



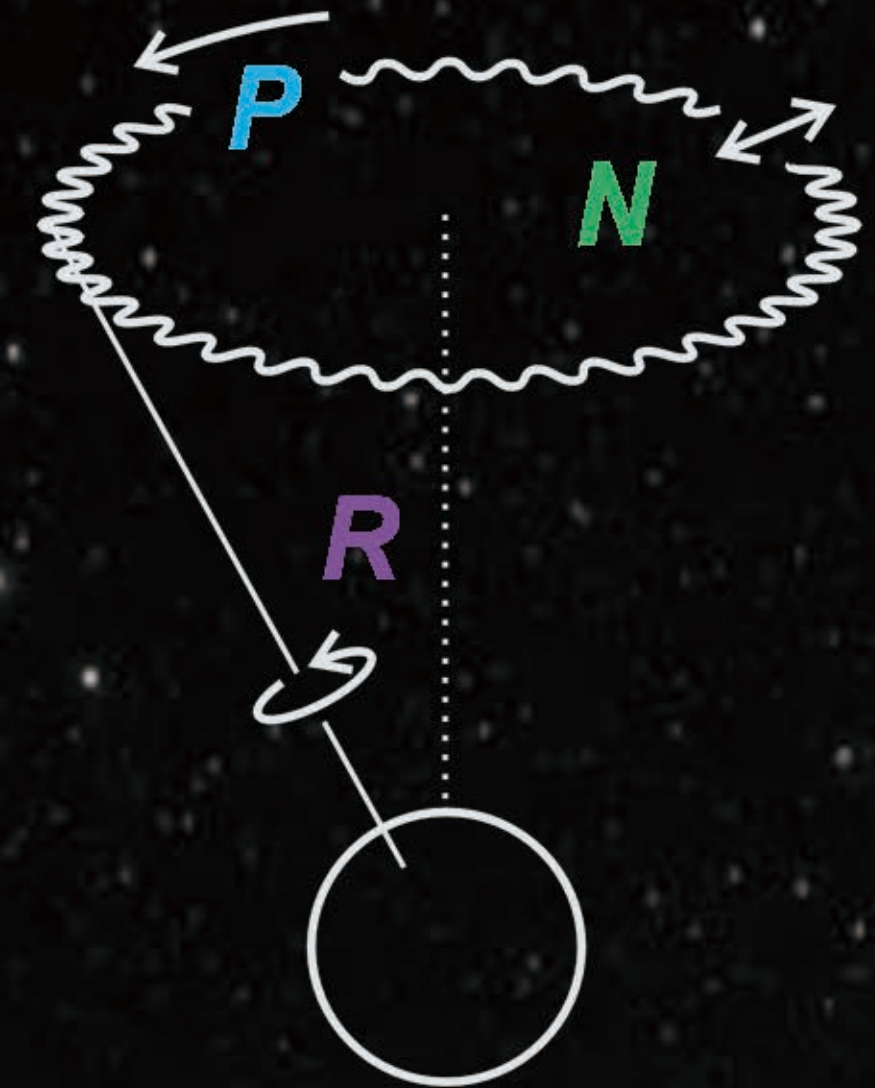
Analyses of Celestial Pole Offsets

with VLBI, LLR and Optical Observations

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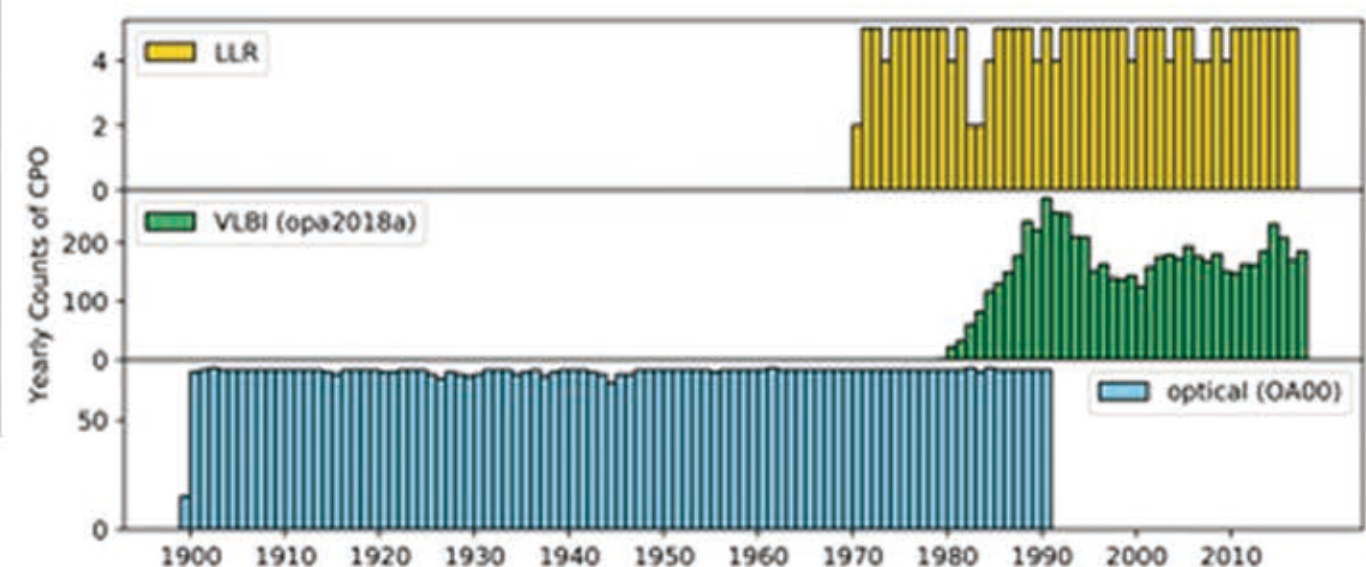
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The CIP axis is used as a rotation axis to orient the international terrestrial reference system with respect to a kinematically nonrotating celestial coordinate system, the latter being given by a group of distant radio source positions. This work aims to explore the possibilities of determining the long-period part of the precession-nutation of the Earth with techniques other than very long baseline interferometry (VLBI). Lunar laser ranging (LLR) is chosen for its relatively high accuracy and long period. Results of previous studies could be updated using the latest data with generally higher quality, which would also add ten years to the total time span. Historical optical data are also analyzed for their rather long time-coverage to determine whether it is possible to improve the current Earth precession-nutation model.

Time distribution of the data used in this work (number of CPO every year)



1. A parabola, that is, a quadratic function of t .
 2. A linear term and 18.6-year nutation term:
- $$dX, dY = A_0 + A_1 t + A_8 \sin \Omega_1 + A_c \cos \Omega_1,$$
3. A linear term, and 18.6-year and 9.3-year nutation terms:
- $$dX, dY = A_0 + A_1 t + A_{s1} \sin \Omega_1 + A_{c1} \cos \Omega_1 + A_{s2} \sin \Omega_2 + A_{c2} \cos \Omega_2,$$

Equations of the empirical models used in analyses of the CPO series

We fit all of the three models to the residuals and show the results in Table 1; the two data sources are presented for comparison. The uncertainties of the fit parameters are smaller than those presented in Capitaine et al. (2009) because the data have accumulated over ten more years and also because they have a higher quality. A parabola clearly is not an effective model to fit the curvature because the secular and quadratic term are strongly correlated (-0.9). For the second model, the weighted root mean square (WRMS) is reduced by about 15%. The correlation coefficient between the secular term and the sine term of the 18.6-year nutation (0.4) of model 3 stands out among others. The short time-span of the VLBI observations (38 years) leads to an evident correlation between the 18.6-year nutation term and long-period terms (t_1 and t_2).

