SPHERICAL RECTANGULAR EQUAL-AREA GRID (SREAG)— SOME FEATURES

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ABSTRACT. A new method Spherical Rectangular Equal-Area Grid (SREAG) is proposed in Malkin (2019) for splitting spherical surface into equal-area rectangular cells with the latitude/longitude-oriented boundaries. In this work, supplement investigation of the SREAG properties is presented. The maximum number of rings that can be achieved with SREAG for coding with 32-bit integer is N_{ring}=41068, which corresponds to the smallest resolution of $\sim16''$. This satisfies all the applications known in the literature. Computational precision of the SREAG is tested. It was found that the worst level of precision is $7 \cdot 10^{-12}$ for maximum N_{ring} without special efforts in coding of the algorithm.

1. INTRODUCTION

A new approach to pixelization of a spherical surface Spherical Rectangular Equal-Area Grid (SREAG) was proposed in Malkin (2019). It is aimed at constructing of a grid that best satisfies the following properties:

1. it consists of rectangular cells with the boundaries oriented along the latitudinal and longitudinal circles;
2. it has uniform cell area over the sphere;
3. it has uniform width of the latitudinal rings;
4. it has near-square cells in the equatorial rings;
5. it allows simple realization of basic functions such as computation of the cell number given object position, and computation of the cell center coordinates given the cell number.

In this presentation, some more details of this method are discussed in addition to Malkin (2019).

2. SREAG METHOD

The basic parameter of the SREAG method is the number of rings N_{ring}. The sphere is first split into latitudinal N_{ring} rings of constant width $dB = 180^\circ/N_{ring}$. Then each ring is split into several cells of equal size. The longitudinal span of the cells in each ring is computed as $dL_i = dBb'^{\prime i}_0$, where i is the ring number, and b^i_0 is the central latitude of the ring. This provides near-square cells in the equatorial rings. Then the number of cells in each ring equal to $360/dL_i$ is rounded to the nearest integer value. This procedure results in the initial grid.

To provide uniform cell area over the sphere, the latitudinal boundaries of the rings are adjusted as follows. Let A be the cell area computed as $4\pi/N_{cell}$. Let us start from the North pole. Let b^u be the upper (closer to the pole) boundary of the ring in the final (adjusted) grid, and b^l be the lower boundary. Then, taking into account that the cell area is $A = dL * (\sin b^u - \sin b^l)$, the simple loop will allow to compute all the ring boundaries:

$b^u_1 = \pi/2$
do i=1,$N_{ring}/2$
    $b^l_i = \arcsin(\sin b^u_i - A/dL_i)$
    $b^u_{i+1} = b^l_i$
end do

The last value $b^l_{Nring/2}$ must be equal to zero (corresponds to the equator), which verifies the correctness of the computation. After that, the latitudinal boundaries for the rings in the South hemisphere are just copied from the North hemisphere with opposite (negative) sign. Figure 1 presents an examples of grids constructed making use of the proposed method.

Table 1 presents a comparison of the actual and nominal central latitude of the rings for the SREAG and Hierarchical Equal Area isoLatitude Pixelization (HEALPix, Górski et al. (2005)). The nominal central latitude of the ring is the central ring latitude for the grid with the same number of rings of constant width. In other words, the nominal central latitude is the central latitude of the rings in the initial grid (see above). Comparison of the results presented in Table 1 shows that the deviation of the central latitude of the rings from the uniform distribution is much smaller for the SREAG than for the HEALPix.

