Geodetic (Relativistic) rotation of the Mars satellites system

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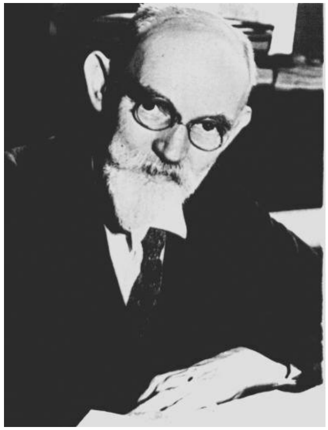
INTRODUCTION

The geodetic rotation of a body is the most essential relativistic effect of its rotation and consist of two effects:



the geodetic precession is the systematic effect,

which was first predicted by Willem de Sitter in 1916, who provided relativistic corrections to the Earth–Moon system's motion,

and

the geodetic nutation is the periodic effect.

The components of the geodetic nutation for the Earth were first calculated by Toshio Fukushima in 1991.

These effects have some analogies with precession and nutation, which are better-known events on the classical mechanics.

Their emergence, unlike the last classical events, **are not depend on from influences of any forces to body,** represents only the effect of the curvature of space-time, **predicted by general relativity**, on a vector of the body rotation axis carried along with an orbiting body.

Aims of the present research:

1.The effect of the geodetic rotation in the rotation of Mars satellites system for the first time are investigated.

2.To define a new high-precision values of the geodetic rotation for Mars dynamically adjusted to JPL DE431/LE431 ephemeris (Folkner et al., 2014) in Euler angles and
for its satellites dynamically adjusted to Horizons On-Line Ephemeris System (Giorgini et al., 2001) in Euler angles and in the perturbing terms of its physical librations.

For these purposes, it will be used, the algorithm of Pashkevich (2016), which is applicable to the study of any bodies of the Solar system that have long-time ephemeris.

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Expressions for the perturbing terms of the physical librations for the fixed ecliptic of epoch J2000:

$$\tau = \varphi + \psi - 180^\circ - L_i$$

$$\rho = \theta - \mathrm{I}$$

$$\sigma = \psi - \Omega$$

where ψ is the longitude of the descending node of epoch J2000 of the body equator,

θ is the inclination of the body equator to the fixed ecliptic J2000,

φ is the body proper rotation angle between the descending node of epoch J2000 and the principal axis A (with the minimum moment of inertia);

L_i is the mean longitude of the body;

I is a constant angle of the inclination of the body equator to the fixed ecliptic J2000;

Ω is the mean longitude of the ascending node of its orbit;

τ, ρ and σ are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.

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Mathematical model of the problem

The problem of the geodetic (relativistic) rotation for Mars and for his satellites (Phobos and Deimos)
is studied over the time span from AD1600 to AD2500 with one hour spacing
with respect to the kinematically non-rotating (Kopeikin et al., 2011) proper coordinate system of the studied bodies:
for Mars (Seidelmann et al., 2005) and
for Mars satellites (Archinal et al., 2018).

The positions, velocities, physical parameters and orbital elements for Phobos and Deimos are taken from the Horizons On-Line Ephemeris System (Giorgini et al., 2001) and
ones for the Sun, the Moon, Pluto and the major planets
are calculated using the fundamental ephemeris JPL DE431/LE431 (Folkner et al., 2014).

Results

As a result of this investigation, in the perturbing terms of the physical librations and in Euler angles **for the Martian satellites (Phobos and Deimos),** and in Euler angles **for Mars** the most significant systematic Δx_s (Table 1) and periodic Δx_p (Table 2) terms of the geodetic rotation are calculated:

$$\Delta x_s = \sum_{n=1}^{N} \Delta x_n t^n,$$

$$\Delta x_p = \sum_{j}\sum_{k=0}^{M} (\Delta x_{Cjk} \cos(\nu_{j0} + \nu_{j1} t) + \Delta x_{Sjk} \sin(\nu_{j0} + \nu_{j1} t)) t^k,$$

where $\Delta x = x_{\mathbf{relativistic}} - x_{\mathbf{Newtonian}}$, $x = \psi,\ \theta,\ \varphi,\ \tau,\ \rho,\ \mathrm{I}\sigma$; Δx_n are the coefficients of the systematic terms; Δx_{Sjk}, Δx_{Cjk} are the coefficients of the periodic terms; ν_{j0}, ν_{j1} are phases and frequencies of the body under study, which are combinations of the corresponding Delaunay arguments and the mean longitudes of the perturbing bodies; the summation index j is the number of added periodic terms, and its value changes for each body under study; t is the time in the Julian days.

