CHANDLER WOBBLE VARIABILITY

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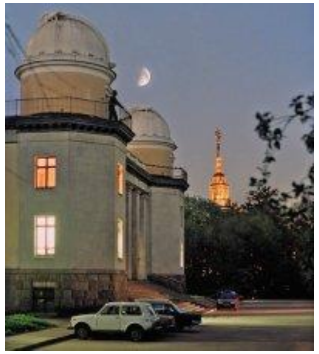

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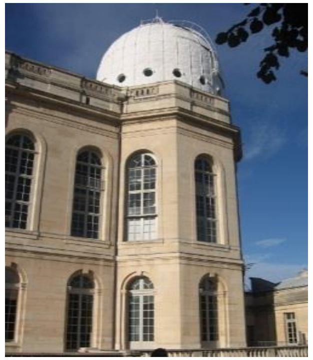

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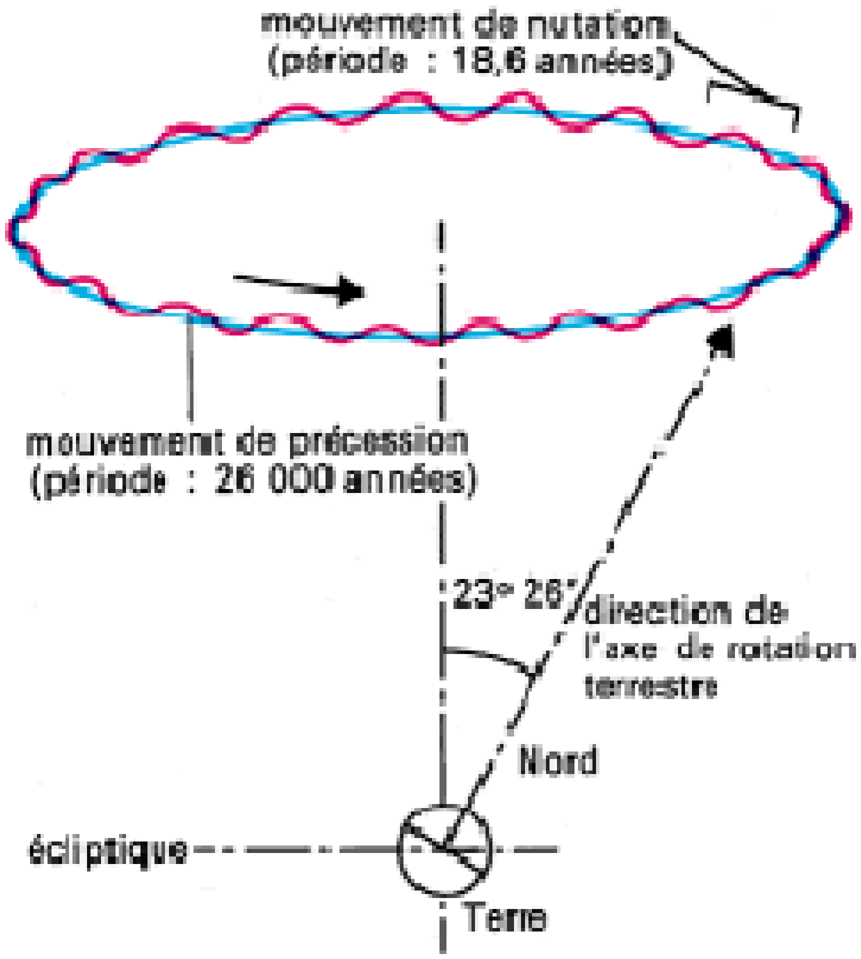
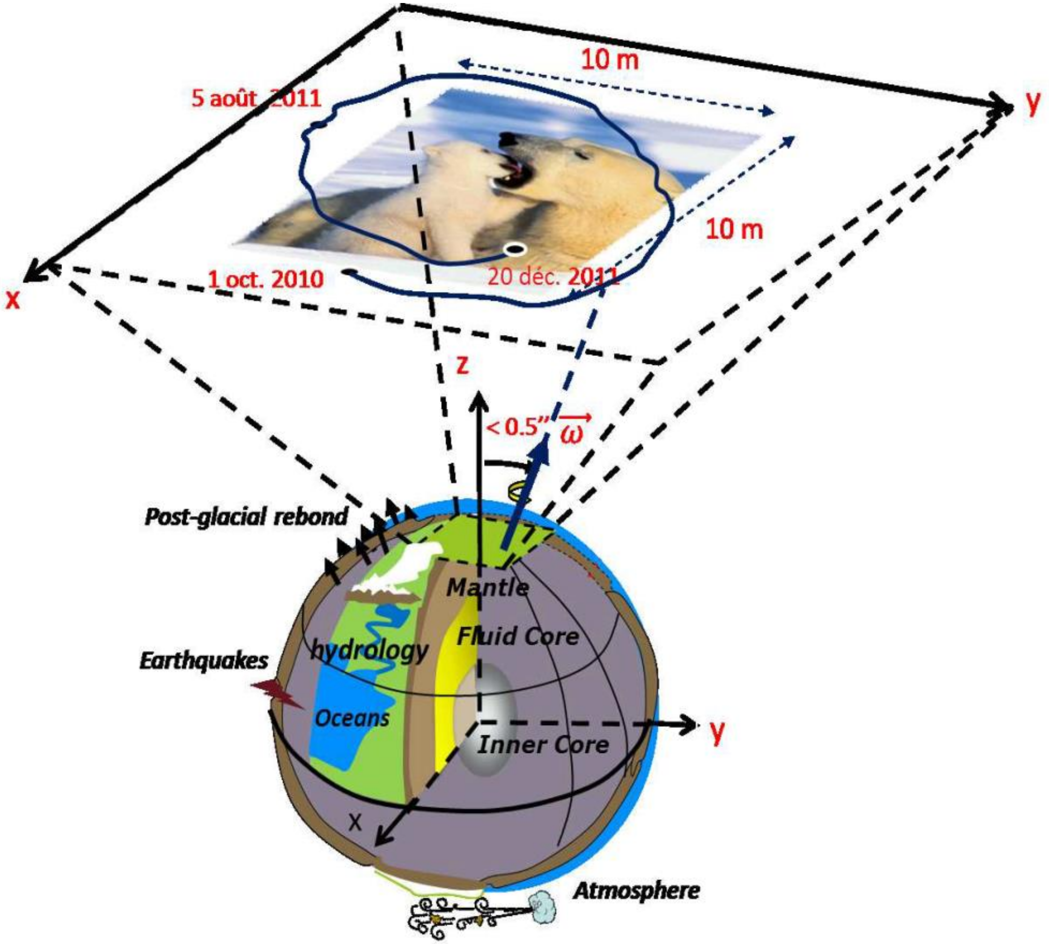
Precession, nutation and polar motion – astronomy or geophysics?