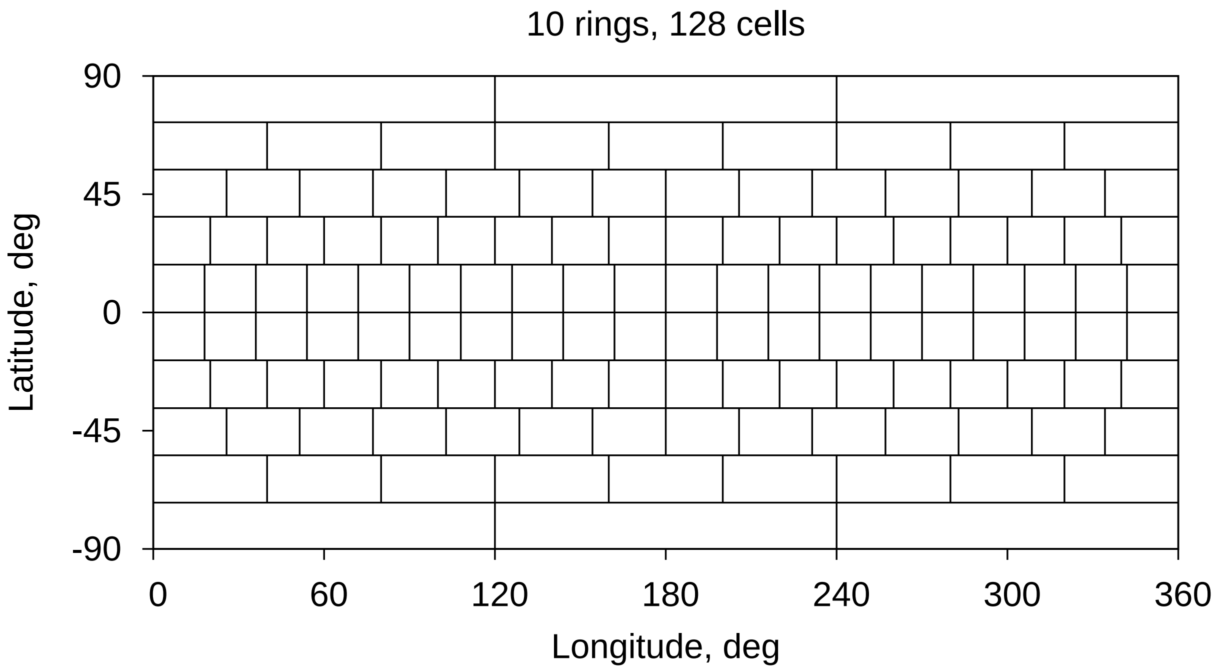

Figure 1. Example: 10-ring SREAG grid.

Table 1. Basic parameters of the HEALPix and SREAG grids, and the maximum difference actual minus nominal central latitude ($B - B_0$), deg.

HEALPix				SREAG		
Nside	Ncell	Nring	(B-Bo)max	Nring	Ncell	(B-Bo)max
1	12	3	18.19	4	20	0.28650
2	48	7	10.70	8	82	0.24336
4	192	15	6.19	16	326	0.33813
8	768	31	4.96	32	1302	0.17710
16	3072	63	4.35	64	5216	0.09530
32	12288	127	4.05	128	20862	0.04800
64	49152	255	3.91	256	83452	0.02408
128	196608	511	3.84	512	333758	0.01201
256	786432	1023	3.80	1024	1335096	0.00601
512	3145728	2047	3.79	2048	5340348	0.00301
1024	12582912	4095	3.78	4096	21361378	0.00150
2048	50331648	8191	3.77	8192	85445660	0.00075
4096	201326592	16383	3.77	16384	341782612	0.00038
8192	805306368	32767	3.77	32768	1367130454	0.00019