Results

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Table 1. The secular terms of geodetic rotation of Mars and its satellites

	Mars	**Phobos** (a=9376 km)		**Deimos** (a=23458 km)	
tjy	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\tau_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\tau_s$ (μas)
T	**–7** 113 935.6683	**–209** 314 864.7430	**–95** 713 236.3800	**–27** 680 096.2268	**–15** 836 815.1715
T^2	9758.6588	43043.9996	22074.5862	14436.8795	1970.5567
T^3	1328.3085				
tjy	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\rho_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\rho_s$ (μas)
T	119866.5547	109821.3069	109821.3069	118932.5546	118932.5546
T^2	–1065.6036	–79913.4426	–79913.4426	–5802.8941	–5802.8941
T^3	–57.9607				
tjy	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta(I\sigma)_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta(I\sigma)_s$ (μas)
T	405134.4944	113601628.3630	94088505.4932	11843281.0553	12076398.3007
T^2	–11482.6140	–20969.4134	32232.9894	–12466.3227	640.6748
T^3	–280.3423				

As can be seen from the Tables 1, the values of the geodetic rotation of Mars satellites decrease with increasing their distance from Mars. **At the same time,** the values of the total geodetic rotation of Phobos and Deimos far exceed the value of the geodetic rotation of Mars. **It is due to the fact** that **Mars has a greater influence on the geodetic rotation of its satellites** than the Sun on its and Mars by reason of the close distances between Mars and **its satellites.**

Results

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Table 1a. The secular terms of geodetic rotation of some planets of Solar system and the Moon (Pashkevich and Vershkov, 2019)

	Mercury	Venus	The Earth	The Moon	**Mars**
tjy	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\tau_s$ (μas)	$\Delta\psi_s$ (μas)
T	**–426** 451 032.8798	**–156** 030 839.3400	**–19** 198 865.6280	**–19** 494 124.5472	**–7** 113 935.6683
T^2	–39215.8785	–687024.3196	50432.5497	–12.3454	9758.6588
T^3	14420.2934	78660.6535	–657.0605	565.0905	1328.3085
tjy	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\rho_s$ (μas)	$\Delta\theta_s$ (μas)
T	36028.3827	–740880.9685	–10.7322	–297.2493	119866.5547
T^2	–2910.6802	60179.7955	–1951.6003	–1779.1546	–1065.6036
T^3	–193.9063	627.4990	–4125.4000	–3128.5299	–57.9607
tjy	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta(I\sigma)_s$ (μas)	$\Delta\varphi_s$ (μas)
T	214 756196.8118	113 009422.3955	–17.8008	6536.9172	405134.4944
T^2	2268.1428	687231.8895	–54775.6865	–36208.8512	–11482.6140
T^3	–12967.5814	–78746.0736	1245.0101	27296.6113	–280.3423

As can be seen from the Tables 1a, the values of the geodetic rotation of planets decrease with increasing their distance from the Sun. The values of the geodetic rotation of the Earth and the Moon are very close. **It is due to the fact** that the Earth and the Moon have very close heliocentric orbit and **the Sun has a greater influence on the geodetic rotation of the Moon** than the Earth.