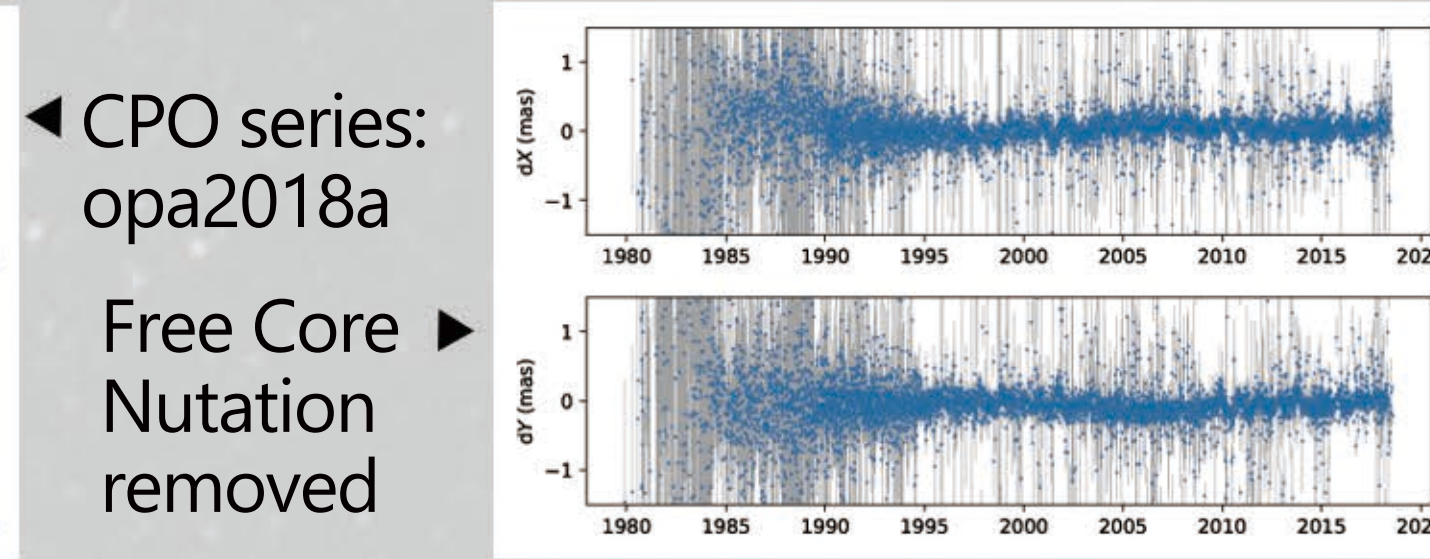
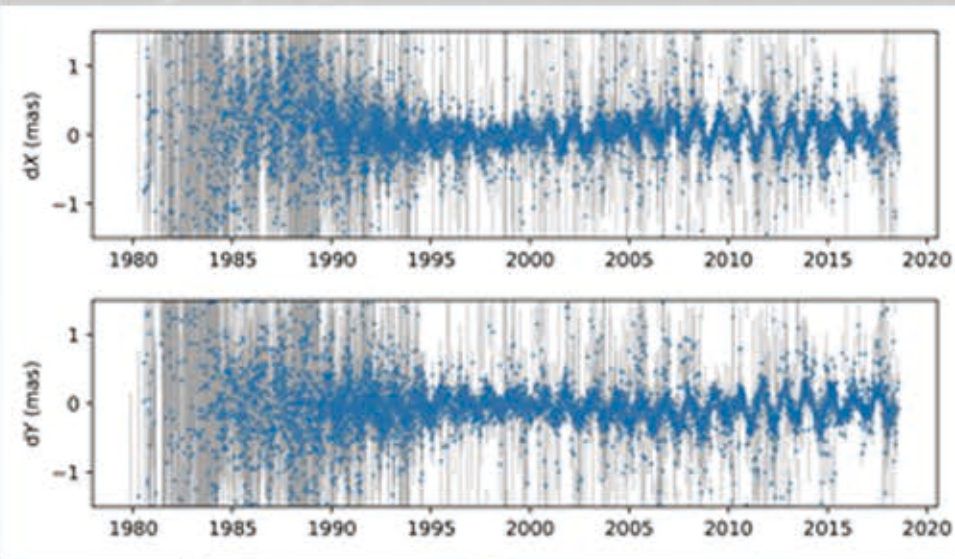
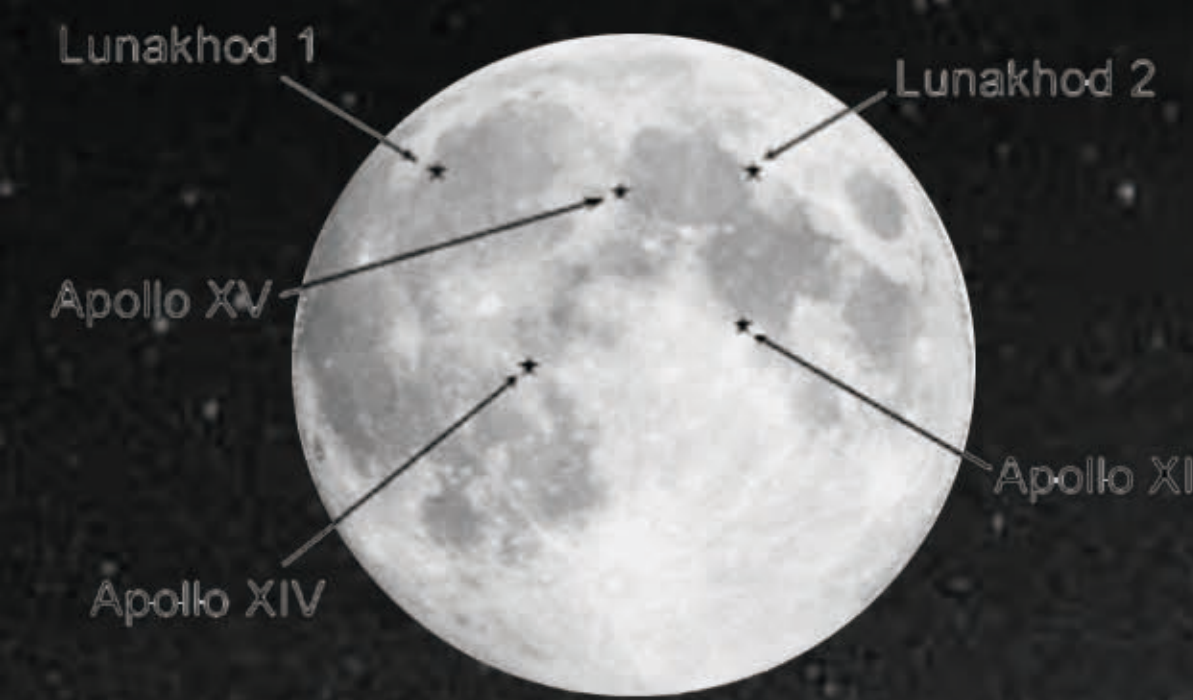


