

# AN EXAMPLE TO ANALYZE DISCRETE VECTOR FIELDS ON THE SPHERE USING QUANTITATIVE AND QUALITATIVE METHODS



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## Abstract:

An accurate correction of massive ground-based catalogs may be an interesting but difficult task. The use of DR2 for a re-reduction of data is not always possible. We would like to compatibilize the correction with the study of properties that are missed in DR2. Two advantages are sufficient to justify these studies. On the one hand, it is usual to suppose that in the correction process, the signal and the noise are accurately detected, but this is not necessary true if the statement "with respect to the adjustment model and a precision order" is not added to the assertion. The improvement of two ground-based catalogs and their comparison may explain a common part in the residuals depending on certain physical properties. On the other hand, we can wonder about some questions: the observation from the Earth involves some intrinsic errors but is it possible to align them to the ICRF while conserving the above-mentioned intrinsically terrestrial properties? That said, this is an impossible task, but we can lower the expectations seeking for quantitative improvements that eliminate bias and determine qualitative properties of the residual vector field on the celestial sphere with radius  $r$  considering magnitudes and spectral types. This arrangement is applied to assign a proper motion vector field in the same domain of work.

**1. INITIAL STEPS:** Choice of a set of stars common to the Hipparcos and the massive catalog 2MASS. Denote this set as  $\Omega_0$ . Now, we consider different properties such as the spectral type (splitting the data into a KM set and a no-KM set), H-magnitudes (splitting the data into  $m_4=[5.750,7.153)$ ,  $m_5=[7.153,8.556)$ ,  $m_6=[8.556,9.959)$  and  $m_7=[9.959,11.363)$ ) and also the distances assigning the data to sets, following Astraatmadja and Bailer-Jones (2016) in DR1. We can build different subsets of stars as, for instance,  $\Omega_{KM,m_4}$  for KM stars with  $m_4$  magnitude; or  $\Omega_{m_5}|_{r=200}$ , containing  $m_5$  stars in the slice [100pc,300pc], for example. The intervals for  $r$  [25pc,200pc], [100pc,300pc], [200pc,400pc], named after their "center"  $r=100, 200, 300, \dots$ pc. We will denote as  $\Omega$  any data work set.

**STEP 2. OBTENTION OF THE VECTOR FIELDS:** From each set of data points  $\Omega$ , we suppose a relation  $Y_i = m(X_i)$  where  $X_i \in \Omega$  is a random vector and we assume that:

$$m(X_i) \approx m(x) + (D^1 m)_x (X_i - x)$$

with  $x$  near some  $X_i$ . We obtain the estimator for the vector field and the estimators for the first derivatives of the vector field:  $\widehat{m}(x), \widehat{m}_1(x), \widehat{m}_2(x), \dots$  as the solution of the problem described in:

$$\{\widehat{m}(x), \widehat{m}_1(x), \widehat{m}_2(x)\} \equiv \{\widehat{b}_{00}(x), \widehat{b}_{11}(x), \widehat{b}_{22}(x)\} = \min_{\{\widehat{b}_k(x)\}} \{Y_i - b_0(x) - b_{11}(x)(X_{i,1} - x_1) - b_{12}(x)(X_{i,2} - x_2)\}^2 K_{h,ix} \quad (1)$$

$$K_{h,ix} = \frac{1}{h_1} K\left(\frac{X_{i,1} - x_1}{h_1}\right) \frac{1}{h_2} K\left(\frac{X_{i,2} - x_2}{h_2}\right) \quad (2)$$