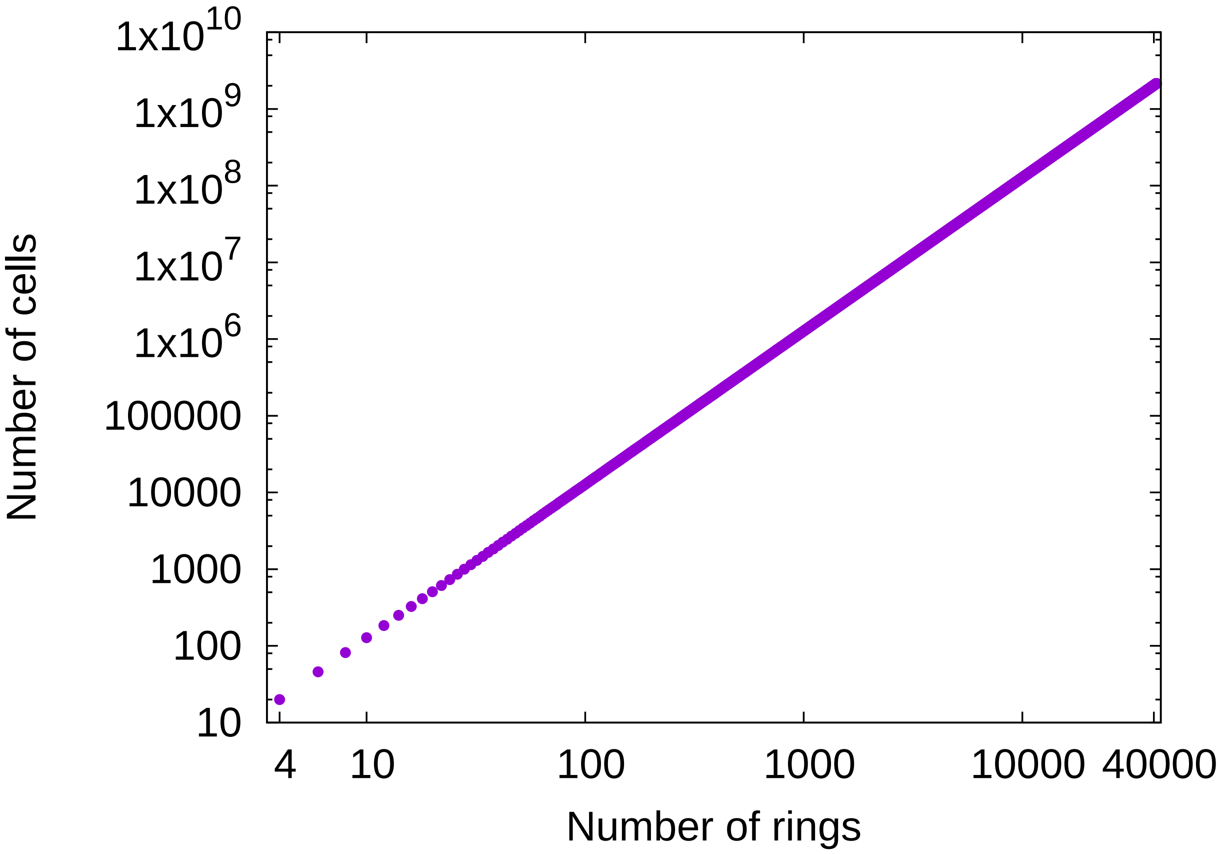

Figure 2. The number of the cells in the grid as function of N_{ring}.

The number of the cells in the grid depending on N_{ring} is shown in Fig. 2. For 32-bit integer, maximum available N_{ring} is 41068. The SREAG method provides much more detailed choice of the grid resolution than HEALPix: more than 20'000 SREAG grids vs. 14 for HEALPix.

For $N_{ring} = 4 \ldots 41'068$ grid resolution varies from $\sim 45^\circ$ to $\sim 16''$ (Fig 3). Analysis of the literature showed that the resolution used in practice lies in the range 7.3° to $26''$, which is fully covered by the SREAG resolution range. Indeed, the latter can be extended using 64-bit integer.

Figure 4 shows the precision of the computation that is defined by the deviation of the absolute value of the last (equatorial) latitude in the computational loop above from zero.

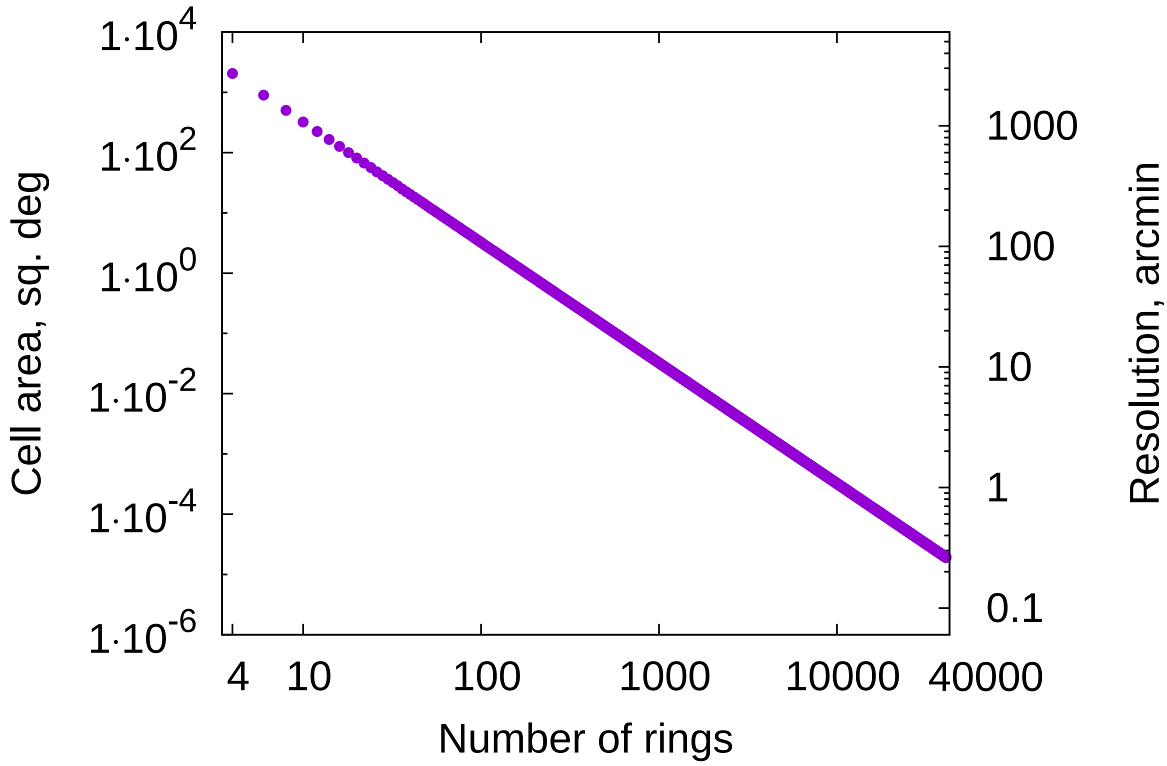

Figure 3. Cell area and grid resolution.

