

Polar motion resonance in the retrograde diurnal band

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Introduction

The resonance parameters (T_{PM} , Q_{PM}) of the polar motion (PM) are generally considered as the Chandler wobble period $T_c = 430 - 432$ days and its quality factor $Q_c = 56 - 255$ respectively [5]. But this coincidence is only insured in the broad frequency band surrounding the Chandler wobble, including annual period. Actually, in the retrograde diurnal band of the polar motion, that is the nutation band in a non-rotating frame, these resonance parameters become $T_{PM} \approx 383$ d, $Q_{PM} \approx -11$ [6]. We propose a refine modeling of this phenomenon by accounting the response of a dynamical ocean and anelastic solid Earth to the pole tide potential.

Resonant period of the common polar motion

Notations: C : Earth axial principal moment of inertia; A and A_m : equatorial principal moments of inertia of the Earth and of the mantle respectively; $e = (C - A)/A \approx 1/304.5$: Earth dynamical flattening; $\sigma_e = e\Omega$: Euler frequency.

Resonant angular frequency of the PM given by [2]

$$\sigma_{PM} = \sigma_e \frac{A}{A_m} \left(1 - \frac{\tilde{k}}{k_s} + O(e^2)\right) = \frac{A}{A_m} \Omega(e - \kappa + O(e)), \quad (1)$$

where $k_s = 0.938$: the secular Love number, and \tilde{k} : coefficient accounting for Earth response to the pole tide potential, $\kappa = e\tilde{k}/k_s$: compliance. Actually \tilde{k} is composed of two parts:

$$\tilde{k} = \tilde{k}_2 + \tilde{k}_o. \quad (2)$$

Here \tilde{k}_2 : body Love number of degree 2 accounting for the solid Earth response to the pole tide tesseral potential, and \tilde{k}_o : oceanic Love number describing the ocean response to the same potential. For the common polar motion (beyond 2 days) the ocean response is considered at equilibrium. This leads to $\tilde{k}_o = 0.0477$. The solid Earth response is assumed as quasi-elastic, described by the body Love number $\tilde{k}_2 = 0.307 - i0.0035$ [7]. These values determine the resonance parameters ($T_{PM} = 433.6$, $Q_{PM} = 85$) in conformity with the observations.

Contribution of the dynamical ocean response

Below 10 days the ocean response is no more hydrostatic, and k_o changes accordingly. In the diurnal band, this issue can be solved in light of the diurnal ocean tides. For, as the pole tide potential has the same form than the luni-solar tesseral potential and concerns the same frequency band, the Earth response should be formally the same. The tidal height produces matter term $H(t)$. The observed diurnal ocean tide height is smaller than the theoretical equilibrium tide ξ , and strongly out-of-phased with respect to the tidal potential. Meanwhile, dynamical processes produce currents, in turn a relative angular momentum $h(t)$. The tesseral tidal potential is

$$W = -\frac{\Omega^2 r^2}{3} \operatorname{Re} \left[\tilde{\phi}(t) \mathcal{D}_2^{-1} \right]$$

where $\tilde{\phi}(t) = 3gN_2^1/(\Omega^2 R_e^2) \sum_{\sigma \geq 0} \xi_\sigma e^{-i(\theta_\sigma(t) - \pi/2)}$ is formally equivalent to $m(t)$ in pole tide potential. So, in

Liouville equation, the tidal excitation $\chi_o(t)$ is proportional to $\tilde{\phi}(t)$, as the rotational excitation is proportional to $m(t)$:

$$\chi_o = \tilde{k}_o/k_s \tilde{\phi}, \quad (3)$$

The components of χ_o are computed from FES 2012 reported in [3]. For tesseral tides J_1 , K_1 , P_1 , O_1 , Q_1 , we extract the retrograde diurnal components H^- and h^- . Then, accounting for loading effect, for a tidal constituent at frequency σ we have $\chi_o(t) = \frac{H_\sigma^-(t)(1+k'_2) + h_\sigma^-(t)}{(C-A)\Omega} = \frac{H_\sigma^-(1+k'_2) + h_\sigma^-}{(C-A)\Omega} e^{-i(\theta+\chi)}$ where θ is the tidal argument. It results

$$\tilde{k}_o = k_s \frac{H_\sigma^-(t)(1+k'_2) + h_\sigma^-(t)}{(C-A)\Omega \tilde{\phi}_\sigma} = -k_s \frac{H_\sigma^-(1+k'_2) + h_\sigma^-}{C-A} \frac{\Omega R_e^2}{3gN_2^1 \xi_\sigma}. \quad (4)$$

