Solid body tides in the dynamical model of the Moon

Dmitry Pavlov
Laboratory of Ephemeris Astronomy, IAA RAS
dpavlov@iaaras.ru

(with thanks to James Williams from NASA JPL)

iaaras.ru/en

Journées 2019
Paris
“We are not there yet”

— Veronique Dehant, 2019
Earth, Moon, and LLR

Moon study is different from the Earth:

➢ Dynamical model of orbit and rotation, integrated over the timespan of LLR observations.
➢ Lunar spacecraft orbits are not tied directly to the surface. Indirect tie via ephemeris: Spacecraft — Earth — Moon

We need to improve the dynamical model in order to:

➢ Improve the Earth-Moon reference frame
  EPM2019 5-point lunar frame has an accuracy of 0.2–2 m
➢ Study the interior of the Moon

Thanks to a single 50 year orbit, LLR is very sensitive to:

➢ Degree 2 tides on the Earth (Williams, Boggs, 2016)
➢ Low degree harmonics of the Moon
➢ Earth’s orbit plane
LLR Solution: Determined Parameters

- Geocentric position and velocity of the Moon at epoch;
- Euler angles ($\phi$, $\theta$, $\psi$) and their derivatives at epoch;
- Angular velocity of the lunar liquid core at epoch;
- GM of the Earth–Moon system;
- Ratios of undistorted lunar moments of inertia: $(C - A)/B, (B - A)/C$
- Stokes coefficients of undistorted lunar gravitational potential: $C_{32}, S_{32}, S_{33}$
- Lunar Love number $h_2$;
- Lunar core flattening coefficient;
- Lunar tidal delay;
- Rotational delays $\tau_{1R}$ and $\tau_{2R}$ of Earth diurnal and semidiurnal tides;
- Amplitudes of 365 d, 206 d, and 1095 d kinematic terms;
- Selenocentric coordinates of five retroreflectors;
- Terrestrial coordinates of all LLR stations;
- Velocities of McDonald/MLRS1/MLRS2 and Grasse stations;
- 25 biases for chosen stations at chosen periods of time.
## Statistics of Residuals

<table>
<thead>
<tr>
<th>Station</th>
<th>Timespan</th>
<th>Used</th>
<th>Rejected</th>
<th>One-way wrms, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>McDonald</td>
<td>1970-1985</td>
<td>3552</td>
<td>52</td>
<td>20.1</td>
</tr>
<tr>
<td>Crimea</td>
<td>1982-1984</td>
<td>25</td>
<td>0</td>
<td>11.1</td>
</tr>
<tr>
<td>MLRS1</td>
<td>1983-1988</td>
<td>588</td>
<td>43</td>
<td>11.0</td>
</tr>
<tr>
<td>MLRS2</td>
<td>1988-2015</td>
<td>3224</td>
<td>429</td>
<td>3.4</td>
</tr>
<tr>
<td>Haleakala</td>
<td>1984-1990</td>
<td>751</td>
<td>19</td>
<td>5.8</td>
</tr>
<tr>
<td>OCA (Ruby)</td>
<td>1984-1986</td>
<td>1109</td>
<td>79</td>
<td>16.9</td>
</tr>
<tr>
<td>OCA (YAG)</td>
<td>1987-2005</td>
<td>8273</td>
<td>51</td>
<td>1.5</td>
</tr>
<tr>
<td>OCA (MeO)</td>
<td>2009-2019</td>
<td>1814</td>
<td>22</td>
<td>1.61</td>
</tr>
<tr>
<td>OCA (IR)</td>
<td>2015-2019</td>
<td>2797</td>
<td>43</td>
<td>1.30</td>
</tr>
<tr>
<td>APO</td>
<td>2006-2016</td>
<td>2610</td>
<td>38</td>
<td>1.50</td>
</tr>
<tr>
<td>Matera</td>
<td>2003-2019</td>
<td>219</td>
<td>14</td>
<td>3.1</td>
</tr>
<tr>
<td>Wettzell</td>
<td>2018-2019</td>
<td>42</td>
<td>0</td>
<td>1.06</td>
</tr>
</tbody>
</table>
Two cycles of observations, both under three months:
2. Extended mission: 30 August – 14 December 2012