Results

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Table 1b. Comparison the secular terms of geodetic rotation of …

	Mercury	Venus	The Earth	**Phobos**	**Deimos**
tjy	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)	$\Delta\psi_s$ (μas)
T	**–426** 451 032.8798	**–156** 030 839.3400	**–19** 198 865.6280	**–209** 314 864.7430	**–27** 680 096.2268
T^2	–39215.8785	–687024.3196	50432.5497	43043.9996	14436.8795
T^3	14420.2934	78660.6535	–657.0605		
tjy	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)	$\Delta\theta_s$ (μas)
T	36028.3827	–740880.9685	–10.7322	109821.3069	118932.5546
T^2	–2910.6802	60179.7955	–1951.6003	–79913.4426	–5802.8941
T^3	–193.9063	627.4990	–4125.4000		
tjy	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)	$\Delta\varphi_s$ (μas)
T	214 756196.8118	113 009422.3955	–17.8008	113601628.3630	11843281.0553
T^2	2268.1428	687231.8895	–54775.6865	–20969.4134	–12466.3227
T^3	–12967.5814	–78746.0736	1245.0101		

As can be seen from the Table 1b, the value of the geodetic rotation of Phobos greater than ones values of the Earth and Venus; the value of the geodetic rotation of Deimos greater than one value of the Earth. **It is due to the fact** that **Mars has a greater influence on the geodetic rotation of its satellites** than the Sun on some near planets by reason of very close distances between Mars and **its satellites.**

Notes to tables

In Tables 1, 1a, 1b, 1c: *T* is means the Dynamical Barycentric Time (TDB) measured in thousand Julian years (tjy) (of 365250 days) from J2000;

a is mean orbital semi-major axis of Mars satellites (**https://ssd.jpl.nasa.gov/?sat_elem).**

Mars and its satellites Phobos and Deimos (like the Earth and the Moon) are on average at the same distance from the Sun. As a result, their coefficients in $\Delta\psi$ and $\Delta\tau$ for periodic with argument λ_4 (Table 2) components are quite close to each other.

Table 2. The periodic terms of geodetic rotation of Mars and its satellites

Body	Angle	Period	Arg	Coefficient of sin(Arg) (μas)	Coefficient of cos(Arg) (μas)
Mars	**$\Delta\psi_p$**	**1.881 yr**	**λ_4**	**$-543.438-22.455T+\dots$**	**$-241.415+40.433T+\dots$**
	$\Delta\theta_p$	1.881 yr	λ_4	$9.157+0.241T+\dots$	$4.068-0.742T+\dots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$30.949-0.392T+\dots$	$13.748-3.045T+\dots$
Phobos	**$\Delta\psi_p$**	**1.881 yr**	**λ_4**	**-537.291**$+\dots$	**-238.028**$+\dots$
		2.262 yr	Ω_{L41}	$-125.461+\dots$	$680.758+\dots$
		7.657 h	D_{41}	$-58.679+0.204T+0.880T^2+\dots$	$59.450-0.022T+0.783T^2+\dots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.448+\dots$	$3.305+\dots$
		2.262 yr	Ω_{L41}	$4.099+\dots$	$1.339+\dots$
		7.657 h	D_{41}	$-8.480+0.017T-0.098T^2+\dots$	$-9.228-0.036T-0.392T^2+\dots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$-29.698+\dots$	$14.013+\dots$
		2.262 yr	Ω_{L41}	$-1.139+\dots$	$-3.219+\dots$
		7.657 h	D_{41}	$21.359-0.129T-1.005T^2+\dots$	$-22.722-0.036T-0.319T^2+\dots$
Deimos	**$\Delta\psi_p$**	**1.881 yr**	**λ_4**	**-544.598**$+\dots$	**-241.408**$+\dots$
		54.537 yr	Ω_{L42}	$-2879.646+\dots$	$757.953+\dots$
		1.265 d	D_{42}	$-51.777+0.142T-1.000T^2+\dots$	$11.009+0.007T+0.532T^2+\dots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.220+\dots$	$3.792+\dots$
		54.537 yr	Ω_{L42}	$28.678+\dots$	$106.025+\dots$
		1.265 d	D_{42}	$-1.214+0.009T+0.231T^2+\dots$	$-7.571-0.090T+0.675T^2+\dots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$32.026+\dots$	$13.676+\dots$
		54.537 yr	Ω_{L42}	$195.066+\dots$	$-216.931+\dots$
		1.265 d	D_{42}	$19.149-0.045T-0.069T^2+\dots$	$-4.576-0.042T-0.310T^2+\dots$