Q_1	$-0.037 + i0.039$	O_1	$-0.030 + i0.038$
P_1	$-0.023 + i0.042$	K_1	$-0.023 + i0.042$
		J_1	$-0.022 + i0.047$

Table 1: Oceanic Love number for some prominent waves of the retrograde diurnal band.

We estimate \tilde{k}_o for each tidal components H_σ^- , h_σ^- and corresponding tidal height ξ_σ . The resonance of the loading love number k'_2 at FCN frequency does not impact significantly \tilde{k}_o in retrograde the diurnal band. For $k'_2 = -0.3075$, the obtained values differ strikingly from the oceanic Love number $k_o = 0.0477$ estimated for an equilibrium pole tide: at K_1 $k_o = -0.023 + i0.042$ or compliance $\tilde{\kappa}_o = \tilde{k}_o e/k_s = (-7.9 + i14.6) 10^{-5}$ in agreement with $(-6.9 + i11.5) 10^{-5}$ of [4]. The values of Table 1 allow to model $k_o(\sigma)$ through a degree 2 polynomial of the frequency in the diurnal retrograde band $k_o(f) = (-0.716 + i0.721)f^2 + (-1.483 + i1.337)f + (-0.791 + i0.658)$, where f is in cpd. Then we see that the resonance frequency (1) becomes frequency dependent:

$$\sigma_{PM}(\sigma) = \sigma_e \frac{A}{A_m} \frac{\tilde{k}_2 + \tilde{k}_o(\sigma)}{k_s}, \quad (5)$$

where \tilde{k}_2 is taken as the Love number of an anelastic Earth, slightly varying with frequency. In the diurnal domain we have $\tilde{k}_2 = 0.299 - i0.00144$ [7]. The corresponding period and quality factor, namely $T_{PM}(\sigma) = 2\pi/\operatorname{Re}(\sigma_{PM})$ and $Q_{PM}(\sigma) = \operatorname{Re}(\sigma_{PM})/(2\operatorname{Im}(\sigma_{PM}))$ are displayed in Figure 1.

