Estimation of Earth rotation resonance parameters through VLBI analysis

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Summary

The VLBI observations enable the investigation of the Earth rotation resonances, associated with Free Core Nutation, Free Inner Core Nutation, and Polar Motion Resonance, in the retrograde diurnal band with respect to a terrestrial frame. The resonance parameters (period and quality factor) are estimated by confronting the theoretical rigid Earth nutation terms to the corresponding nutation terms observed by VLBI. We revisit this problem by considering 40 years of VLBI observations and more complete atmospheric + oceanic non-tidal perturbations, that have to be removed from the observed nutation terms before the estimation. The inclusion of complete atmosphere and ocean correction produce a significant change in the period and quality factor of Free Core Nutation and Free Inner Core Nutation. On the other hand, the nutation analysis allows us to investigate the frequency dependence of Polar Motion Resonance in the diurnal retrograde band. We analyzed the frequency sensitivity of Polar Motion resonance and found that this resonance is mostly determined by the prograde nutation terms of period smaller than 386 days.

Methods and Data

Following the procedure of Mathews et al. (2002), the Earth resonance parameters are estimated through a relation between the observed nutation $\tilde{\eta} = \mathrm{X} + \mathrm{i}\,\mathrm{Y}$ and corresponding rigid Earth nutation $\tilde{\eta}_R$ in the frequency domain as follows

$$\tilde{\eta}(\sigma') = \tilde{T}(\sigma'; \tilde{\sigma'}_j)\tilde{\eta}_R(\sigma') \qquad (1)$$

where the transfer function $\tilde{T}(\sigma'; \tilde{\sigma'}_j)$ is represented by

$$\tilde{T}(\sigma'; \tilde{\sigma'}_j) = \frac{e - \sigma'}{e + 1}\Big(1 + (1 + \sigma')\sum_{j=1}^{3}\frac{\tilde{N}_j}{\sigma' - \tilde{\sigma'}_j}\Big). \qquad (2)$$

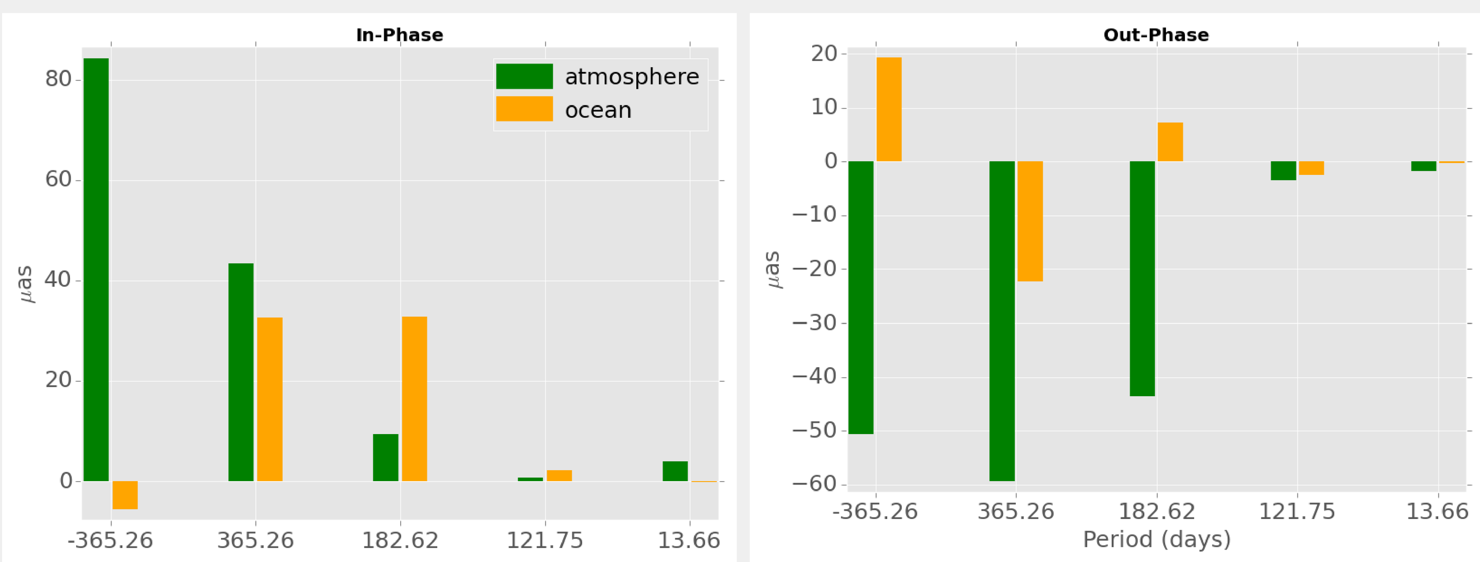
Here σ' is the frequency in cpd as seen from a celestial frame, e is the dynamical ellipticity of the Earth, $\tilde{N}_j$ and $\tilde{\sigma}'_j = 2\pi/P'_j(1 + n(i/2Q'_j))$ are the coefficient and frequency specifying the resonance modes in celestial frame, n is '+1' for prograde and '-1' for retrograde, P'_j and Q'_j are the space-referred period and quality factor of the resonance terms respectively. The celestial frequencies $\tilde{\sigma}'_j$ are mapped into terrestrial frequencies $\tilde{\sigma}_j = \tilde{\sigma}'_j - \Omega$ with corresponding period and quality factor P_j and Q_j, where Ω is the mean angular velocity of the Earth. The index 1 until 3 is for representing the resonances associated with Polar Motion Resonance (PMR), Free Core Nutation (FCN), and Free Inner Core Nutation (FICN) respectively. We estimate $\tilde{N}_j$ and $\tilde{\sigma}'$ from 42 observed nutation terms (see Table 1) and the corresponding set of rigid Earth terms by performing a weight least square inversion. The observed nutation terms are estimated through VLBI observation from 1979 to 2017. Before the inversion is performed, some corrections have to be applied. First, the nutation terms have to be referred to a dynamical celestial reference frame by removing the geodetic nutation and then non-linear effects are suppressed (see Table 7 of Mathews et al. (2002)). Finally, we eventually get rid of the atmospheric + oceanic contribution, which cannot be related to rigid Earth nutation caused by luni-solar tides.