Great sensitivity to high-degree harmonics, mascons etc.
Not so much to tidal variations of low-degree harmonics.
Results are supposed to be in the PA (principal axes) frame, but:

<table>
<thead>
<tr>
<th>Solution</th>
<th>Team</th>
<th>Ephemeris</th>
<th>C21</th>
<th>S21</th>
<th>S22</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL0660b</td>
<td>JPL</td>
<td>DE421</td>
<td>0.123e-9</td>
<td><strong>0.101e-8</strong></td>
<td>-0.249e-9</td>
</tr>
<tr>
<td>GRGM900c</td>
<td>GSFC</td>
<td>DE421</td>
<td>0.223e-9</td>
<td><strong>0.101e-8</strong></td>
<td>-0.105e-9</td>
</tr>
<tr>
<td>GRGM1200a</td>
<td>GSFC</td>
<td>DE430</td>
<td>0.147e-10</td>
<td><strong>0.117e-8</strong></td>
<td>0.908e-9</td>
</tr>
<tr>
<td>GL1500e</td>
<td>JPL</td>
<td>DE430</td>
<td>0.173e-9</td>
<td><strong>0.104e-8</strong></td>
<td>-0.102e-9</td>
</tr>
</tbody>
</table>
Known problems with the lunar model

Detected few mas amplitudes in libration in longitude:
- $\cos l'$ (1 yr)
- $\cos(2l - 2D)$ (206 d)
- $\cos(2F - 2l)$ (3 yr)
- $\sin(F - l)$ (6 yr)

Nonzero S21:
Detected even in LLR solution: $(0.74 \pm 0.05^{(1\sigma)}) \times 10^{-9}$

Suspected causes:
- Dependency of tidal $k_2$ and/or $Q$ on frequency
- Something with the inner core
What to do with GRAIL?

GRAIL solution is consistent with DE430 ephemeris. It is tied to the ephemeris rather than to the Moon itself. Using it in other ephemeris probably “drags” it towards DE430.

Use GRAIL coefficients in DE430 frame?
   Very bad orbit

Use kinematic model of lunar tides (Konopliv & Williams, 2015)?
   Also bad orbit

Proposal:
1. Deal with lunar dynamics without GRAIL’s $C_{20}, C_{32}, S_{32}, S_{33}$ until the longitude libration terms are gone
2. After that, maybe refit GRAIL data to new ephemeris or even make a single GRAIL-LLR solution
What to do with lunar dynamics?

Tilted core?

No effect ([Pavlov, 2018](#), in Russian)

Including the triaxial tilted core case ([Viswanathan et al, 2019](#))

Aligned triaxial core as well as shifted core ([Wieczorek et al, 2019](#)): worth trying in near future.

Multi-delay integrator: tried in this work.

Other dynamical approaches that account for dependency of tidal dissipation on frequency?
Delayed tidal dissipation (JPL model)

\[
\frac{I}{m} = \frac{2R_M^2 J_2}{2\beta - \gamma + \beta \gamma} \left[ \begin{array}{ccc} 1 - \beta \gamma & 0 & 0 \\ 0 & 1 + \gamma & 0 \\ 0 & 0 & 1 + \beta \end{array} \right] - \frac{I_c}{m}
\]

\[- k_2 \frac{\mu_E}{\mu_M} \left( \frac{R_M}{r} \right)^5 \left[ \begin{array}{ccc} x^2 - \frac{1}{3} r^2 & xy & xz \\ xy & y^2 - \frac{1}{3} r^2 & yz \\ xz & yz & z^2 - \frac{1}{3} r^2 \end{array} \right] + k_2 \frac{R_M^5}{3\mu_M} \left[ \begin{array}{ccc} \omega_x^2 & \frac{1}{3} \omega^2 - n^2 & \omega_x \omega_y \\ \omega_y \omega_y & \frac{1}{3} \omega^2 - n^2 & \omega_y \omega_z \\ \omega_x \omega_z & \omega_x \omega_z & \frac{1}{3} (\omega^2 + 2n^2) \end{array} \right] \]

\[
C_{20} = \frac{1}{R_M^2} \left[ \frac{1}{2} \left( \frac{I_{11}^*}{m} + \frac{I_{22}^*}{m} \right) - \frac{I_{33}^*}{m} \right]
\]

\[
C_{22} = \frac{1}{4R_M^2} \left[ \frac{I_{22}^*}{m} - \frac{I_{11}^*}{m} \right]
\]

\[
C_{21} = \frac{1}{R_M^2} \omega_z
\]

\[
S_{21} = - \frac{1}{R_M^2} \omega_z
\]

\[
S_{22} = - \frac{1}{2R_M^2} \frac{I_{21}^*}{m}
\]

