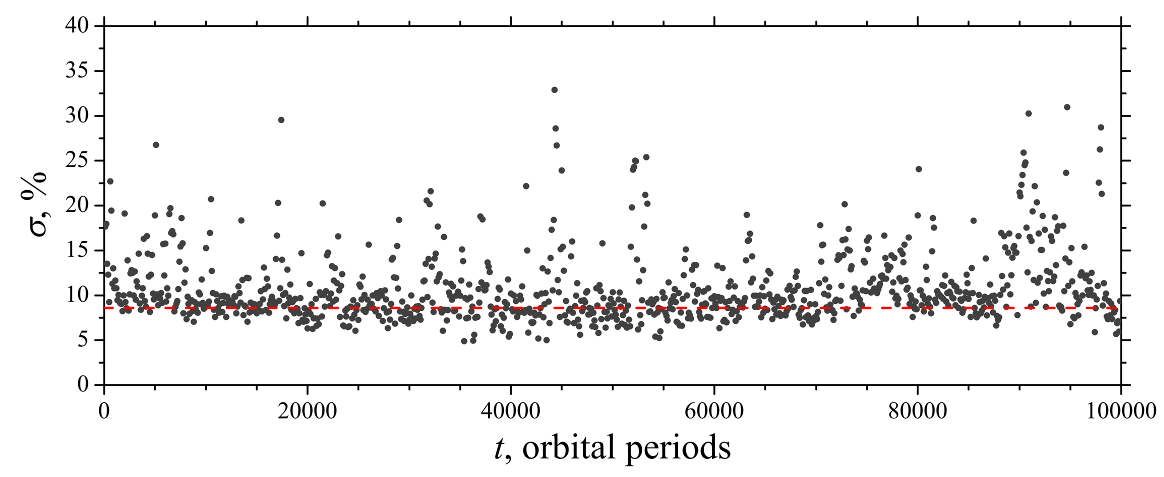

Figure 3: The relative percentage of time σ during which, with chaotic rotation, the largest axis of the figure of Prometheus (S16) is oriented to Saturn. Under "isotropic" chaotic rotation $\sigma \simeq 9\%$ (red dashed horizontal line).

The geodetic rotation of a body is the most essential relativistic effect of its rotation and consist of two effects: the geodetic precession is the systematic effect and the geodetic nutation is the periodic effect. These effects have some analogies with precession and nutation, which are better-known events on the classical mechanics. Their emergence, unlike the last classical events, are not depend on from influences of any forces to body, represents only the effect of the curvature of space-time, predicted by general relativity [3], on a vector of the body rotation axis carried along with an orbiting body.

Using the technique developed in [11], we obtained estimates of the values of the systematic terms in the Euler angles (ψ, θ, ϕ) and the rates of their change for the geodesic rotation of a some of planetary satellites. Satellites were considered whose rotation parameters were well established [1]. Table 1 shows the differences $\Delta x = x_{\mathrm{r}} - x_{\mathrm{N}} = \Delta x_1 T + \Delta x_2 T^2 + \ldots$ of the relativistic (x_{r}) and Newtonian Euler angles (x_{N}) of the body under study, where $x = \psi, \theta, \phi$. Only the first two terms of the expansion are presented, making the main contribution to the geodesic precession; Δx_i values are given in arc seconds per thousand years. Fig. 4 shows the rate of change of the magnitude of the geodesic rotation.