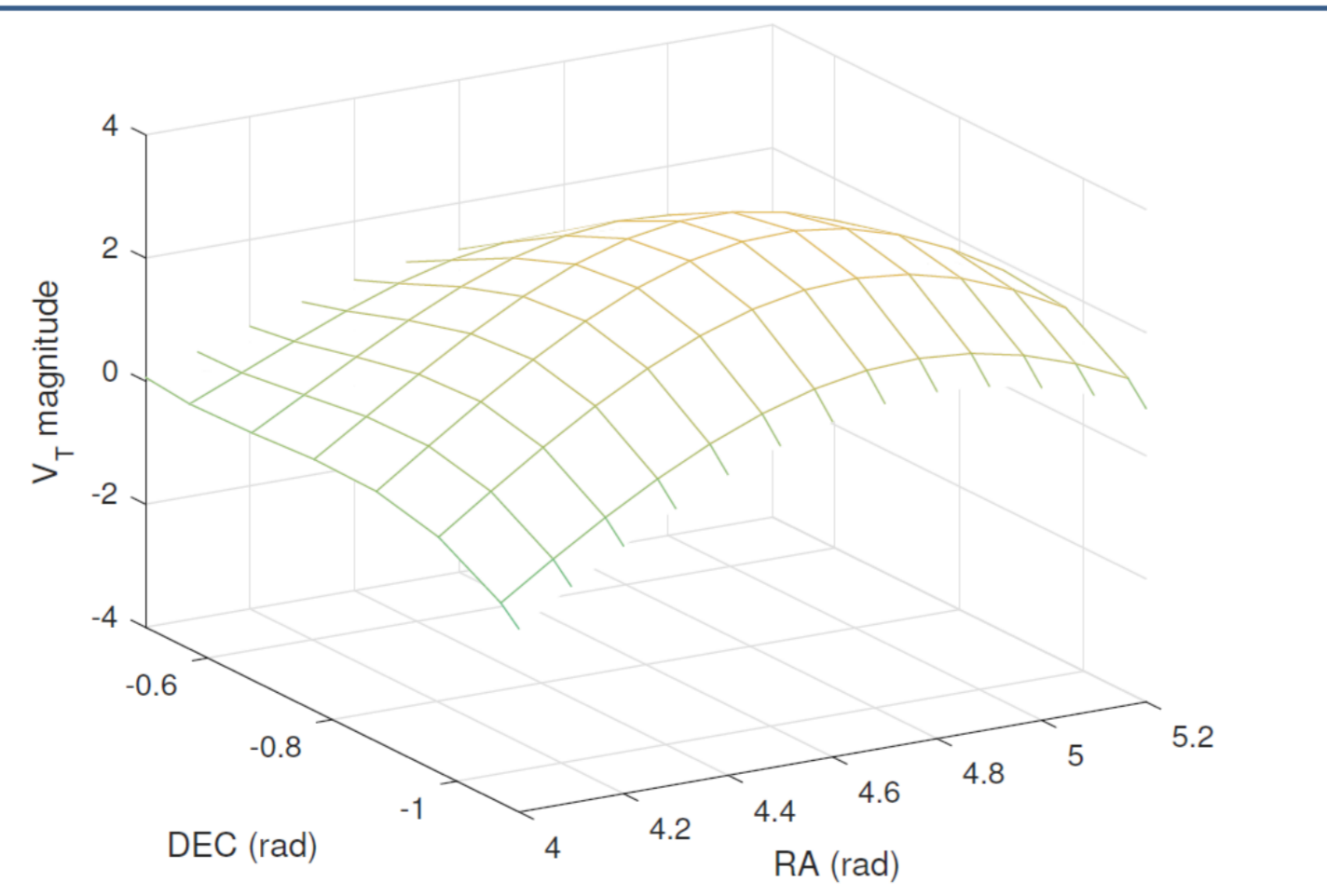
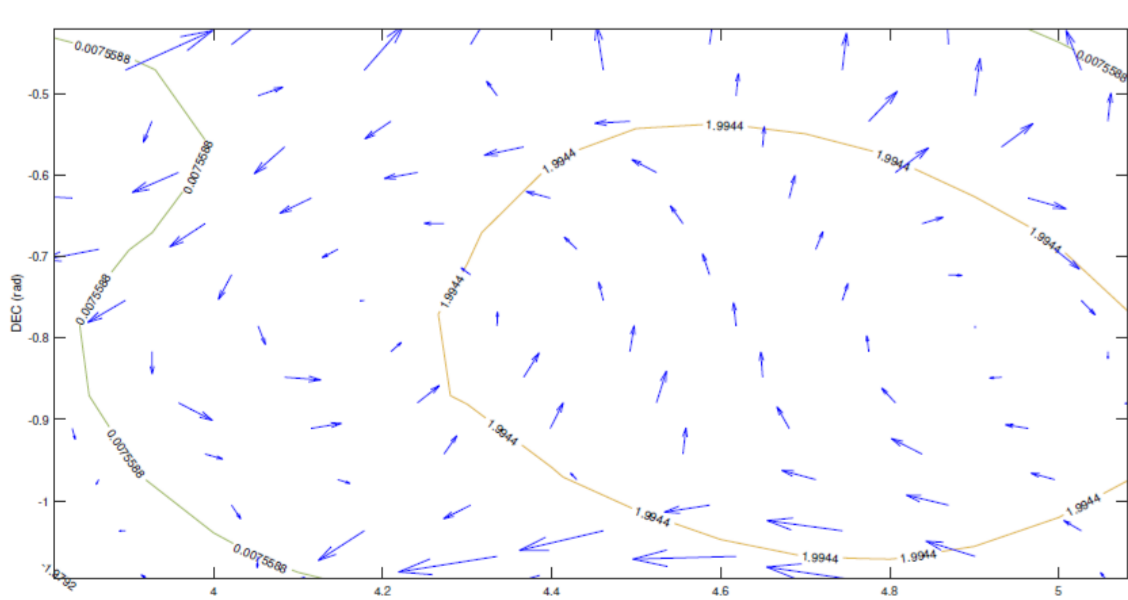
**STEP 3: Using vector fields to obtain quantitative data:** Suppose that the vector of the residuals is developed in vector spherical harmonics depending on  $r, \alpha, \delta$  by means of:

