Improved geodetic-hydrological residual time series constrained by polar motion observations and geophysical models

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Introduction

The mass transport occurring in the Earth has a large influence on the excitation of polar motion. However, determination of the accurate time series of geophysical Earth rotation excitations coming from atmosphere, ocean and land hydrology models is far from being completely explain observed variations in polar motion. The major contributors to changes in the Earth's rotation are atmosphere and ocean, but land hydrology is important as well. Current global water storage models differ significantly from one another and are unable to fully close the so called geodetic budget: the agreement between observed geodetic excitations on polar motion and geophysical ones.
Here, we compare the effects of several hydrological excitation functions calculated by removing modelled atmospheric and oceanic effects from space observations of full polar motion excitations GAM (Geodetic Angular Momentum). We called that functions as geodetic-hydrological residual time series. The estimation of hydrological effects on Earth's rotation differs when using one atmospheric (Atmospheric Angular Momentum – AAM) and one oceanic model (Oceanic Angular Momentum – OAM). The goal of this study is to build and objective criterion that justifies the use of one combination of AAM+OAM to estimate geodetic-hydrological residuals time series. To do that, we determine the quality of each series by making an estimation of their noise level, using a generalized formulation of the " three cornered-hat" method. After that, we construct a combined $Res\ Comb.$ (GAM-(AAM+OAM)) series, which the noise level of combined geophysical time series will be minimal. These geodetic residuals time series will be analyzed and compared with hydrological excitation functions determined from hydrological models and from Gravity Recovery and Climate Experiment GRACE satellite mission.

Methodology

To determine the quality of geodetic-hydrological residual time series, we have to estimate their noise level. We choose to use the *three-cornered hat method* which allows an estimation of the noise level of the series only by comparing them against each other.
We take a difference between each series and one of them (*Res1*), arbitrary chosen as reference:

$$Y^{iN} = X^i - X^N = \epsilon^i - \epsilon^N, \qquad i = 1, ..., N-1 \tag{1}$$

where X^N is a reference time series. The samples of the $N-1$ solution centers differences are concatenated in an $M\times(N-1)$ matrix as:

$$Y = \left[Y_{1N} \quad Y_{2N} \quad ... \quad Y_{(N-1)N\cdot}\right] \tag{2}$$

The covariance matrix S of the series of differences is computed as $S{=}cov(Y)$. We introduce the $N\times N$ Allan covariance matrix of the individual noises R, whose elements are the unknows of the problem and will be determined by relating to S as:

$$S = [I-u]\begin{bmatrix}\widehat{R} & r\\ r^T & r_{NN}\end{bmatrix}\begin{bmatrix} I\\ -u^T\end{bmatrix}, \tag{3}$$

where I is the identity matrix and u is the $[11...1]^T$ vecor. We have isolated the N free parameters of equation (3) by the minimization of the global correlation among the noises of the individual time series using objective function, according to Kuhn-Tucker theorem:

$$F(r, r_{NN}) = \sum \frac{r_{ij}^2}{(det(S))^{\frac{2}{N-1}}}, \tag{4}$$

with a constraint function:

$$G(r, r_{NN}) = -\frac{r_{NN} - [r - r_{NN}u]^T \cdot S^{-1} \cdot [r - r_{NN}u]}{(det(S))^{\frac{1}{N-1}}} < 0. \tag{5}$$

The initial conditions were selected as:

$$r_{iN}^{(0)} = 0 \quad i < N \quad and \quad r_{NN}^{(0)} = (2\cdot u^T \cdot S^{-1} \cdot u)^{-1}. \tag{6}$$

After the noise level of each geodetic-hydrological tiime series were found, the combined GAO time series was computed as:

$$\begin{bmatrix}\chi 1\\ \chi 2\end{bmatrix} = \sum_{i=1}^{4} w_i(t)\begin{bmatrix}\chi_1^i(t)\\ \chi_2^i(t)\end{bmatrix} \tag{7}$$

and is called $Res\ Comb.$.