Table 1: VLBI fitting parameters

Term	μ^0	μ^1	μ^2	$\sin(18.6 \text{ yr})$	$\cos(18.6 \text{ yr})$	$\sin(9.3 \text{ yr})$	$\cos(9.3 \text{ yr})$	WRMS _{pre}	WRMS _{post}
Unit	μas	$\mu\text{as/cy}$	$\mu\text{as/cy}^2$	μas	μas	μas	μas	μas	μas
opa2018a	dX	18 ± 1	266 ± 21	-4338 ± 143	-0.37 ± 0.01	-0.30 ± 0.01	-0.30 ± 0.01	126	119
	dY	-86 ± 1	-519 ± 22	5325 ± 145	-0.35 ± 1	40 ± 1	40 ± 1	147	130
gsf2016a	dX	49 ± 1	215 ± 19	540 ± 120	43 ± 1	-11 ± 1	134 ± 1	134	104
	dY	-101 ± 1	-477 ± 20	5492 ± 122	-34 ± 1	47 ± 1	131 ± 1	131	109
opa2018a	dX	13 ± 1	381 ± 12	36 ± 1	-18 ± 1	126	116		
	dY	-71 ± 1	-34 ± 12	-35 ± 1	40 ± 1	147	128		
gsf2016a	dX	49 ± 1	441 ± 10	43 ± 1	-11 ± 1	134	101		
	dY	-87 ± 1	61 ± 10	-34 ± 1	47 ± 1	131	107		
opa2018a	dX	13 ± 1	334 ± 12	34 ± 1	-24 ± 1	126	116		
	dY	-70 ± 1	4 ± 12	-38 ± 1	46 ± 1	147	126		
gsf2016a	dX	49 ± 1	413 ± 11	41 ± 1	-14 ± 1	134	101		
	dY	-84 ± 1	40 ± 11	-43 ± 1	48 ± 1	20 ± 1	23 ± 1	131	105