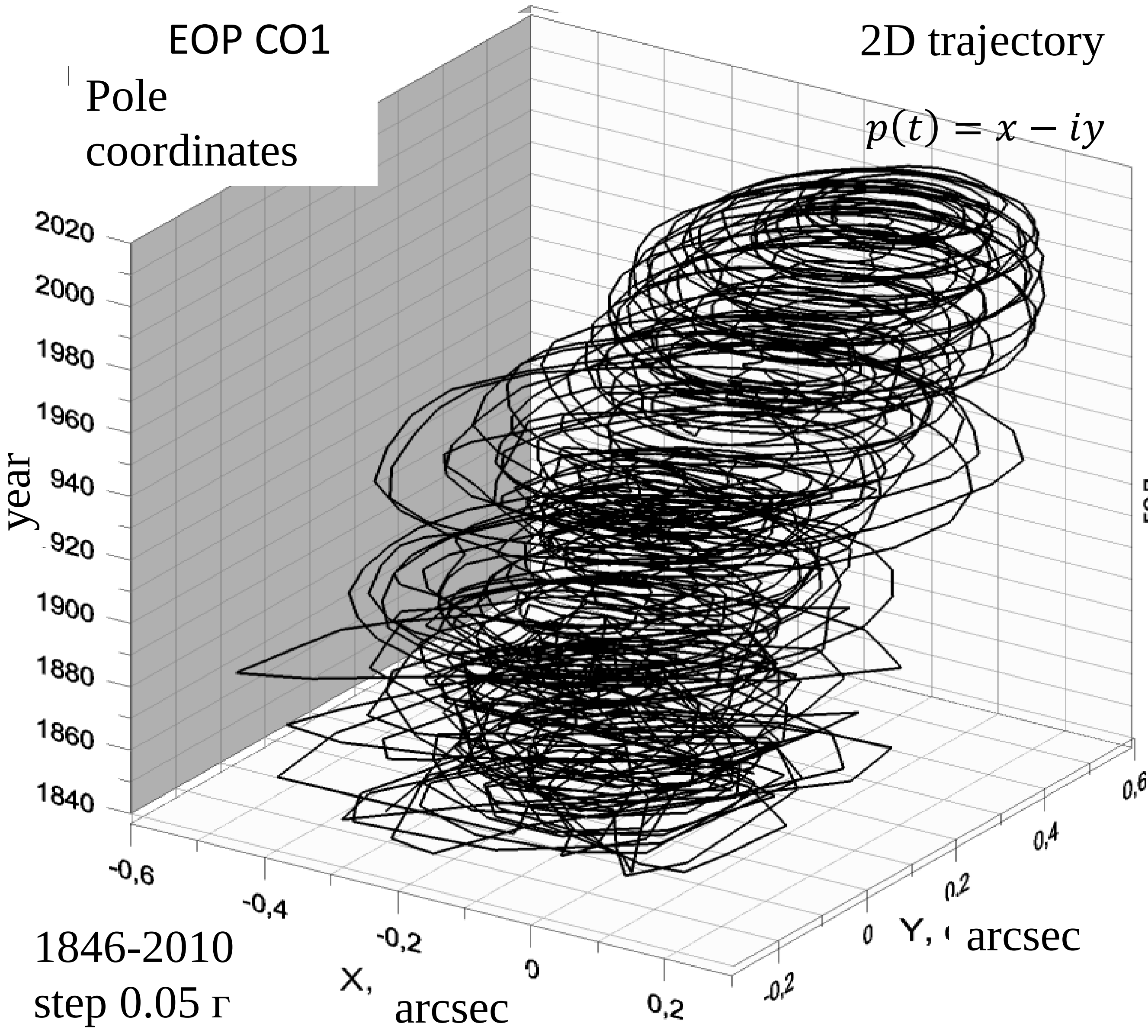
Precession and Nutation 岁差和章动
of the Earth axis 26 000, 18.6 years,
1, 0.5 year caused by Sun and Moon tides