All geodetic residuals were decomposed into seasonal and non-seasonal components using least squares method and smoothed using Gaussian filter with full width at half maximum $FWHM{=}20$. All considered geophysical time series were decomposed into complex Fourier series and the forward (+) and backward (-) terms both of χ_1 and χ_2 components were separated.

Hydrological - geodetic time series comparison of prograde and retrograde oscillations, frequency band > 400 days

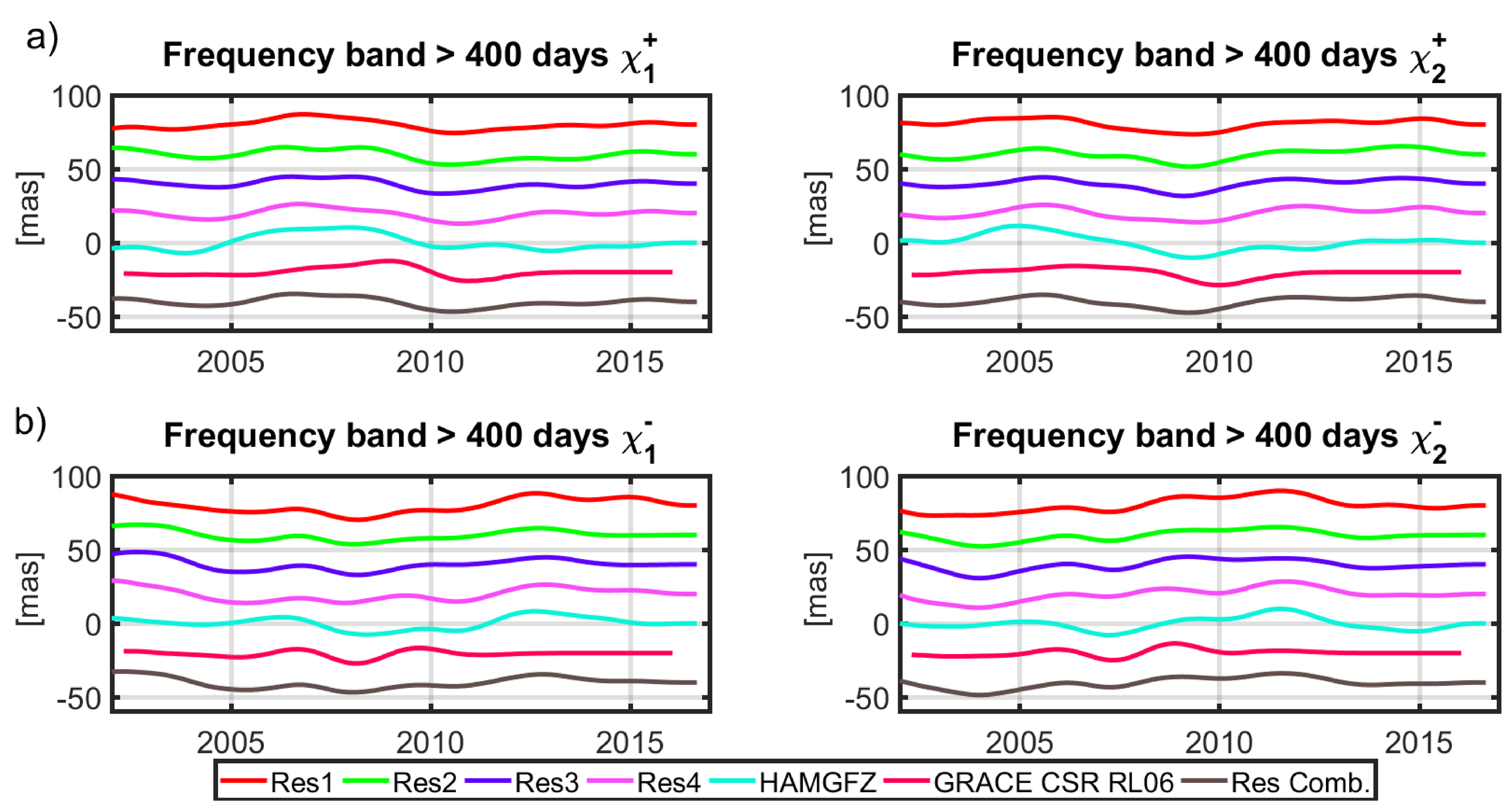

Figure 1: Comparison of non-seasonal components of prograde (+) (Fig 1a) and retrograde (-) (Fig 1b) oscillations, χ_1 and χ_2, of various geodetic residuals defined as the difference in the geodetic excitation and the sum of mass and motion terms in different AAM and OAM excitation functions of polar motion (Res1, Res2, Res3, Res4), HAM GFZ excitations, gravimetric hydrological excitation functions computed from GRACE CSR RL06 data, and hydrological-geodetic residuals computed as combined Res1, Res2, Res3 and Res4 time series ($Res\ Comb.$), taking their quality into account (combined series has a noise level as low as possible). The frequency band for each considered time series is above 400 days.

Table 2: Correlation coefficients between hydrological - geodetic and geophysical excitation functions of polar motion, separately for prograde and retrograde oscillations, $\chi_1^+, \chi_2^+, \chi_1^-, \chi_2^-$.

Non-seasonal time series of prograde oscillations, frequency band > 400 days						
χ_1^+	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.82	0.89	0.92	0.85	0.62
Res Comb.	0.93	0.97	0.98	0.99	0.30	0.44
χ_2^+	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.85	0.89	0.87	0.83	0.57
Res Comb.	0.92	0.97	0.99	0.97	0.67	0.51
Non-seasonal time series of retrograde oscillations, frequency band > 400 days						
χ_1^-	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.82	0.79	0.88	0.77	0.45
Res Comb.	0.89	0.98	0.97	0.96	0.64	0.48
χ_2^-	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.94	0.88	0.92	0.73	0.66
Res Comb.	0.95	0.99	0.97	0.96	0.61	0.66

Hydrological - geodetic time series comparison of prograde and retrograde oscillations, frequency band < 400 days

