Dynamical model of lunar core and observational constraint by LLR

V. Viswanathan\(^1,^2,^3\), N. Rambaux\(^1,^2\), A. Fienga\(^4\), J. Laskar\(^2\), and M. Gastineau\(^2\)

1 Sorbonne Université, Paris, France
2 IMCCE, Obs. Paris, France
3 Goddard Space Flight Center, Greenbelt, USA
4 AstroGéo/Geozur – Obs. de la Côte d'Azur, Valbonne, France

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Open questions

• The presence of a lunar fluid core has been revealed by **dynamical**, magnetic, and seismic data
  (e.g. Yoder 1981, Hood et al. 1999, Williams et al. 2001, Weber et al. 2011, Garcia et al. 2011)

• However, the knowledge of its interior properties is still challenging:
  – Size/density of the fluid core
  – Presence of an inner core?
  – Presence of a Low Viscosity Zone?

• How the rotational dynamics and LLR experiment can access to the lunar interior properties?
LLR measurements

- Time span: 1969-today
- Number of NP: ~ 26000
- Apollo station telescope of 3.5 meters
- Grasse-OCA laser: green and IR
- Earth-Moon distance accuracy of ~ cm
- Theoretical accuracy: few mm
Moon’s rotation

- **Physical librations** are departure from a uniform rotational motion
- **Cassini state** is an equilibrium state where the spin axis, normal to the orbital plane and the normal to the ecliptic plane are aligned
- The obliquity is **constant**.

Lunar orbit
Ecliptic
Lunar Equator

The three planes intersect along the same line of nodes

Lunar poles motion

18.6 years

I = 5.2°  I = 1.5°

Asymmetric bulge in the lunar equatorial plane

Lunar Orbit

Influence of the fluid core

\[
\vec{\omega} \quad \frac{d \vec{H}}{dt} = \vec{\Gamma} 
\]

Angular momentum equation

\[
\vec{H} = I \vec{\omega} 
\]

With a fluid core

\[
\vec{H} = \vec{H}_c + I_m \vec{\omega} 
\]

(e.g. Williams et al 2001, Richard, Rambaux, Charnay 2014 (extension), Dumberry andWieczorek 2016....)
Lunar-Laser Ranging Experiment and ephemerides

- Numerical planetary and lunar ephemerides DE, EPM, IfE and INPOP (e.g. Williams et al., Pavlov et al., Hoffman et al. Fienga et al.)
  - Lunar accuracy \(\sim 2 \text{ cm}\) and \(1 \text{ mas}\) in rotation over \(50 \text{ years}\).
  - Fundamental physics, geophysics, selenophysics and interior of the Moon.

- These models (DE, EPM, INPOP) are joint numerical integration of the orbits of the Moon, the Earth, the planets and asteroids, and of the lunar rotation

- Dynamical partial derivatives of the orbits and lunar Euler angles with respect to solution parameters such as moment of inertia, gravity field, tides, dissipation, CMB flattening, and initial conditions.
Dynamical signature of the lunar core

- Departure of the spin of the Moon to the Cassini state of \(~260\) mas.
- Attributed to the fluid core dissipation (Yoder 1981)
- Tidal dissipation and core-mantle friction (Williams et al. 2001)

\[ \mathbf{N}^c = K (\mathbf{\omega} - \mathbf{\omega}^c) \]

- CMB flattening (axi-symmetric) estimation (Williams et al. 2008, Williams et al. 2014)

(Williams et al 2001)
Weighted root-mean-square of LLR post-fit residuals w/o and with fluid core
Direct approach

1. Fixed a value of $R_{CMB}$
2. From INPOP17a geophysical parameters built a new reference lunar interior (density profile)
3. Fit the polar flattening with LLR data
4. Iteration to step (2) with the new set of parameters to converge towards a solution at the fixed $R_{CMB}$

Full dynamical equations with the triaxiality and here the inner core is neglected.
The LLR-fitted value of the lunar core oblateness ($f_c$) (in black dots with region of uncertainty in red) intersects the theoretical hydrostatic values of $f_c$ (solid lines in violet and blue corresponding to models with lunar crustal thickness of 34 and 43 km, respectively) at a lunar CMB radius of $R_{CMB} \approx 384 \pm 12$ km (in gray region). The LLR-fitted mean values here are obtained by assuming a mean value of lunar crustal thickness estimates (Wieczorek et al., 2013) ($T_{crust} = \frac{34 + 43}{2} = 38.5$ km) in the LLR dynamical model. A model with $T_{crust} = 43$ km tends to increase the LLR-fitted mean value by 8.7 to 5.5%, while a $T_{crust} = 34$ km tends to decrease the same by 7.4% to 5.3%, for $R_{CMB}$ varying from 320 to 440 km, respectively.