Table 1: In-phase and out-phase coefficients of 42 observed nutation terms.

Period (days)	X (mas)	Y (mas)	Period (days)	X (mas)	Y (mas)
-6798.38	-8024.74	1.41	31.81	3.18	0.00
6798.38	-1180.44	-0.14	-27.55	-13.82	-0.04
-3399.19	86.14	-0.04	27.55	14.48	-0.00
3399.19	3.63	-0.01	-23.94	0.05	-0.00
-1615.75	-0.01	-0.01	23.94	1.18	-0.00
1615.75	-0.12	-0.01	-14.77	-1.20	-0.00
-1305.48	0.30	0.01	14.77	1.32	-0.00
1305.48	2.12	0.00	-13.78	-0.55	-0.00
-1095.18	0.22	0.00	13.78	0.61	-0.00
1095.18	-0.23	0.00	-13.66	-3.66	-0.02
-386.00	-0.16	-0.00	13.66	-94.20	0.13
386.00	-0.71	-0.00	-9.56	-0.09	-0.00
-365.26	-33.01	0.34	9.56	-2.46	0.01
365.26	25.65	0.14	-9.13	-0.46	-0.00
-346.64	-0.59	0.00	9.13	-12.45	0.04
346.64	-0.07	-0.00	-9.12	-0.29	0.00
-182.62	-24.58	-0.04	9.12	-2.34	0.00
182.62	-548.46	-0.51	-7.10	-0.06	0.00
-121.75	-0.94	0.00	7.10	-1.59	0.01
121.75	-21.49	-0.02	-6.86	-0.04	-0.00
-31.81	-3.06	-0.01	6.86	-1.28	0.01

