Solid body tides in the dynamical model of the Moon

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(with thanks to James Williams from NASA JPL)

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Tagline

“We are not there yet”

– Veronique Dehant, 2019

Earth, Moon, and LLR

Moon study is different from the Earth:

- Dynamical model of orbit and rotation, integrated over the timespan of LLR observations.
- Lunar spacecraft orbits are not tied directly to the surface.

  Indirect tie via ephemeris: Spacecraft — Earth — Moon

We need to improve the dynamical model in order to:

- Improve the Earth-Moon reference frame

  EPM2019 5-point lunar frame has an accuracy of 0.2–2 m
- Study the interior of the Moon

Thanks to a single 50 year orbit, LLR is very sensitive to:

- Degree 2 tides on the Earth (Williams, Boggs, 2016)
- Low degree harmonics of the Moon
- Earth's orbit plane

LLR Solution: Determined Parameters

- Geocentric position and velocity of the Moon at epoch;
- Euler angles (φ, θ, ψ) and their derivatives at epoch;
- Angular velocity of the lunar liquid core at epoch;
- GM of the Earth–Moon system;
- Ratios of undistorted lunar moments of inertia: $(C - A)/B$, $(B - A)/C$
- Stokes coefficients of undistorted lunar gravitational potential: C_{32}, S_{32}, S_{33}
- Lunar Love number h_2;
- Lunar core flattening coefficient;
- Lunar tidal delay;
- Rotational delays τ_{1R} and τ_{2R} of Earth diurnal and semidiurnal tides;
- Amplitudes of 365 d, 206 d, and 1095 d kinematic terms;
- Selenocentric coordinates of five retroreflectors;
- Terrestrial coordinates of all LLR stations;
- Velocities of McDonald/MLRS1/MLRS2 and Grasse stations;
- 25 biases for chosen stations at chosen periods of time.

Statistics of Residuals

Station	Timespan	Used	Rejected	One-way wrms, cm
McDonald	1970-1985	3552	52	20.1
Crimea	1982-1984	25	0	11.1
MLRS1	1983-1988	588	43	11.0
MLRS2	1988-2015	3224	429	3.4
Haleakala	1984-1990	751	19	5.8
OCA (Ruby)	1984-1986	1109	79	16.9
OCA (YAG)	1987-2005	8273	51	1.5
OCA (MeO)	2009-2019	1814	22	1.61
OCA (IR)	2015-2019	2797	43	1.30
APO	2006-2016	2610	38	1.50
Matera	2003-2019	219	14	3.1
Wettzell	2018-2019	42	0	1.06

GRAIL mission

Two cycles of observations, both under three months:

1. Primary mission: 1 March – 29 May 2012
2. Extended mission: 30 August – 14 December 2012

Great sensitivity to high-degree harmonics, mascons etc.

Not so much to tidal variations of low-degree harmonics.

Results are supposed to be in the PA (principal axes) frame, but:

Solution	Team	Ephemeris	C21	S21	S22
GL0660b	JPL	DE421	0.123e-9	**0.101e-8**	-0.249e-9
GRGM900c	GSFC	DE421	0.223e-9	**0.101e-8**	-0.105e-9
GRGM1200a	GSFC	DE430	0.147e-10	**0.117e-8**	0.908e-9
GL1500e	JPL	DE430	0.173e-9	**0.104e-8**	-0.102e-9

Known problems with the lunar model

Detected few mas amplitudes in libration in longitude:

- cos l' (1 yr)
- cos$(2l - 2D)$ (206 d)
- cos$(2F - 2l)$ (3 yr)
- sin$(F - l)$ (6 yr)

Nonzero S21:

Detected even in LLR solution: $(0.74 \pm 0.05^{(1\sigma)}) \times 10^{-9}$

Suspected causes:

- Dependency of tidal k_2 and/or Q on frequency
- Something with the inner core

What to do with GRAIL?

GRAIL solution is consistent with DE430 ephemeris.
It is tied to the ephemeris rather than to the Moon itself.
Using it in other ephemeris probably “drags” it towards DE430.

Use GRAIL coefficients in DE430 frame?

Very bad orbit

Use kinematic model of lunar tides (Konopliv & Williams, 2015)?

Also bad orbit

Proposal:

1. Deal with lunar dynamics without GRAIL’s C_{20}, C_{32}, S_{32}, S_{33} until the longitude libration terms are gone
2. After that, maybe refit GRAIL data to new ephemeris or even make a single GRAIL-LLR solution

What to do with lunar dynamics?

Tilted core?