Table 2. The periodic terms of geodetic rotation of Mars and its satellites

Body	Angle	Period	Arg	Coefficient of sin(Arg) (μas)	Coefficient of cos(Arg) (μas)
Mars	**$\Delta\psi_p$**	**1.881 yr**	**λ_4**	**$-543.438-22.455T+\ldots$**	**$-241.415+40.433T+\ldots$**
	$\Delta\theta_p$	1.881 yr	λ_4	$9.157+0.241T+\ldots$	$4.068-0.742T+\ldots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$30.949-0.392T+\ldots$	$13.748-3.045T+\ldots$
Phobos	**$\Delta\tau_p$**	**1.881 yr**	**λ_4**	**–507.594**+...	**–224.015**+...
		2.262 yr	Ω_{L41}	–126.600+...	677.538+...
		7.657 h	D_{41}	$-37.319+0.074T-0.125T^2+\ldots$	$36.728-0.058T+0.464T^2+\ldots$
	$\Delta\rho_p$	1.881 yr	λ_4	9.448+...	3.305+...
		2.262 yr	Ω_{L41}	4.099+...	1.339+...
		7.657 h	D_{41}	$-8.480+0.017T-0.098T^2+\ldots$	$-9.228-0.036T-0.392T^2+\ldots$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	250.815+...	114.078+...
		2.262 yr	Ω_{L41}	–176.168+...	935.392+...
		7.657 h	D_{41}	$26.377-0.044T-0.408T^2+\ldots$	$-26.723-0.035T-0.358T^2+\ldots$
Deimos	**$\Delta\tau_p$**	**1.881 yr**	**λ_4**	**–512.573+...**	**–227.733+...**
		54.537 yr	Ω_{L42}	–2684.581+...	541.022+...
		1.265 d	D_{42}	$-32.628+0.097T-1.069T^2+\ldots$	$6.433-0.035T+0.223T^2+\ldots$
	$\Delta\rho_p$	1.881 yr	λ_4	9.220+...	3.792+...
		54.537 yr	Ω_{L42}	28.678+...	106.025+...
		1.265 d	D_{42}	$-1.214+0.009T+0.231T^2+\ldots$	$-7.571-0.090T+0.675T^2+\ldots$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	237.604+...	105.749+...
		54.537 yr	Ω_{L42}	–5381.398+...	1058.129+...
		1.265 d	D_{42}	$22.601+0.174T-1.924T^2+\ldots$	$-4.805-0.053T+0.261T^2+\ldots$

Notes to table

In Table 2: T is means the Dynamical Barycentric Time (TDB) measured in thousand Julian years (tjy) (of 365250 days) from J2000;

Ω_{L41} , Ω_{L42} are longitudes of the ascending node (Mars satellites orbits) on the Laplace plane for Phobos and Deimos, respectively;

$D_{41} = \lambda_{41} - \lambda_4 + 180^{\circ}$, $D_{42} = \lambda_{42} - \lambda_4 + 180^{\circ}$ are mean elongations of Phobos and Deimos from the Sun, respectively;

λ_4 is mean longitude of Mars;

λ_{41}, λ_{42} are mean marsocentric longitudes of Phobos and Deimos, respectively.

The mean longitude of Mars was taken from (Brumberg and Bretagnon, 2000).

The mean longitudes of the Martian satellites, their longitudes of the ascending node on the Laplace plane and mean elongations from the Sun are calculated using data from the Horizons On-Line Ephemeris System (Giorgini et al., 2001).

The geodetic rotations of Phobos and Deimos are determined not only by the Sun, but also by Mars. This fact is confirmed by the appearance of a harmonic with the argument D_{41} for Phobos and D_{42} for Deimos (Table 2).