The region of uncertainty of the LLR-fitted $f_c$ (in red region) encompasses the cumulative errors from lunar core density (Garcia et al., 2011), crustal thickness variations (Wieczorek et al., 2013), degree-2 potential Love number (Konopliv et al., 2013), and other parameters listed in Table B2 in the order of decreasing precedence. Previously reported (Williams et al., 2009) $f_c (2.0 \pm 2.3 \times 10^{-4})$ is in agreement but with much larger error bars (in white dot). A more recent estimate (Williams et al., 2014) $(2.42 \pm 1.4 \times 10^{-4})$ covers plausible value of $f_c$ obtained for $R_{CMB} \approx 320$ to 440 km (in green region). The estimated value of $R_{CMB} \approx 384 \pm 12$ km (in gray region) is obtained by the intersection of the lower and upper bounds of LLR-fitted $f_c$ with the hydrostatic models of $T_{crust} = 34$ and 43 km, respectively (see SI). The CMB radius agree within one of an Apollo seismic data analysis (Garcia et al., 2011) (in hatched region). Within these limits, the value of lunar core oblateness ($f_c$) is estimated as $(2.13 \pm 0.53) \times 10^{-4}$.
**LLR-fitted values of the lunar core oblateness**

\[ f_c = (2.2 \pm 0.6) \times 10^{-4} \]

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Weighted root-mean-square of LLR post-fit residuals with fluid core
Inclination of the lunar core

Goldreich 1983
Meyer and Wisdom 2011
Estimation of the FCN

Lunar core-mantle boundary oblateness ($f_c \times 10^{-4}$)

Free Core Nutation (years)

FCN = 367 ± 100 years

Period FCN $\sim 27.32 / f_c$ (Rambaux and Williams 2011)
Conclusion

- Lunar Laser Ranging continues to provide new results because of improving range with station (APOLLO, New Mexico, USA), updated station (Grasse, OCA, France), new stations, data analysis accuracies (DE430, EPM, IfE, INPOP), and echoes from the lost retroreflector!

- The fluid core friction controls \((1/3)\) the departure of the Cassini state;
- The oblateness of the core has been determined in the LLR fit = \((2.2 \pm 0.6) \times 10^{-4}\);
- Constrain on the size of the core \((381 \pm 12 \text{ km})\) assuming that the CMB is at the hydrostatic equilibrium

- Estimation of the FCN period \((367 \pm 100 \text{ years})\) and its detection in libration series is still in progress.
- The core mass fraction is in the range of 1.63-2.06%.
- Signature of an inner core is not yet observed.

- New retroreflectors or active laser transponders settled to the surface of the Moon will offer improved accuracies to mm and new results…
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1. Outbound pulses start out 3.5 meters in diameter, 2 cm thick
2. Atmosphere causes beam to diverge by one arcsecond or more
3. At the moon, 1 arcsecond is 1.8 km, so beam at moon is about 2 km across
4. Only about 1 in 30 million photons in this 2 km beam hit the suitcase-sized reflector
5. Each outgoing laser pulse contains 300 quadrillion photons

6. Returning beam expands due to corner–cube diffraction
7. Returning beam divergence is about 8 arcseconds
8. Return beam footprint on earth is about 15 km across
9. About 1 in 30 million of the returning photons hit 3.5 m mirror
10. APOLLO launches 20 pulses per second
11. The round-trip time is about 2.5 seconds
12. There are about 50 pulses en-route at any moment in time
LLR measurements

Retroreflector A15

Station Laser - Lune Grasse, OCA

Lunakhod

Yunnan
Reference lunar interior model

Three layer model:
crust, mantle and fluid core

Constrained from INPOP17a (radius, mass, moment of inertia)

\[ U(r_j) = W_j(r_j) + W_{cent}(r_j) + W_{tidal}(r_j). \]

Non-hydrostatic
(measured by LRO)

Gravity coefficient
(measured by GRAIL)

Assumption that CMB at Hydrostatic eq.

(method as Meyer and Wisdom 2011, Dumberry and Wieczoreck 2016; agreement with Wieczoreck et al. 2019)