Atmospheric and Oceanic contribution

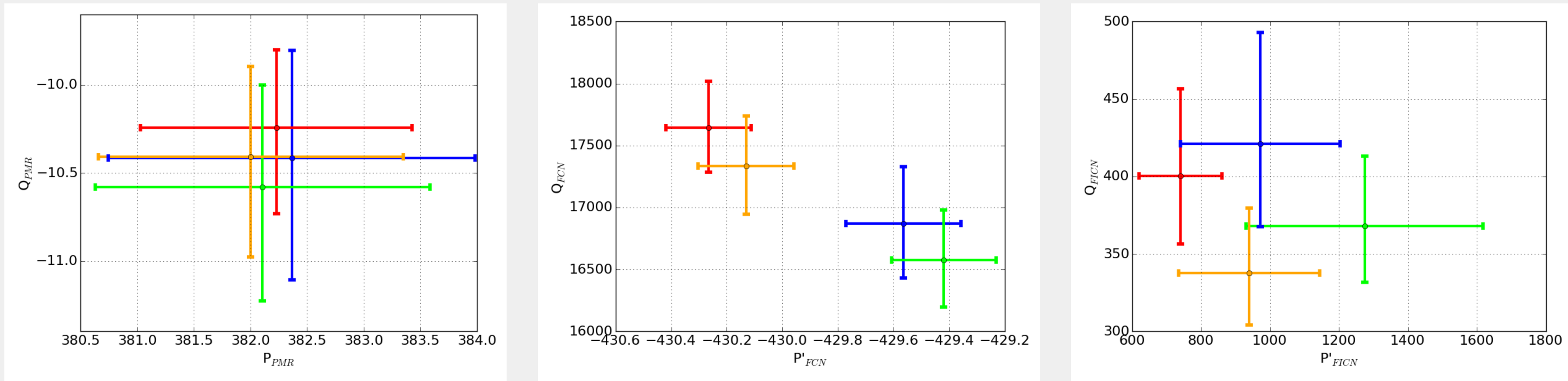
The analysis done in Mathews et al. (2002) was restricted to the atmospheric effect on the annual prograde nutation. Actually, a more complete treatment of the fluid layer perturbations has to include other nutation components (-365 d, $+182.6$ d, $+121.75$ d, $+13.66$ d) and consider the contribution of the non-tidal circulation in the ocean. The corresponding effects were evaluated by using the non-inverted barometer version of atmospheric angular momentum time series from TU Vienna (based upon ECMWF model) (Schindelegger et al. 2011) and oceanic angular momentum series from ERA40 and ERA operational (based upon OMCT model) (Dobslaw & Thomas 2007) from 1984 until 2017. This calculation is done through Celestial Angular Momentum Functions, and their effects on the nutation are estimated by using the formula reported in Brzeziński (1994). The results are shown in Figure 1.



(a) (b)

Figure 1: The contribution of atmosphere and non-tidal ocean to the nutation terms. Here (a) is for in-phase and (b) is for out-phase components.

Results



(a) (b) (c)

Figure 2: The comparison of atmospheric and oceanic contribution to the Earth resonance parameters. Here (a) is PMR, (b) is FCN, and (c) is FICN. The period of PMR is in TRF and the others are in CRF. Here •: no atmospheric and oceanic correction, •: atmospheric correction, •: oceanic correction, and •: atmospheric + oceanic correction.

The results of Earth resonances estimation is displayed in Figure 2. The excitation of atmosphere and ocean has no impact in the polar motion resonance. However, the inclusion of these correction could make a significant change in the FCN and FICN parameters. We found that the inclusion of atmospheric + oceanic correction would increase the value of FCN quality factor for about 500 and decrease the period in half days. For FICN, the inclusion of complete excitation correction has diminished the quality factor by about 100 and the period for about 10 days. For the purpose of comparison, Table 2 shows the result of ours and Mathews et al. (2002). It shows that our results have a good agreement with Mathews et al. (2002) except for the quality factor of FCN and FICN.

Table 2: Resonance parameters of PMR, FCN, and FICN. The FICN and FCN periods are given in CRF and other is in TRF. Periods are in mean solar days.

Parameter	Mathews et al. (2002)	This study (atmosphere + ocean)
P_{PMR}	$(381.9, 385.0)$	$(380.7, 383.4)$
Q_{PMR}	-10.4	$(-11.0, -9.9)$
P'_{FCN}	$(-429.9, -430.5)$	$(-430.3, -430.0)$
Q_{FCN}	20000	$(16946, 17736)$
P'_{FICN}	$(930, 1140)$	$(735.6, 1143.5)$
Q_{FICN}	677	$(304, 380)$