Table 2. The periodic terms of geodetic rotation of Mars and its satellites

Body	Angle	Period	Arg	Coefficient of sin(Arg) (μas)	Coefficient of cos(Arg) (μas)
Mars	$\Delta\psi_p$	1.881 yr	λ_4	$-543.438-22.455T+\ldots$	$-241.415+40.433T+\ldots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.157+0.241T+\ldots$	$4.068-0.742T+\ldots$
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		7.657 h	$\boldsymbol{D_{41}}$	$\boldsymbol{-58.679+0.204T+0.880T^2+\ldots}$	$\boldsymbol{59.450-0.022T+0.783T^2+\ldots}$
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		54.537 yr	Ω_{L42}	$-2879.646+\ldots$	$757.953+\ldots$
		1.265 d	$\boldsymbol{D_{42}}$	$\boldsymbol{-51.777+0.142T-1.000T^2+\ldots}$	$\boldsymbol{11.009+0.007T+0.532T^2+\ldots}$
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	$\Delta\varphi_p$	1.881 yr	λ_4	$32.026+\ldots$	$13.676+\ldots$
		54.537 yr	Ω_{L42}	$195.066+\ldots$	$-216.931+\ldots$
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Phobos	$\Delta\tau_p$	1.881 yr	λ_4	$-507.594+\ldots$	$-224.015+\ldots$
		2.262 yr	Ω_{L41}	$-126.600+\ldots$	$677.538+\ldots$
		7.657 h	$\boldsymbol{D_{41}}$	$\mathbf{-37.319+0.074}\boldsymbol{T}\mathbf{-0.125}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{36.728-0.058}\boldsymbol{T}\mathbf{+0.464}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$
	$\Delta\rho_p$	1.881 yr	λ_4	$9.448+\ldots$	$3.305+\ldots$
		2.262 yr	Ω_{L41}	$4.099+\ldots$	$1.339+\ldots$
		7.657 h	$\boldsymbol{D_{41}}$	$\mathbf{-8.480+0.017}\boldsymbol{T}\mathbf{-0.098}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{-9.228-0.036}\boldsymbol{T}\mathbf{-0.392}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	$250.815+\ldots$	$114.078+\ldots$
		2.262 yr	Ω_{L41}	$-176.168+\ldots$	$935.392+\ldots$
		7.657 h	$\boldsymbol{D_{41}}$	$\mathbf{26.377-0.044}\boldsymbol{T}\mathbf{-0.408}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{-26.723-0.035}\boldsymbol{T}\mathbf{-0.358}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$
Deimos	$\Delta\tau_p$	1.881 yr	λ_4	$-512.573+\ldots$	$-227.733+\ldots$
		54.537 yr	Ω_{L42}	$-2684.581+\ldots$	$541.022+\ldots$
		1.265 d	$\boldsymbol{D_{42}}$	$\mathbf{-32.628+0.097}\boldsymbol{T}\mathbf{-1.069}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{6.433-0.035}\boldsymbol{T}\mathbf{+0.223}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$
	$\Delta\rho_p$	1.881 yr	λ_4	$9.220+\ldots$	$3.792+\ldots$
		54.537 yr	Ω_{L42}	$28.678+\ldots$	$106.025+\ldots$
		1.265 d	$\boldsymbol{D_{42}}$	$\mathbf{-1.214+0.009}\boldsymbol{T}\mathbf{+0.231}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{-7.571-0.090}\boldsymbol{T}\mathbf{+0.675}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	$237.604+\ldots$	$105.749+\ldots$
		54.537 yr	Ω_{L42}	$-5381.398+\ldots$	$1058.129+\ldots$
		1.265 d	$\boldsymbol{D_{42}}$	$\mathbf{22.601+0.174}\boldsymbol{T}\mathbf{-1.924}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$	$\mathbf{-4.805-0.053}\boldsymbol{T}\mathbf{+0.261}\boldsymbol{T}^{\mathbf{2}}\mathbf{+\ldots}$

Results

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In contrast to Phobos, located closer to the planet, **the Sun has a greater influence** on the geodetic rotation of Deimos.

It is easy to see that the closer a satellite is located to the planet, the more the harmonic contribution depends on the mean longitude of the planet (see Phobos in Table 2). **Therefore,** the harmonic with a period of **1.881 years** and the argument of λ_4 (Table 2) becomes predominant for Phobos.