Station name	Observation duration	WRMS(cm)	N_{total}	N_{rejected}
APOLLO	2006.04.07–2016.11.25	1.03	2648	336
Haleakala	1984.11.13–1990.08.30	5.40	770	202
Matera	2003.02.22–2017.11.10	8.44	105	26
McDonald	1970.07.20–1985.06.30	77.54	3575	117
MLRS1	1983.08.02–1988.01.27	40.64	631	58
MLRS2	1988.02.29–2015.03.25	7.41	3669	510
OCA(IR)	2015.03.11–2017.12.21	0.92	2839	105
OCA(MeO)	2009.11.11–2017.12.21	1.33	1836	32
OCA(Ruby)	1984.06.11–1986.06.12	36.58	1112	3
OCA(YAG)	1987.10.12–2005.07.30	7.27	8316	493
Total	1970.07.20–2017.12.21	2.06	25501	1882

$$D = SE + EM + MR$$

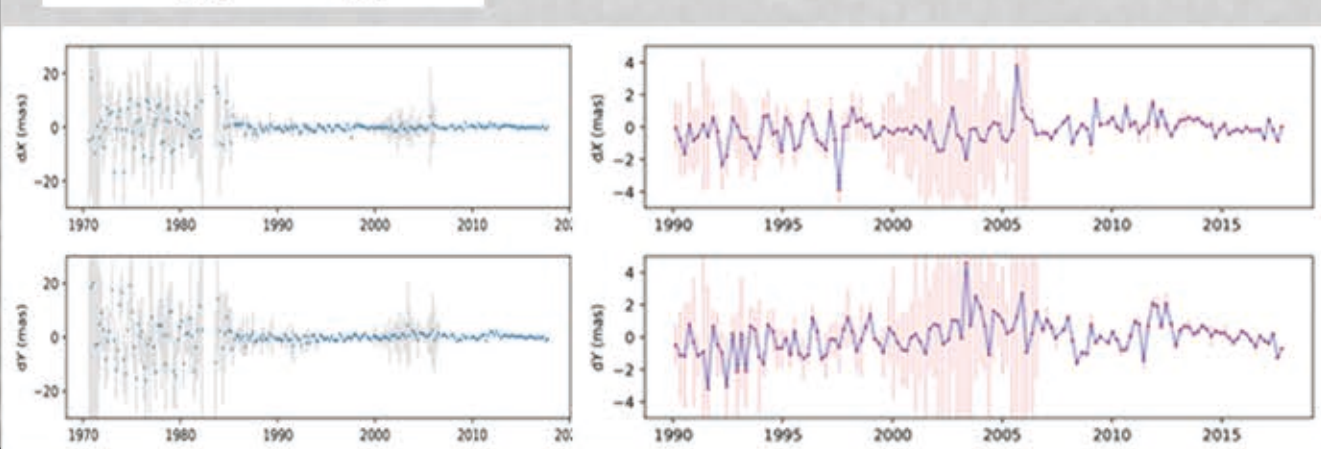
SE: station, E for Earth, M for Moon, R for reflector