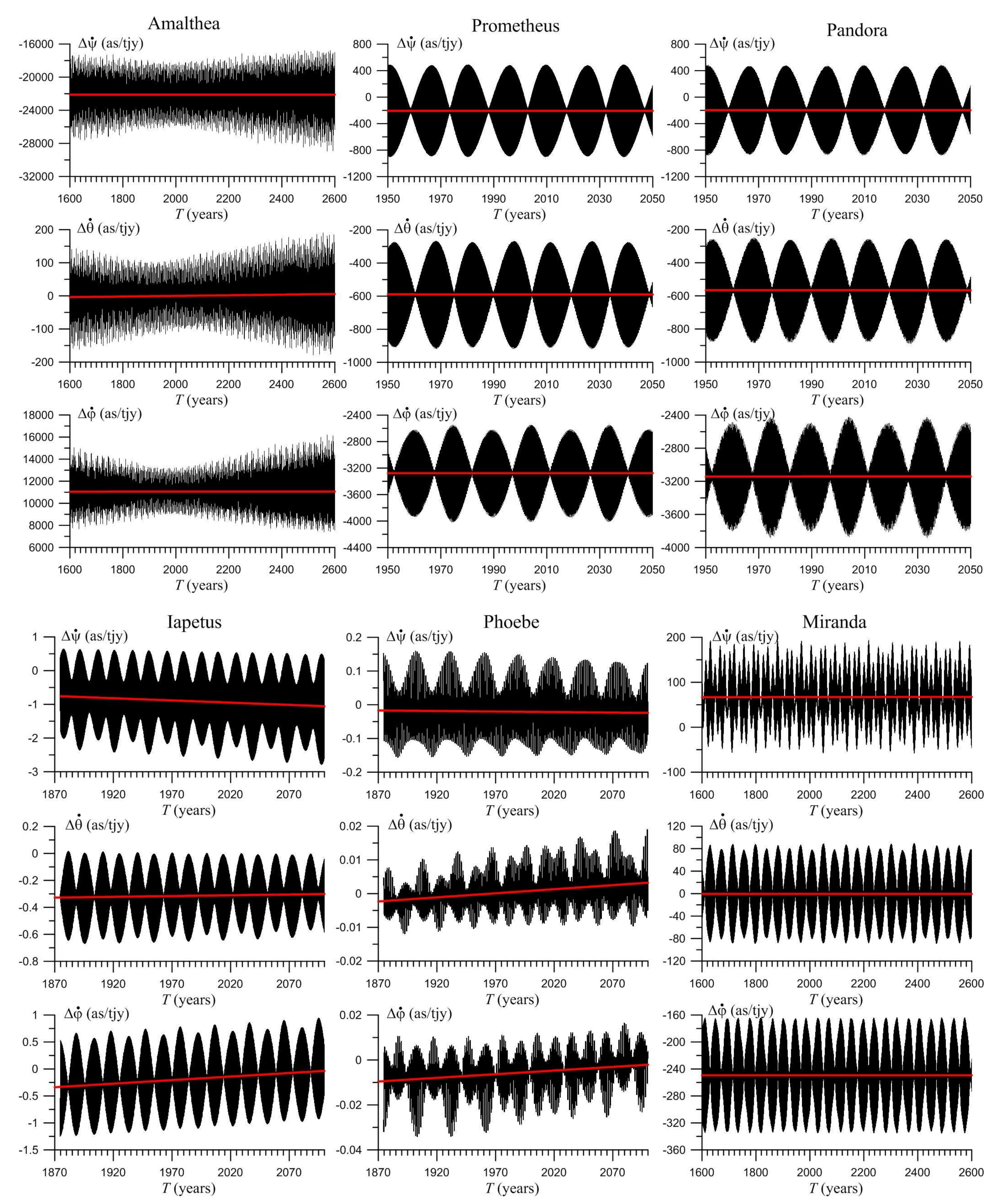

Figure 4: The values of the velocities of the change in full effect of the geodetic rotation (black) and in only geodetic precession of some planet satellites (red line).

Table 1: The systematic terms of geodetic rotation of some planet satellites (in as/tjy).

	Mercury	Amalthea (J5)	Prometheus (S16)	Pandora (S17)	Iapetus (S8)	Phoebe (S9)	Miranda (U5)
$\Delta\psi_1$	-426.4	-22118.2	-205.6	-197.2	-0.924	-0.021	67.04
$\Delta\psi_2$	-0.039	-0.756	-1.623	-1.679	-0.656	-0.016	0.365
$\Delta\theta_1$	0.036	-0.092	-590.1	-566.0	-0.313	0.001	-0.472
$\Delta\theta_2$	-0.003	4.735	-3.283	-3.205	0.058	0.012	-0.0002
$\Delta\varphi_1$	214.8	11055.2	-3275.7	-3142.2	-0.167	-0.005	-249.4
$\Delta\varphi_2$	0.002	0.579	1.986	2.374	0.659	0.016	0.001

Despite the fact that the Sun (1000 times) is more massive than Jupiter, the value of the geodetic precession of the fifth satellite of Jupiter Amalthea is 50 times greater than the value of the geodetic precession of Mercury, which is the largest among ones values of major planets of Solar system [11]. Almatea (J5) is 300 times closer to Jupiter than Mercury is to the Sun. Therefore, in this case, the smaller distance of the satellite to its central body has a greater effect in calculating the geodetic precession of the satellite than effect other more massive central body in calculating the geodetic precession of a more distant planet. In the case of Amalthea, the geodetic precession we obtained is about 0.006 degrees of arc per year and must be taken into account in the existing ephemeris [1].

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