$$X(r, \alpha, \delta) = \sum_{n, |m| \leq n} [r_{nm} R_{nm} + s_{nm} S_{nm} + t_{nm} T_{nm}] \quad (3)$$

where  $R_{nm} = Y_{nm} \mathbf{r}$ ,  $S_{nm} = r \nabla Y_{nm}$ ,  $T_{nm} = -\mathbf{r} \times \nabla Y_{nm}$

Using inner product  $\langle, \rangle$  in the Hilbert Space of the spherical  $S^2$ -vector, the coefficient for the normalized basis vector  $\phi_k$ , are  $\alpha_k = \langle \widehat{m}, \phi_k \rangle$ .

**Figure 1.** Slice 25-200. Area III. KM-m5, Shrink (1.6,1); On the left, the potential vector field. On the right, the corresponding VT surface.



**Table:** Comparison between the proper motions from DR1, PMA and our obtained results for some stars in the neighborhood of some singular points of their vector field. Hip stands for the number of the star in the Hipparcos catalogue, DR1 for the DR1 identifier. Subindex (1) means DR1 and (2) means PMA. The proper motions are given in mas, being the last two columns  $\mu_\alpha^*$ ,  $\mu_\delta$  our obtained results.

Hip	DR1	$\alpha_{(1)}$	$\delta_{(1)}$	$\mu_\alpha^*_{(1)}$	$\mu_\delta_{(1)}$	$\mu_\alpha^*_{(2)}$	$\mu_\delta_{(2)}$	$\mu_\alpha^*$	$\mu_\delta$
28951	1008018207212849024	91.6628127275	63.4538738263	-35.558	8.594	-34.73	21.84	-36.35	21.33
30031	998050069154382592	94.7912065760	56.5264619828	-37.640	-31.332	-	-	-39.10	-31.05
24771	188796557490084480	79.6695937289	-32.3228506874	4.963	-13.891	3.08	-20.75	4.47	-23.68
27047	4756115082715073920	86.0379453784	-65.1018594755	-2.259	21.541	-4.01	26.0	-2.24	27.01
23865	279443525899748352	76.9426318743	55.7590984639	26.115	-14.343	22.01	-16.58	29.28	-21.70
89345	6721441368029854976	273.4528707722	-43.2043919512	-9.007	-13.989	-8.55	-17.27	-9.91	-18.69
44931	3841861165533628672	137.2857737710	-1.5880525106	-27.981	-35.349	-28.80	-45.08	-23.34	-43.16
23863	279443525899748352	76.9426318743	55.7590984639	26.115	-14.3434	22.01	-16.58	28.47	-22.17

In Marco et al (2019) one can see an exhaustive set of coefficients of developments for each set  $\Omega$ . There it can be observed that the values of the obtained parameters and their evolution (in the distance) depend on both magnitudes and spectral types, so that the corrections must be more specific than what is usually considered. This step, in itself, was qualitatively and quantitatively finer and more precise than other more usual procedures.

**STEP 4: Using vector fields to obtain qualitative data:** Before applying any correction (which is in J2000), we study stars where (or very close where) the residual field is singular and, in addition, the rotational component of the field is irrelevant. These points have the particularity of being maximum or minimum of the function of magnitude VT of Tycho, used in the reduction of the 2MASS.