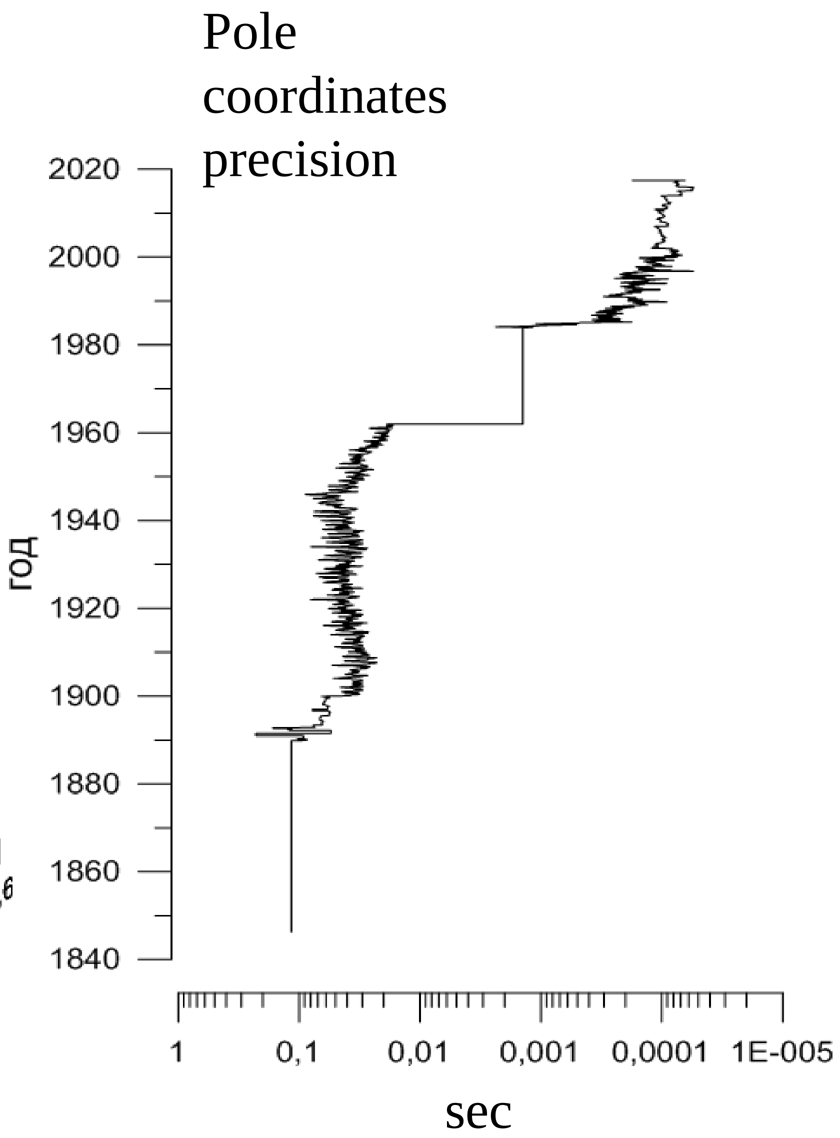


Earth's pole motion **极点运动** up to 10 meters
caused by geophysical effects –
momentum exchange between the Ocean,
Atmosphere, and Solid Earth