If the satellite is farther away from the planet, then more harmonic contribution depends on **the precession orbit node in the Laplace plane**[1] (see Deimos in Table 2). **Therefore,** the harmonic with a period of **54.537 years** and the argument of Ω_{42} (Table 2) becomes predominant for Deimos.

[1] **The Laplace plane** is the plane normal to the satellite's orbital precession pole. It is a kind of "average orbital plane" of the satellite (between their planet's equatorial plane and the plane of its solar orbit), around which the instantaneous orbital plane of the satellite precesses, and to which it has a constant additional inclination (P. Kenneth Seidelmann (ed.), 1992).

Table 2. The periodic terms of geodetic rotation of Mars and its satellites

Body	Angle	Period	Arg	Coefficient of sin(Arg) (μas)	Coefficient of cos(Arg) (μas)
Mars	$\Delta\psi_p$	1.881 yr	λ_4	$-543.438-22.455T+\ldots$	$-241.415+40.433T+\ldots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.157+0.241T+\ldots$	$4.068-0.742T+\ldots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$30.949-0.392T+\ldots$	$13.748-3.045T+\ldots$
Phobos	$\Delta\psi_p$	**1.881 yr**	$\boldsymbol{\lambda_4}$	**$-537.291+\ldots$**	**$-238.028+\ldots$**
		2.262 yr	$\boldsymbol{\Omega_{L41}}$	**$-125.461+\ldots$**	**$680.758+\ldots$**
		7.657 h	D_{41}	$-58.679+0.204T+0.880T^2+\ldots$	$59.450-0.022T+0.783T^2+\ldots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.448+\ldots$	$3.305+\ldots$
		2.262 yr	Ω_{L41}	$4.099+\ldots$	$1.339+\ldots$
		7.657 h	D_{41}	$-8.480+0.017T-0.098T^2+\ldots$	$-9.228-0.036T-0.392T^2+\ldots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$-29.698+\ldots$	$14.013+\ldots$
		2.262 yr	Ω_{L41}	$-1.139+\ldots$	$-3.219+\ldots$
		7.657 h	D_{41}	$21.359-0.129T-1.005T^2+\ldots$	$-22.722-0.036T-0.319T^2+\ldots$
Deimos	$\Delta\psi_p$	**1.881 yr**	$\boldsymbol{\lambda_4}$	**$-544.598+\ldots$**	**$-241.408+\ldots$**
		54.537 yr	$\boldsymbol{\Omega_{L42}}$	**$-2879.646+\ldots$**	**$757.953+\ldots$**
		1.265 d	D_{42}	$-51.777+0.142T-1.000T^2+\ldots$	$11.009+0.007T+0.532T^2+\ldots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.220+\ldots$	$3.792+\ldots$
		54.537 yr	Ω_{L42}	$28.678+\ldots$	$106.025+\ldots$
		1.265 d	D_{42}	$-1.214+0.009T+0.231T^2+\ldots$	$-7.571-0.090T+0.675T^2+\ldots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$32.026+\ldots$	$13.676+\ldots$
		54.537 yr	Ω_{L42}	$195.066+\ldots$	$-216.931+\ldots$
		1.265 d	D_{42}	$19.149-0.045T-0.069T^2+\ldots$	$-4.576-0.042T-0.310T^2+\ldots$