Note that from Helmholtz decomposition, a vector field is split into two components (rotational and irrotational) by means of  $\vec{X} = \nabla \phi + \nabla \times \vec{u}$ , where  $X$  is a spherical vector field. Taking divergence operator, we deduce the relation  $div(\vec{X}) = \Delta \phi$  and assuming  $\phi(\alpha, \delta) = \sum_{n,m} a_{nm} Y_{nm}(\alpha, \delta)$ , from the application of properties of the Laplace-Beltrami operator we deduce:

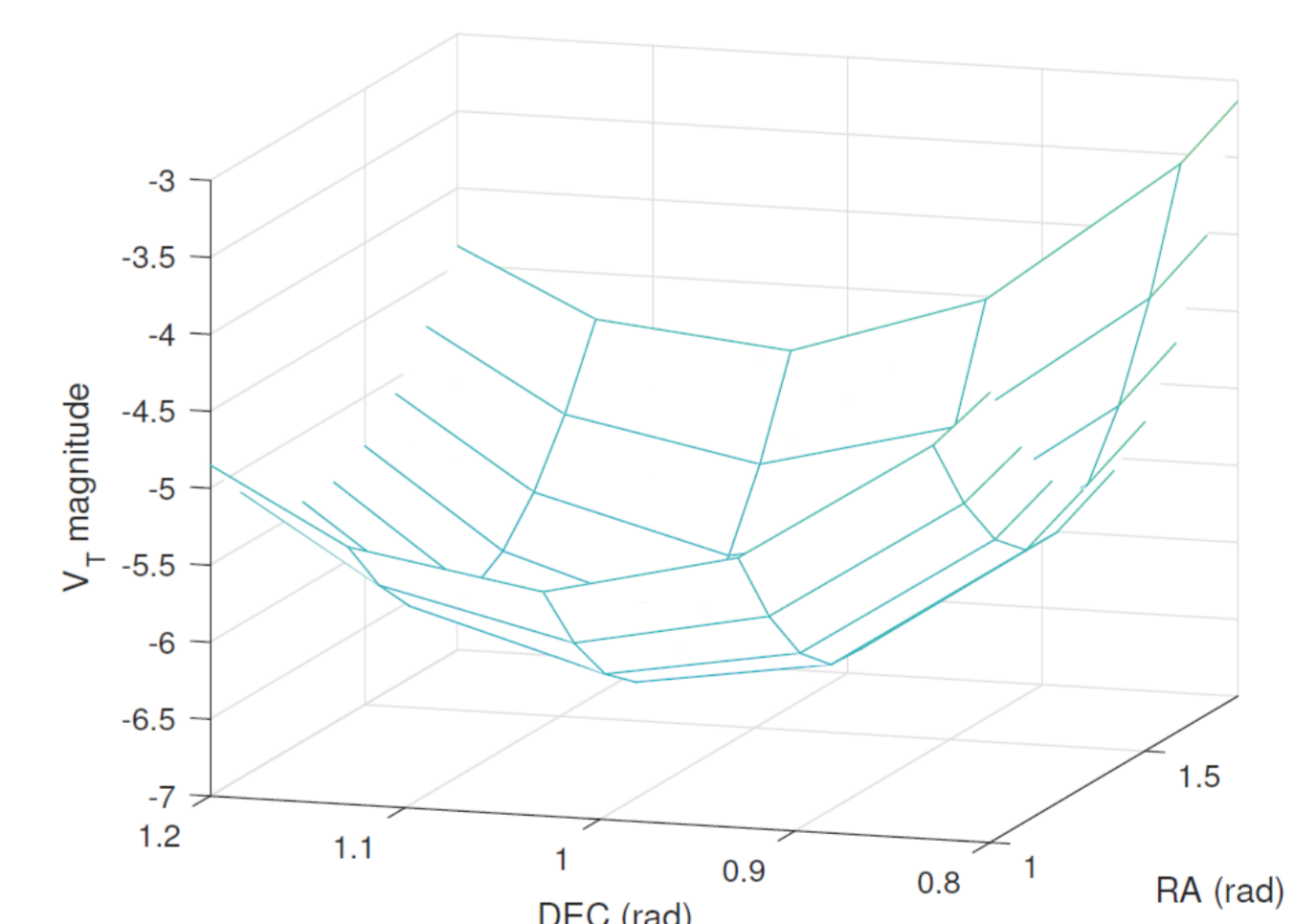
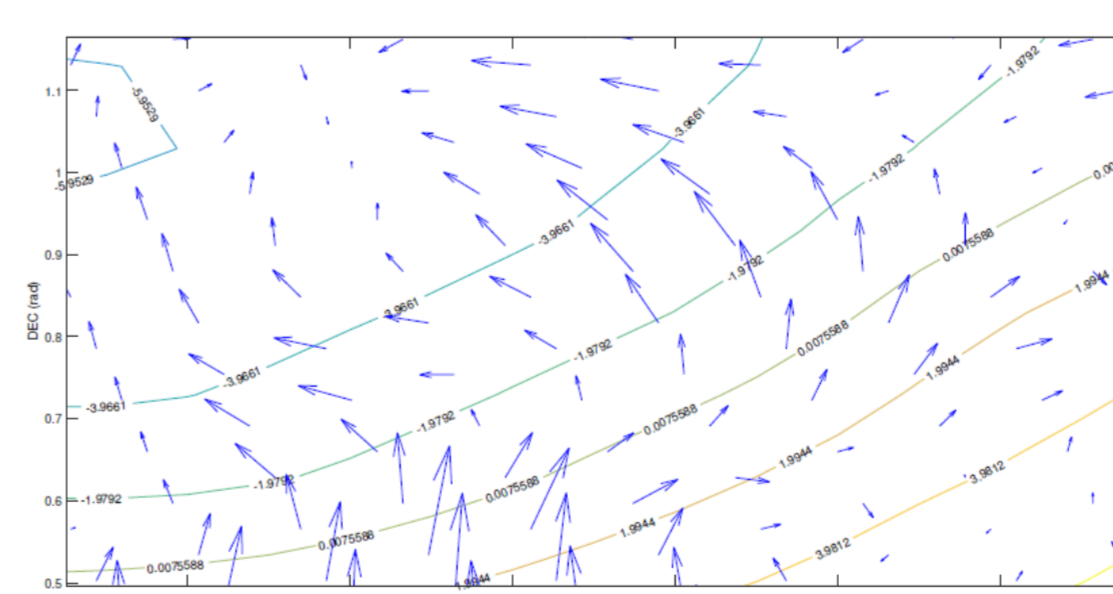
$$\Delta \phi = \sum_{n,m} a_{nm} \Delta Y_{nm} = \sum_{n,m} [-n(n+1)a_{nm}] Y_{nm}$$