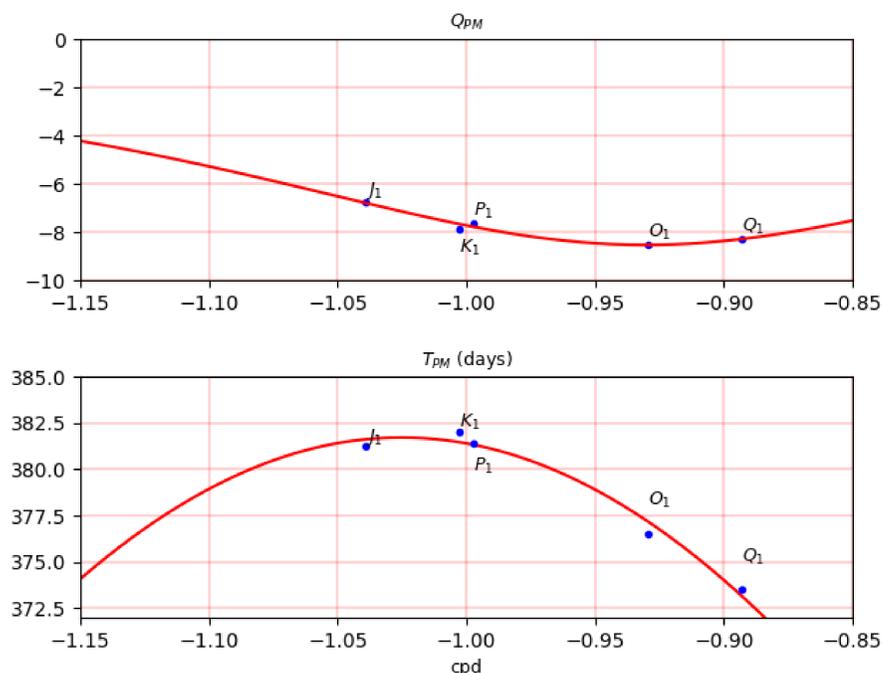


Figure 1: PM Resonance parameters in the diurnal retrograde band for an anelastic solid Earth covered by oceans.

So, in the frequency band $[-1.15$ cpd, -0.85 cpd], the dynamical ocean response leads to the resonance parameters lying in the intervals $374 \text{ d} < T_{PM} \leq 382.5 \text{ d}$ and $-4 \leq Q_{PM} \leq -10$, confirmed by $T_{PM} = 382.0 \pm 1.3$ days and $Q_{PM} = -10.4 \pm 0.5$ obtained from nutation analysis in [6].

Influence of the fluid core

Close to the retrograde diurnal frequency $\sigma_{FCN} = -1.00506$ cycle/day (cpd) of the free core nutation, the solid Earth tide departs from the one of a quasi-elastic Earth. Other perturbations, of much lesser amplitude (100 times less), occur because of the free inner core nutation (FICN) mode at $\sigma_{FICN} \sim 1.0017$ cpd in the TRF, and because of the Polar motion resonance appearing at the period ~ 380 days, as justified in the former section. From IERS Conventions 2010 [7], Table 6.4, Eq. 6.9 and 6.10, the "diurnal" body Love number has the form

$$k_2(\sigma) = 0.29954 - i0.1412 \cdot 10^{-2} - \frac{L_{PM}}{\sigma - \tilde{\sigma}_{PM}} - \frac{L_{FCN}}{\sigma - \tilde{\sigma}_{FCN}} - \frac{L_{FICN}}{\sigma - \tilde{\sigma}_{FICN}}, \quad (6)$$

with the dominant term $L_{FCN} = (0.91 \cdot 10^{-4} - i0.30 \cdot 10^{-5})$ cpd. Here $\sigma_{PM} \approx 1/383$ cpd. Replacing in (5) the pure anelastic value of k_2 by its resonant version (6), we get

$$\sigma_{PM}(\sigma) = \sigma_e \frac{A}{A_m} \frac{\tilde{k}_2(\sigma) + \tilde{k}_o(\sigma)}{k_s}. \quad (7)$$

The resonance parameters deduced from (7) are plotted in Figure 2 over the band $[-1.15$ cpd, -0.85 cpd] (denoted band I). This theoretical curve is compared with the estimated values from different sets of dominant luni-solar nutation terms, as reported in [6]. In average, far from the resonance at σ_{FCN} , the theoretical curve corresponds grossly to the estimated value obtained for the whole band I ($T_{PM} = 382 \pm 1.3$ d, $Q_{PM} = -10 \pm 1$) [6].

The resonance produced by the free core nutation strongly is partly confirmed by nutation inversion for restricted bands:

- K_1 - precession and long period nutation terms (6798 d, 1095 d, 3399 d) in the CRF (band III₃). Modeled values ($T_{PM} = 360$ d, $Q_{PM} = -10$) at K_1 quite well predict the estimated value (382 ± 8 d, -8.5).
- Ψ_1 - retrograde nutation terms in 365.25 d, 386 d in the CRF (band III₂). Enhancement of T_{PM} up to 470 d at Ψ_1 , matching $T_{PM}^{observed} = 487 \pm 58$ d. The value $Q_{PM}^{observed} = 13 \pm 31$ too uncertain for confirming the modeled value -5 at Ψ_1 .
- far from K_1 and Ψ_1 , the resonance parameters rejoin the curves obtained for an anelastic Earth covered by oceans. At the right part of the spectrum corresponding to band III₄, covering tidal lines S_1 and O_1 , the estimates ($T_{PM} = 381.8 \pm 1.3$ d, $Q_{PM} = -10.4 \pm 0.5$) are close to the modeled parameters. For the opposite band (III₁), the estimated period is longer (418 days), as expected from the asymmetry of the resonance.