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Body	Angle	Period	Arg	Coefficient of sin(Arg) (μas)	Coefficient of cos(Arg) (μas)
Mars	$\Delta\psi_p$	1.881 yr	λ_4	$-543.438-22.455T+\ldots$	$-241.415+40.433T+\ldots$
	$\Delta\theta_p$	1.881 yr	λ_4	$9.157+0.241T+\ldots$	$4.068-0.742T+\ldots$
	$\Delta\varphi_p$	1.881 yr	λ_4	$30.949-0.392T+\ldots$	$13.748-3.045T+\ldots$
Phobos	$\Delta\tau_p$	**1.881 yr**	$\boldsymbol{\lambda_4}$	$\mathbf{-507.594+\ldots}$	$\mathbf{-224.015+\ldots}$
		2.262 yr	$\boldsymbol{\Omega_{L41}}$	$\mathbf{-126.600+\ldots}$	$\mathbf{677.538+\ldots}$
		7.657 h	D_{41}	$-37.319+0.074T-0.125T^2+\ldots$	$36.728-0.058T+0.464T^2+\ldots$
	$\Delta\rho_p$	1.881 yr	λ_4	$9.448+\ldots$	$3.305+\ldots$
		2.262 yr	Ω_{L41}	$4.099+\ldots$	$1.339+\ldots$
		7.657 h	D_{41}	$-8.480+0.017T-0.098T^2+\ldots$	$-9.228-0.036T-0.392T^2+\ldots$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	$250.815+\ldots$	$114.078+\ldots$
		2.262 yr	Ω_{L41}	$-176.168+\ldots$	$935.392+\ldots$
		7.657 h	D_{41}	$26.377-0.044T-0.408T^2+\ldots$	$-26.723-0.035T-0.358T^2+\ldots$
Deimos	$\Delta\tau_p$	**1.881 yr**	$\boldsymbol{\lambda_4}$	$\mathbf{-512.573+\ldots}$	$\mathbf{-227.733+\ldots}$
		54.537 yr	$\boldsymbol{\Omega_{L42}}$	$\mathbf{-2684.581+\ldots}$	$\mathbf{541.022+\ldots}$
		1.265 d	D_{42}	$-32.628+0.097T-1.069T^2+\ldots$	$6.433-0.035T+0.223T^2+\ldots$
	$\Delta\rho_p$	1.881 yr	λ_4	$9.220+\ldots$	$3.792+\ldots$
		54.537 yr	Ω_{L42}	$28.678+\ldots$	$106.025+\ldots$
		1.265 d	D_{42}	$-1.214+0.009T+0.231T^2+\ldots$	$-7.571-0.090T+0.675T^2+\ldots$
	$\Delta(I\sigma)_p$	1.881 yr	λ_4	$237.604+\ldots$	$105.749+\ldots$
		54.537 yr	Ω_{L42}	$-5381.398+\ldots$	$1058.129+\ldots$
		1.265 d	D_{42}	$22.601+0.174T-1.924T^2+\ldots$	$-4.805-0.053T+0.261T^2+\ldots$

Results

In this investigation it was also carried out a study on the mutual relativistic influence of Mars satellites on each other and on Mars (i.e., the inclusion of another satellite in the number of perturbing bodies).

So, the change in Deimos geodetic rotation from Phobos relativistic influence:

in the longitude of the node ψ is –0.22 μas/tjy, in the longitude τ is $-9.5\cdot10^{-2}$ μas/tjy,
in the inclination θ is $-9.3\cdot10^{-6}$ μas/tjy, in the inclination ρ is $-9.3\cdot10^{-6}$ μas/tjy,
in the proper rotation angle φ is 0.12 μas/tjy; in the node longitude $I\sigma$ is $9.4\cdot10^{-2}$ μas/tjy;

the change in Phobos geodetic rotation from Deimos relativistic influence:

in the longitude of the node ψ is $-5.3\cdot10^{-2}$ μas/tjy, in the longitude τ is $-2.4\cdot10^{-2}$ μas/tjy,
in the inclination θ is $6.2\cdot10^{-6}$ μas/tjy, in the inclination ρ is $6.2\cdot10^{-6}$ μas/tjy,
in the proper rotation angle φ is $2.9\cdot10^{-2}$ μas/tjy; in the node longitude $I\sigma$ is $2.4\cdot10^{-2}$ μas/tjy;

and

the change in Mars geodetic rotation from its satellites relativistic influence:

in the longitude of the node ψ is –0.62 μas/tjy,
in the inclination θ is $-1.2\cdot10^{-4}$ μas/tjy,
in the proper rotation angle φ is 0.35 μas/tjy.