Analysis of Polar Motion resonance

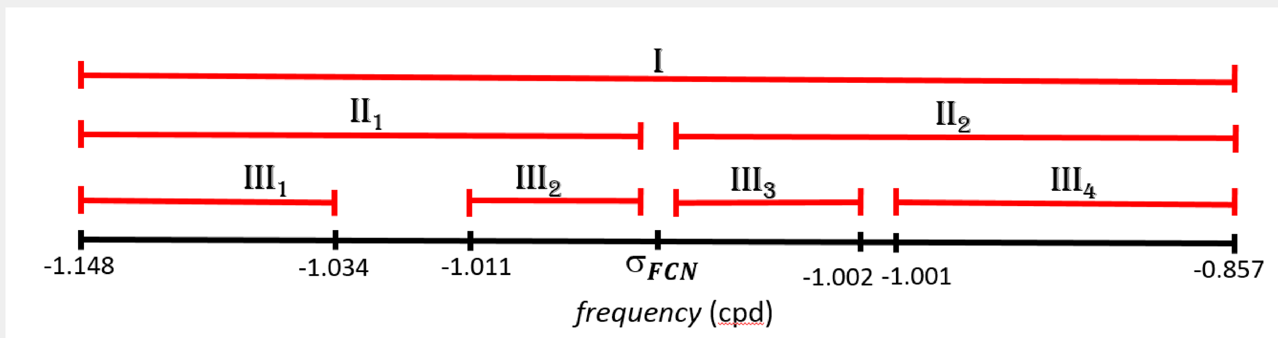


Figure 3: Selected frequency band in the terrestrial frame for least-square adjustment of the PMR parameters. 18.6 year nutation term is belong to the Band III_3. Their limits are precisely reported in Table 3.

The estimated parameters of the PMR ($P_{PMR} = 382$ d, $Q_{PMR} = -10$) are at discrepancy with the ones corresponding to the common polar motion ($P_{PMR} = 433$ d, $Q_{PMR} = 100$). This modification results from the dynamical oceanic response in the diurnal band (more insight about this topic is given in poster of Bizouard et al, Journées 2019). Mathews et al. (2002) assumed that the estimated values ($P_{PMR} = 383$ d, $Q_{PMR} = -10$) are mostly constrained by the 18.6 year ($\sigma = -1.00288$ cpd) nutation term. To check this assumption, $\tilde{\sigma}_1$ is re-estimated over restricted sets of nutation terms sweeping frequency band -1.15 cpd, -0.85 cpd of the polar motion (see Figure 3). It turns out that the estimates $\bar{P}_{PMR} = 382$, $\bar{Q}_{PMR} = -10$ over the whole set (band I) are at discrepancy with the ones obtained over restricted bands, as shown by Table 3. In particular, for frequencies smaller than σ_{FCN} (Band II_1, III_1, III_2) we get P_{PMR} significantly larger than 415 days. Although the 18.6 year retrograde nutation prevails, the estimates of PMR parameters are quite loose in the K_1 band (band III_3) ($P_{PMR} = 382 \pm 8$, $Q_{PMR} = -10 \pm 3$) and better constrained by the prograde short period terms (band III_4) ($P_{PMR} = 382 \pm 1$, $Q_{PMR} = -10 \pm 1$).

Table 3: The period and quality factor of PMR determined over certain band of frequencies.

Band	frequency (cpd)	P_{PMR}	Q_{PMR}
I	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega + 1/6.86)$	382.0 ± 1.3	-10.4 ± 0.5
II_1	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega - 1/386)$	418.5 ± 7.2	-8.24 ± 1.7
II_2	$(-\Omega - 1/1095.18 \leq \sigma \leq -\Omega + 1/6.86)$	381.8 ± 1.2	-10.4 ± 0.5
III_1	$(-\Omega - 1/6.86 \leq \sigma \leq -\Omega - 1/31.81)$	415.1 ± 3.3	-7.7 ± 0.7
III_2	$(-\Omega - 1/121.75 \leq \sigma \leq -\Omega - 1/386)$	486.8 ± 58.4	$+13.4 \pm 30.7$
III_3	$(-\Omega - 1/1095.18 \leq \sigma \leq -\Omega + 1/1095.18)$	381.7 ± 7.6	-10.2 ± 2.9
III_4	$(-\Omega + 1/386 \leq \sigma \leq -\Omega + 1/6.86)$	381.8 ± 1.3	-10.4 ± 0.5

Conclusions

- The inclusion of complete atmospheric + oceanic correction has significantly affected the estimates of the FCN and of the FICN parameters.
- PMR parameters are mostly determined by the prograde short period nutation terms, in contrast to the assumption of Mathews et al. (2002).
- The PMR period and quality factor seem to be frequency dependent in the diurnal band. (explored in more details in poster of Bizouard et al, Journées 2019)

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