$$a_{nm} = -\frac{1}{n(n+1)} \frac{\langle \Delta \phi, Y_{nm} \rangle}{\langle Y_{nm}, Y_{nm} \rangle}$$

We show here only two examples of singular points of the vector field near singular points of the potential (see Figures 1 and 2). For more examples, see the above mentioned paper.

**STEP 5: INDUCED PROPER MOTIONS FROM CORRECTED COMMON STARS TO HIPPARCOS2 AND 2MASS:** At this stage we compare a selection of these stars, corrected after step 3 and contained in the table, with their positions in DR1 and we deduce the induced proper motion. Finally we compare the previous proper motions with those given by DR1 and by PMA (in J2015.0), in order to indirectly check the quality of the positions adjusted in J2000 (since, obviously, those of DR1 are in J2015.0 and does not provide those of J2000.0 more than via its proper motions). The results are promising, but we must continue to deepen the method and explaining some phenomena related to vector fields, the potentials of its irrotational component and surfaces of VT magnitude from Tycho. See table for examples of proper motion.

**Figure 2.** Slice 25-200. Area I. KM-m4, Sources (4.9,-0.8). On the left, the potential vector field. On the right, the corresponding VT surface.



**CONCLUSIONS:** Returning to the text of the Abstract, after the correction process, we obtain the possibility of exploring the aforementioned "second advantage". On the other hand, regarding the "first advantage": both "ground based" catalogs will be improved using Hipparcos2 so that by comparing two improved catalogs with each other, we can launch the hypothesis that the Hipparcos effect has only affected the improvement and not the residual component related to the "Earth-observed" character of both catalogs that may remain. Extension to Tycho-2 stars must be performed preserving the Hipparcos-2 corrections.

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