$$SE_{\text{OCS}} = Q(t)R(t)SE_{\text{TRS}}$$

Q: PN(X, Y)R_c(s) Precession, Nutation, Rotation of the Earth

$$\frac{\partial \Delta t}{\partial X} = \frac{1}{c} \left(\frac{D_1}{D_1} \left(-\frac{\partial \mathbf{M}}{\partial X} (t_1, t_2) ES_{\text{TRS}} \right) + \frac{D_2}{D_2} \left(-\frac{\partial \mathbf{M}}{\partial X} (t_3, t_4) ES_{\text{TRS}} \right) \right)$$
$$\frac{\partial \Delta t}{\partial Y} = \frac{1}{c} \left(\frac{D_1}{D_1} \left(-\frac{\partial \mathbf{M}}{\partial Y} (t_1, t_2) ES_{\text{TRS}} \right) + \frac{D_2}{D_2} \left(-\frac{\partial \mathbf{M}}{\partial Y} (t_3, t_4) ES_{\text{TRS}} \right) \right)$$

$d\Delta t = \frac{\partial \Delta t}{\partial X} dX + \frac{\partial \Delta t}{\partial Y} dY$ obtain dX and dY by least-square fits



The LLR observations are presented as so-called normal points. They refer to lines of data that contain the emission time of the laser, the observed round trip time in UTC, the telescope and reflector ID, and some atmospheric parameters of each observation. These data can be used to calculate the round-trip times and then the residuals of the round-trip time [observation minus calculation (O-C)], which can be converted into residuals in oneway distance in centimeters. Finally, we obtain CPO series based on these residuals.

Opinions of underestimation in former works:

Formal error multiplied by: ZC09: 2; H18: 3

H18: checked by analyses of sub-sets of LLR data

Tests in this work:

- signal propagation through the troposphere and stratosphere (for 532 nm): **0.82 cm** (Mendes & Pavlis 2003)
- Differences in ephemerides (DE430 – INPOP17a): **0.11 cm**
- Typical observational error: **0.61 cm**
- Factor: 3

In our results, the fit coefficients of the 18.6-year nutation term with and without the 9.3-year term are not consistent. This feature is different from the results obtained by VLBI analyses. Meanwhile, the correlation coefficients between two nutation terms are over 0.7, revealing the incapability of LLR data to separate the components effectively. This is probably because that LLR observations are directly related with the motions of the Moon, which is also the most important excitation of the 18.6-year and 9.3-year nutations. Nevertheless, the correlation coefficients between the secular term and the nutation terms are generally smaller than those of VLBI, probably benefiting from the longer time span of ten years. Furthermore, the correlation coefficient between the secular term and the sine term of 18.6 yr nutation remains larger than its counterparts, which may reveal a common problem shared by the VLBI and LLR techniques.

	μ^0	μ^1	μ^2	$\sin(18.6 \text{ yr})$	$\cos(18.6 \text{ yr})$	$\sin(9.3 \text{ yr})$	$\cos(9.3 \text{ yr})$	WRMS _{pre}	WRMS _{post}
Unit	mas	mas/cy	mas	mas	mas	mas	mas	mas	mas
This work	dX	-0.38 ± 0.02	1.43 ± 0.18	-0.26 ± 0.02	-0.37 ± 0.01	-0.30 ± 0.01	-0.30 ± 0.01	0.526	0.463
	dY	-0.36 ± 0.03	-0.54 ± 0.19	-0.81 ± 0.02	-0.30 ± 0.01	-0.30 ± 0.01	-0.30 ± 0.01	0.672	0.581
ZC09	dX	0.27 ± 0.13	5.77 ± 3.25	0.00 ± 0.22	0.01 ± 0.13	0.01 ± 0.13	0.01 ± 0.13		
	dY	-0.17 ± 0.13	1.07 ± 3.11	-0.02 ± 0.21	-0.22 ± 0.12	-0.22 ± 0.12	-0.22 ± 0.12		
This work	dX	-0.28 ± 0.03	1.77 ± 0.19	-0.14 ± 0.04	-0.26 ± 0.02	0.20 ± 0.02	0.01 ± 0.02	0.526	0.458
	dY	-0.12 ± 0.04	0.02 ± 0.19	-0.25 ± 0.05	-0.24 ± 0.02	-0.05 ± 0.02	-0.40 ± 0.03	0.672	0.562
ZC09	dX	0.16 ± 0.15	3.52 ± 3.84	0.17 ± 0.27	0.12 ± 0.14	0.12 ± 0.16	0.32 ± 0.14		
	dY	-0.22 ± 0.14	-0.16 ± 3.67	0.08 ± 0.26	-0.24 ± 0.14	0.10 ± 0.15	-0.01 ± 0.14		
H18	dX	-0.22 ± 0.14	-0.16 ± 3.67	0.08 ± 0.26	-0.24 ± 0.14	0.10 ± 0.15	-0.01 ± 0.14		
	dY	-0.22 ± 0.14	-0.16 ± 3.67	0.08 ± 0.26	-0.24 ± 0.14	0.10 ± 0.15	-0.01 ± 0.14		
	dX	-0.22 ± 0.14	-0.16 ± 3.67	0.08 ± 0.26	-0.24 ± 0.14	0.10 ± 0.15	-0.01 ± 0.14		
	dY	-0.22 ± 0.14	-0.16 ± 3.67	0.08 ± 0.26	-0.24 ± 0.14	0.10 ± 0.15	-0.01 ± 0.14		