Gravity coefficients respond accordingly:

\[
x, y, z \text{ is the position of the Earth taken with a delay } \tau
\]

Using a delay-capable integrator (next talk)
Proposed modifications

1. Include tides from the Sun
2. Fit $C_{20}$, ignoring GRAIL value for the time being
3. Introduce different delays for diagonal and off-diagonal terms of inertia tensor

(still will not entirely separate the bands, but may reduce the disorder)

Two delays:
$\tau_{02}, \tau_{1}$

\[
\begin{bmatrix}
  x^2 - \frac{1}{3}r^2 & xy & xz \\
  xy & y^2 - \frac{1}{3}r^2 & yz \\
  xz & yz & z^2 - \frac{1}{3}r^2
\end{bmatrix}
\]

\[
C_{20} = \frac{1}{R_M^2} \left[ \frac{1}{2} \left( \frac{I_{11}^*}{m} + \frac{I_{22}^*}{m} \right) - \frac{I_{33}^*}{m} \right]
\]

\[
C_{22} = \frac{1}{4R_M^2} \left[ \frac{I_{22}^*}{m} - \frac{I_{11}^*}{m} \right]
\]

\[
C_{21} = -\frac{1}{R_M^2} \frac{I_{13}^*}{m}
\]

\[
S_{21} = -\frac{1}{R_M^2} \frac{I_{32}^*}{m}
\]

\[
S_{22} = -\frac{1}{2R_M^2} \frac{I_{21}^*}{m}
\]
### Results

<table>
<thead>
<tr>
<th>Fitted value</th>
<th>Single delay</th>
<th>Two delays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau / \tau_{02}$, days</td>
<td>$0.098 \pm 0.001$</td>
<td>$0.068 \pm 0.002$</td>
</tr>
<tr>
<td>$\tau_1$, days</td>
<td>N/A</td>
<td>$0.144 \pm 0.002$</td>
</tr>
<tr>
<td>$\cos l'$ amplitude, mas</td>
<td>$4.2 \pm 0.1$</td>
<td>$4.2 \pm 0.1$</td>
</tr>
<tr>
<td>$\cos(2l - 2D)$ amplitude, mas</td>
<td>$1.6 \pm 0.1$</td>
<td>$0.8 \pm 0.1$</td>
</tr>
<tr>
<td>$\cos(2F - 2l)$ amplitude, mas</td>
<td>$-0.8 \pm 0.3$</td>
<td>$-3.4 \pm 0.3$</td>
</tr>
<tr>
<td>$\sin(F - l)$ amplitude, mas</td>
<td>$-9.0 \pm 0.6$</td>
<td>$-7.2 \pm 0.6$</td>
</tr>
<tr>
<td>(separate solution)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{21} \times 10^9$</td>
<td>$0.74 \pm 0.05$</td>
<td>$0.58 \pm 0.05$</td>
</tr>
<tr>
<td>(separate solution)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Given errors are 1σ.  
$S_{21}$ seems to vanish when $\sin(F - l)$ is fitted.
Conclusion

➢ Solid body lunar tides are of no less importance than the core.
➢ GRAIL data should be later reanalyzed with a better ephemeris.
➢ There is a 6 year period in longitude* that is strongly connected to the frame misalignment resulting in nonzero S21.
➢ Three different delays can be detected from LLR data. Separation significantly affects 206 d and 3 yr amplitudes, slightly affects S21 / 6 yr amplitude.
➢ Maybe other approaches exist to separate tidal bands.
➢ Triaxial/shifted core needs to be tested in a dynamical model.

* Credit goes to James Williams