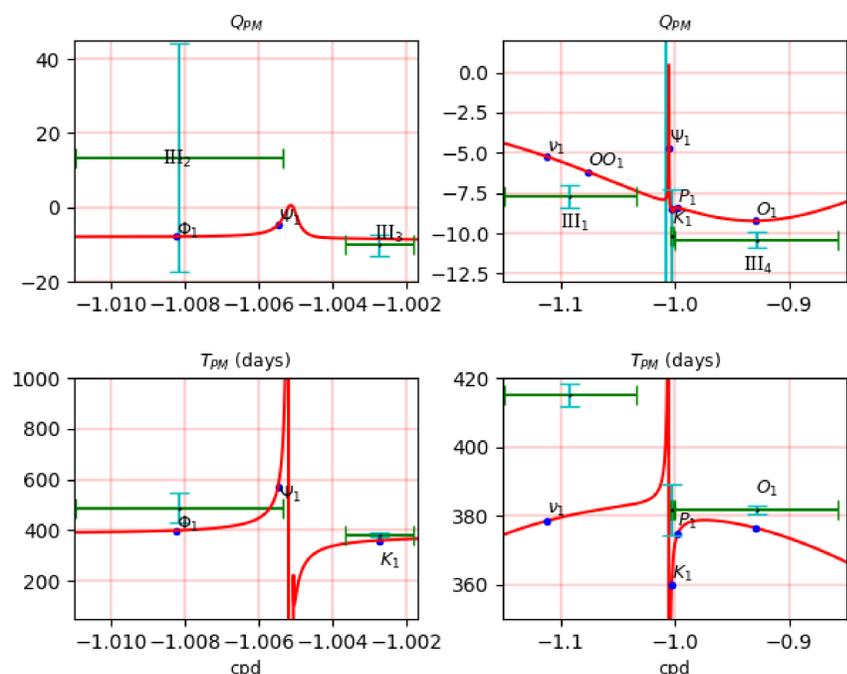


Figure 2: Resonance parameters of the polar motion in the diurnal retrograde band for an anelastic Earth covered by oceans and containing a fluid core. Green crosses specify the values obtained from nutation inversion over the restricted frequency bands III₁ (v_1, OO_1), III₂ (Ψ_1), III₃ (K_1), III₄ (S_1, P_1, O_1, Q_1): the horizontal bar extension gives the frequency band, and the vertical bar the uncertainty of the estimated value.

Conclusion

The dynamical response of the oceans to the pole tide potential is the main factor reducing the polar motion period to about 380 days in the retrograde diurnal band. The associated quality factor ~ -10 reflects the strong phase-shift of this response with respect to the pole tide. In reason of the free core nutation resonance, the body Love number strongly deviates from its mean value of 0.3 in vicinity of the FCN frequency ($\sigma_{FCN} = -1.0050$ cpd). In turn, in the band $[-1.15$ cpd, -0.85 cpd] as observed from the Earth, the resonance period of the polar motion increases above 400 days for frequencies smaller than σ_{FCN} , and remains below this threshold for the band above σ_{FCN} .

In contrast to common polar motion, the excitation at stake, namely the diurnal tidal torque through rigid Earth nutation terms, is almost perfectly known. Despite the remoteness of the polar resonance period from the retrograde diurnal nutation terms in the terrestrial frame, the confrontation of observed nutation terms to those of a rigid Earth, as carried out in [6], amazingly confirms the modeled frequency dependence. So, the luni-solar nutation determined by VLBI reflect the dynamical behavior of the ocean and influence of the fluid core on solid Earth deformation in the retrograde diurnal band.

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