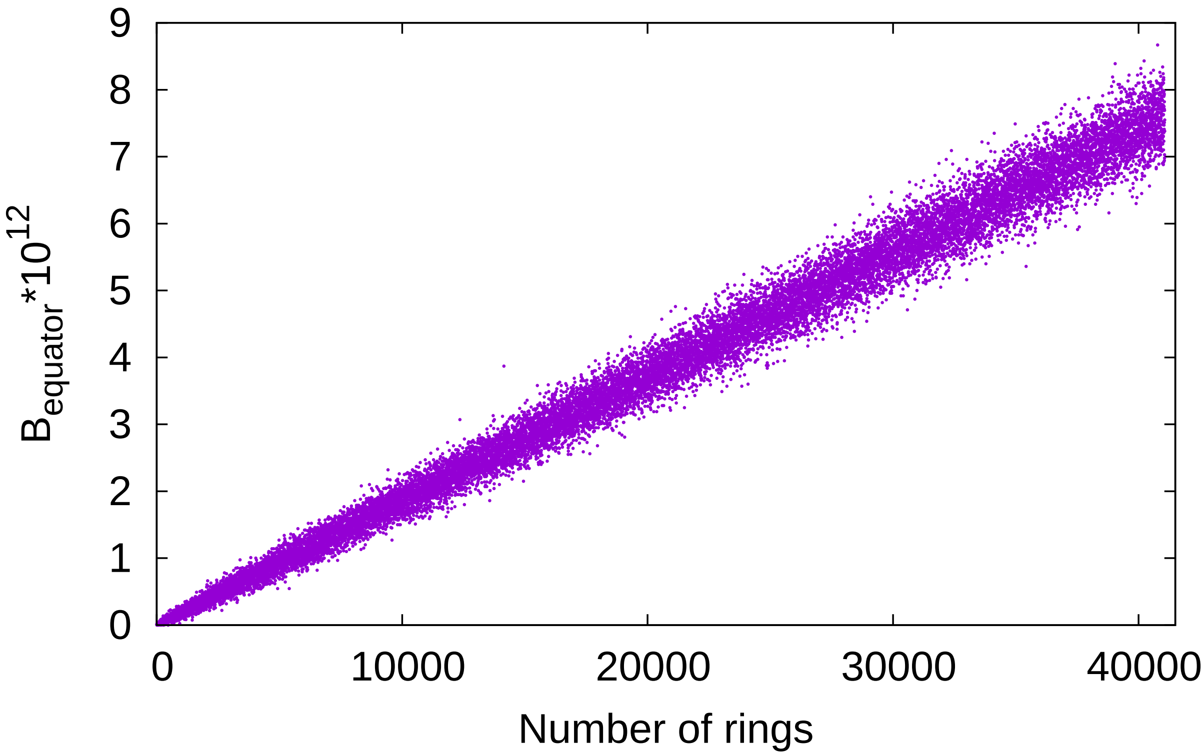

Figure 4. Computational precision.

3. CONCLUSION

The new method SREAG is developed for subdividing a spherical surface into equal-area cells. The main features of the proposed approach are:

- it provides an isolatitudinal rectangular grid cells with the latitude- and longitude-oriented boundaries with near-square cells in the equatorial rings;
- it provides a strictly uniform cell area;
- it provides a near-uniform ring width;
- it provides a wide range of grid resolution with a possibility of detailed choice of desirable cell area;
- the binned data is easy to visualize and interpret in terms of the longitude-latitude (right ascension-declinations) rectangular coordinate system, natural for astronomy and geodesy;
- it is simple in realization and use (Fortran routines for basic operations are provided, see Malkin (2019)).

Proposed approach to pixelization of a celestial or terrestrial spherical surface allows to construct a wide range of grids for analysis of both large-scale and tiny-scale structure of data given on a sphere. The number of cells is theoretically unlimited and is constrained in practice only by the precision of machine calculations.

The SREAG method can be hopefully useful for various practical applications in different research fields in astronomy, geodesy, geophysics, geoinformatics, and numerical simulation. In particular, it can be used in further analyses of the celestial reference frame, for selection of uniformly distributed reference sources in the next ICRF realizations, and for evaluation of the systematic errors of the source position catalogs.

4. REFERENCES

Górski K.M., et al. (2005) ApJ, 622, 759.

Malkin Z. (2019) AJ, 158, 158. preprint: arXiv:1909.04701