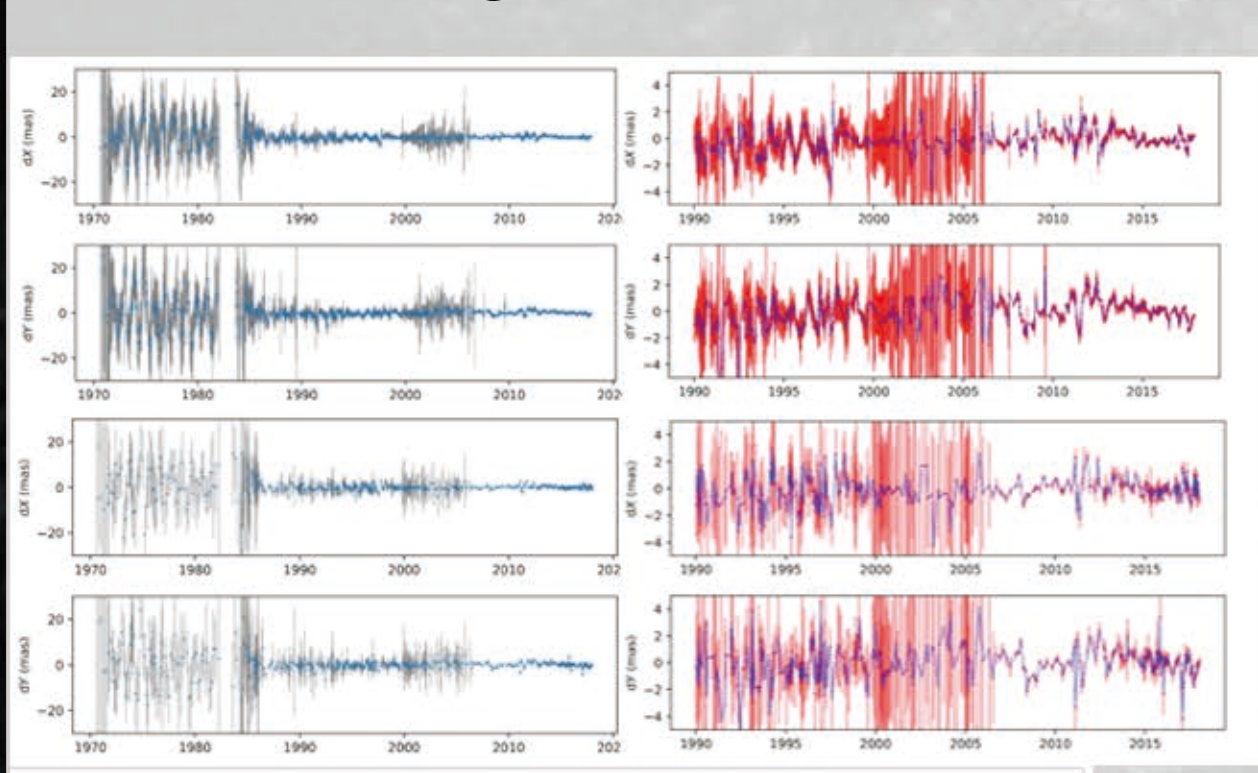
▲ Fitting parameters compared to previous work
ZC09: Zerhouni & Capitaine 2009; H18: Hofmann et al. 2018