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CONCLUSION

1.New high-precision values with the additions periodic terms of the geodetic rotation for Mars in Euler angles were obtained. These values are the dynamically adjusted to the DE431/LE431 ephemeris.

2.In this study, for the first time in the Euler angles and a perturbing term of the physical libration of Martian satellites (Phobos and Deimos) computed their systematic (Table. 1) and periodic (Table. 2) terms of the geodetic rotation. The mutual relativistic influence of the Mars satellites on each other in comparison with the Sun and Mars influences is insignificant. The obtained analytical values for the geodetic rotation of Phobos and Deimos can be used for the numerical study of their rotation in the relativistic approximation.

3.The secular terms of geodetic rotation of Mars satellites depend on their distance from the Sun and Mars, which masses are dominant in the Solar and Mars system, respectively. Mars has a greater influence on the geodetic rotation of its satellites than the Sun on the geodetic rotation of Phobos, Deimos and Mars.

4.The main periodic parts of the geodetic rotations for Mars satellites are determined not only by the Sun but also by Mars, which is the nearest planet to their satellites.

5.The values of the geodetic rotation of Mars satellites decrease with increasing their distance from Mars.

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REFERENCES

- **Pashkevich V.V. (2016) New high-precision values of the geodetic rotation of the major planets, Pluto, the Moon and the Sun, Artificial Satellites, Warszawa, Vol. 51, No. 2 (DOI: 10.1515/arsa-2016-0006), pp. 61-73.**
- **De Sitter W. (1916) On Einstein's Theory of Gravitation and its Astronomical Consequences, Mon. Not. R. Astron. Soc., No. 77, pp. 155–184.**
- **Fukushima T. (1991) Geodesic Nutation, Astronomy and Astrophysics, 244, No.1, pp. L11–L12.**
- **Folkner W.F., Williams J.G., Boggs D.H., Park R.S., and Kuchynka P. (2014) The Planetary and Lunar Ephemerides DE430 and DE431, IPN Progress Report 42-196, February 15, 2014.**
- **Kopeikin S. Efroimsky M. and Kaplan G. (2011) Relativistic Celestial Mechanics in the Solar System, Hoboken, NJ : John Wiley & Sons., pp. 1–894.**
- **Archinal B.A., Acton C.H., A’Hearn M.F. et al. (2018) Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2015, Celest Mech Dyn Astr. 130: 22. https://doi.org/10.1007/s10569-017-9805-5 , pp. 21–46.**
- **Seidelmann P.K., Archinal B.A., A'Hearn M.F., Cruikshank D.P., Hil-ton J.L., Keller H.U., Oberst J., Simon J.L., Stooke P., Tholen D.J., and Thomas P.C. (2005) Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements: 2003, Celestial Mechanics and Dynamical Astronomy, 91, pp. 203–215.**
- **Giorgini J.D., Chodas P.W., Yeomans D.K.(2001) Orbit Uncertainty and Close-Approach Analysis Capabilities of the Horizons On-Line Ephemeris System, 33rd AAS/DPS meeting in New Orleans. LA. Nov 26. 2001 – Dec 01. 2001.**
- **Brumberg V.A., Bretagnon P. (2000) Kinematical Relativistic Corrections for Earth’s Rotation Parameters, in Proc. of IAU Colloquium 180, eds. K.Johnston, D. McCarthy, B. Luzum and G. Kaplan, U.S. Naval Observatory, pp. 293-302.**
- **Eroshkin G.I., Pashkevich V.V. (2007) Geodetic rotation of the Solar system bodies, Artificial Satellites, Vol. 42, No. 1, pp. 59–70.**
- **Pashkevich V.V., Vershkov A.N. New High-Precision Values of the Geodetic Rotation of the Mars Satellites System, Major Planets, Pluto, the Moon and the Sun // *Artificial Satellites*, 2019, Vol. 54, No. 2, pp. 31–42. DOI: https://doi.org/10.2478/arsa-2019-0004**
- **P. Kenneth Seidelmann (ed.) (1992), *Explanatory Supplement to the Astronomical Almanac*, University Science Books, Sausalito (Ca).**

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