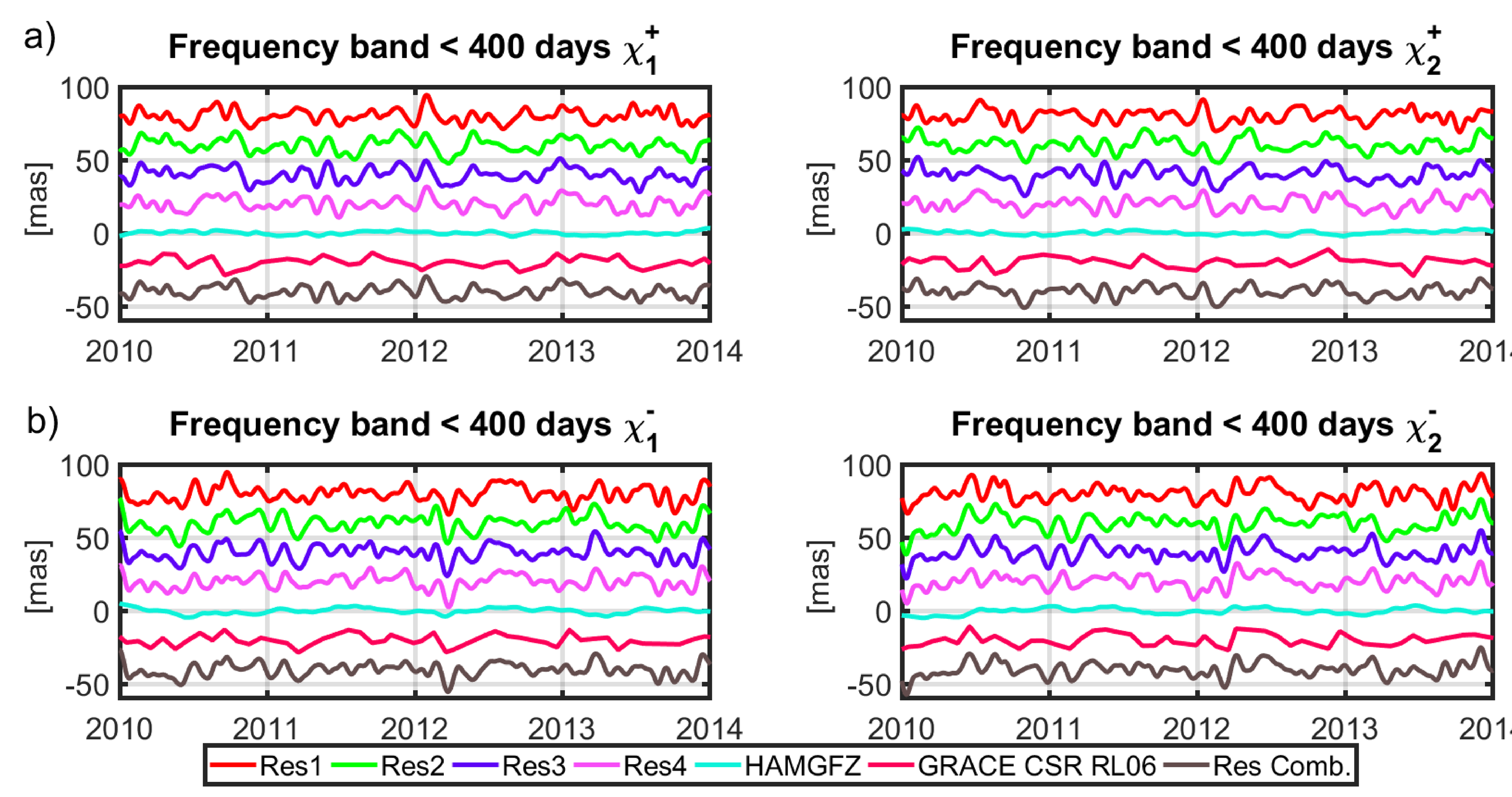

Figure 2: Comparison of non-seasonal components of prograde (+) (Fig 2a) and retrograde (-) (Fig 2b) oscillations, χ_1 and χ_2, of various geodetic residuals defined as the difference in the geodetic excitation and the sum of mass and motion terms in different AAM and OAM excitation functions of polar motion (Res1, Res2, Res3, Res4), HAM GFZ excitations, gravimetric hydrological excitation functions computed from GRACE CSR RL06 data, and hydrological-geodetic residuals computed as combined Res1, Res2, Res3 and Res4 time series ($Res\ Comb.$), taking their quality into account (combined series has a noise level as low as possible). The frequency band for each considered time series is below 400 days.

Table 3: Correlation coefficients between hydrological - geodetic and geophysical excitation functions of polar motion, separately for prograde and retrograde oscillations, $\chi_1^+, \chi_2^+, \chi_1^-, \chi_2^-$.

Non-seasonal time series of prograde oscillations, frequency band < 400 days						
χ_1^+	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.66	0.72	0.79	0.23	-0.05
Res Comb.	0.85	0.91	0.96	0.87	0.27	0.06
χ_2^+	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.62	0.67	0.76	0.30	-0.13
Res Comb.	0.83	0.90	0.95	0.85	0.36	0.00
Non-seasonal time series of retrograde oscillations, frequency Band < 400 days						
χ_1^-	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.0.56	0.70	0.72	0.30	0.43
Res Comb.	0.82	0.87	0.95	0.85	0.36	0.56
χ_2^-	Res1	Res2	Res3	Res4	HAM	GRACE
Res1	1	0.64	0.73	0.75	0.29	0.49
Res Comb.	0.84	0.90	0.96	0.84	0.33	0.60

Conclusions

We compare several geodetic-hydrological time series by computing their noise level and correlation with HAM GFZ and GRACE CSR RL06 gravimetric-hydrological excitation functions. We showed the differences that occur from one geodetic residuals to the other. Generally, when the noise of one geodetic residuals is higher than the others, its correlation with HAM GFZ and GRACE data is usually lower. Time series of decadal and inter-annual oscillations (above 400 days) are better correlated than the non-seasonal time series of frequency band below 400 days.
The combined geodetic-hydrological time series, $Res\ Comb.$, which is a weighted average of the existing ones, doesn't improve significantly correlation between geodetic residuals and hydrological excitation functions of polar motion.

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