Because the OA00 series has a resolution of 5 days (Vondrák & Ron 2000), it is theoretically able to determine the long-periodic nutation terms. Vondrák & Ron (2000) have performed certain analyses of the residual series OA97 and OA99, the predecessors of OA00 and A10, at the end of the 1990s, and compared the results to VLBI observations back then. According to their results, the fit results of OA99 were closer to those of the VLBI series than OA97. The accuracies of fit coefficients of OA99 and VLBI were on the same order owing to the low quality of VLBI data before 1990. Since then, VLBI observations have lasted 20 more years and have been greatly improved. The corresponding accuracy of the fit coefficients of corrections to long-period terms in precession-nutation models has

reached μas level, leaving the optical data an only advantage of the long history. The uncertainties of the CPO derived from OA00 series are 11.33 and 7.66 mas (weighted average) in dX and dY; respectively, about two hundred times that of VLBI data.

Term	μ^0	μ^1	μ^2	$\sin(18.6 \text{ yr})$	$\cos(18.6 \text{ yr})$	WRMS _{pre}	WRMS _{post}
Unit	(mas)	(mas cy ⁻¹)	(mas cy ⁻²)	(mas)	(mas)	(mas)	(mas)
$\Delta\psi \sin \epsilon_A$	19.8 ± 0.3	-72.8 ± 0.8	43.1 ± 3.3			35.6	29.3
$\Delta\epsilon$	-7.1 ± 0.3	-7.8 ± 0.8	34.7 ± 3.3			29.8	29.1
dX	-14.8 ± 0.3	43.6 ± 0.8	49.8 ± 3.3			31.3	29.5
dY	-12.7 ± 0.2	16.0 ± 0.6	23.8 ± 2.3			30.3	28.7
dX	-11.3 ± 0.2	40.2 ± 0.8		1.95 ± 0.2	0.06 ± 0.2	31.3	29.6
dY	-11.2 ± 0.1	13.97 ± 0.6		0.82 ± 0.2	-1.58 ± 0.2	30.3	28.7

Obtaining more CPO from LLR data



- Sliding Window
- 70 days
 - Sliding every 10 points
 - 483 corrections (original:220)

- Changing Window
- 50 points
 - Limited in 70 days
 - 67 (out of 483) windows containing less than 50 points
 - 2513 corrections

- Sliding Vs Original
- Numbers of points improve the quality of the CPO series
- Changing Vs Original
- Influences of different lengths of windows
 - Poor time distribution of observation

The sliding-window method reduces the WRMS of the series, The accuracy of the CPO determined from LLR observations can and the estimated deviations of all coefficients are smaller than be affected by many aspects. Of these, the observational error those in Table 3. The changing-window method results in a lower and frequency are most directly related to the observation itself. We show their changes with time in Fig. 12. The observational accuracy, although it suffers from instability, is improving. By 0.051 mas for the sliding-window method and 0.112 mas for the changing-window method, respectively. These results are within expectations. Observations are both more frequent and more CPO in this period are the consequences of the lower obtain a larger portion of the CPO of smaller uncertainties and frequency of the observations. Therefore, making LLR method, the window durations are different while many observations regular and parameters change with time in the calculation process. sufficiently frequent to achieve a more uniform time distribution of normal points According to these two tests, the limited quantity of obtained CPO corrections is a cause of the estimate uncertainties of the is quite essential in the future, observations and the uncertainties of the models that were used before other necessary developments of related theories in the calculation process are clearly not perfect. are possible.