No effect (Pavlov, 2018, in Russian)

Including the triaxial tilted core case (Viswanathan et al, 2019)

Aligned triaxial core as well as shifted core (Wieczorek et al, 2019): worth trying in near future.

Multi-delay integrator: tried in this work.

Other dynamical approaches that account for dependency of tidal dissipation on frequency?

Delayed tidal dissipation (JPL model)

$$\frac{I}{m} = \frac{2R_M^2 \tilde{J}_2}{2\beta - \gamma + \beta\gamma} \begin{bmatrix} 1-\beta\gamma & 0 & 0 \\ 0 & 1+\gamma & 0 \\ 0 & 0 & 1+\beta \end{bmatrix} - \frac{I_c}{m}$$

$$- k_2 \frac{\mu_E}{\mu_M} \left(\frac{R_M}{r}\right)^5 \begin{bmatrix} x^2 - \frac{1}{3}r^2 & xy & xz \\ xy & y^2 - \frac{1}{3}r^2 & yz \\ xz & yz & z^2 - \frac{1}{3}r^2 \end{bmatrix}$$

x, y, z is the position of the Earth taken with a delay τ

$$+ k_2 \frac{R_M^5}{3\mu_M} \begin{bmatrix} \omega_x^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_x\omega_y & \omega_x\omega_z \\ \omega_x\omega_y & \omega_y^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_y\omega_z \\ \omega_x\omega_z & \omega_y\omega_z & \omega_z^2 - \frac{1}{3}(\omega^2 + 2n^2) \end{bmatrix}$$

Gravity coefficients respond accordingly:

$$C_{20} = \frac{1}{R_M^2}\left[\frac{1}{2}\left(\frac{I_{11}^*}{m} + \frac{I_{22}^*}{m}\right) - \frac{I_{33}^*}{m}\right]$$

$$C_{22} = \frac{1}{4R_M^2}\left[\frac{I_{22}^*}{m} - \frac{I_{11}^*}{m}\right]$$

$$C_{21} = -\frac{1}{R_M^2}\frac{I_{13}^*}{m}$$

$$S_{21} = -\frac{1}{R_M^2}\frac{I_{32}^*}{m}$$

$$S_{22} = -\frac{1}{2R_M^2}\frac{I_{21}^*}{m}$$

Using a delay-capable integrator (next talk)

Proposed modifications

1. Include tides from the Sun
2. Fit C_{20}, ignoring GRAIL value for the time being
3. Introduce different delays for diagonal and off-diagonal terms of inertia tensor

   (still will not entirely separate the bands, but may reduce the disorder)

$$\begin{bmatrix} x^2 - \frac{1}{3}r^2 & xy & xz \\ xy & y^2 - \frac{1}{3}r^2 & yz \\ xz & yz & z^2 - \frac{1}{3}r^2 \end{bmatrix}$$

$$C_{20} = \frac{1}{R_M^2}\left[\frac{1}{2}\left(\frac{I_{11}^*}{m} + \frac{I_{22}^*}{m}\right) - \frac{I_{33}^*}{m}\right]$$

$$C_{22} = \frac{1}{4R_M^2}\left[\frac{I_{22}^*}{m} - \frac{I_{11}^*}{m}\right]$$

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$$S_{22} = -\frac{1}{2R_M^2}\frac{I_{21}^*}{m}$$

Two delays:

τ_{02}, τ_1

Results

Fitted value	Single delay	Two delays
τ / τ_{02} , days	0.098 ± 0.001	0.068 ± 0.002
τ_1, days	N/A	0.144 ± 0.002
cos *l′* amplitude, mas	4.2 ± 0.1	4.2 ± 0.1
cos(*2l* − *2D*) amplitude, mas	1.6 ± 0.1	0.8 ± 0.1
cos(2F − 2l) amplitude, mas	-0.8 ± 0.3	-3.4 ± 0.3
sin(*F* - *l*) amplitude, mas (separate solution)	-9.0 ± 0.6	-7.2 ± 0.6
$S21 \times 10^9$ (separate solution)	0.74 ± 0.05	0.58 ± 0.05

Given errors are 1σ.
S21 seems to vanish when sin(*F* - *l*) is fitted.

Conclusion

- Solid body lunar tides are of no less importance than the core.
- GRAIL data should be later reanalyzed with a better ephemeris.
- There is a 6 year period in longitude* that is strongly connected to the frame misalignment resulting in nonzero S21.
- Tho different delays can be detected from LLR data. Separation significantly affects 206 d and 3 yr amplitudes, slightly affects S21 / 6 yr amplitude.
- Maybe other approaches exist to separate tidal bands.
- Triaxial/shifted core needs to be tested in a dynamical model.

* Credit goes to James Williams