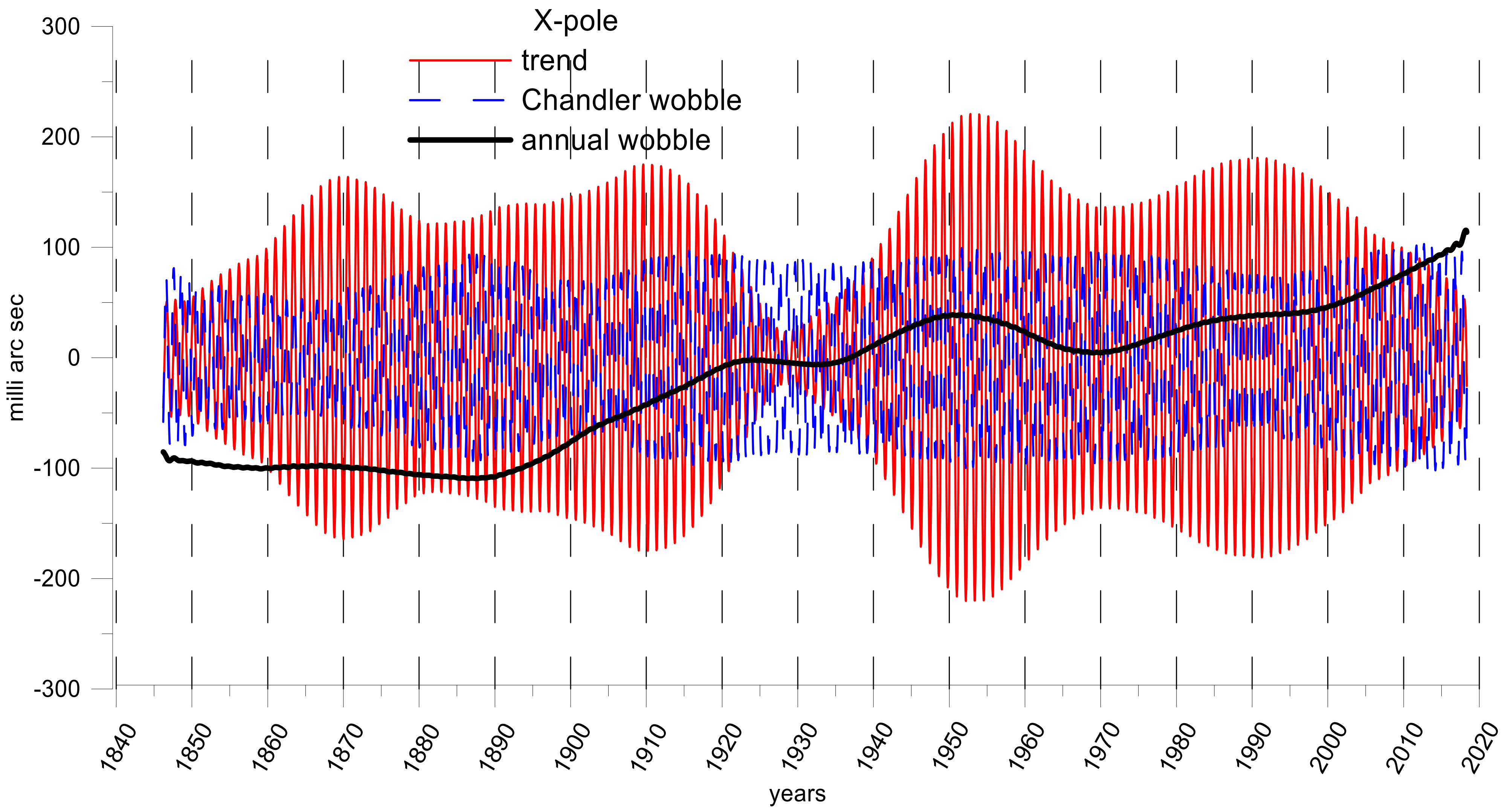


Polar motion trajectory

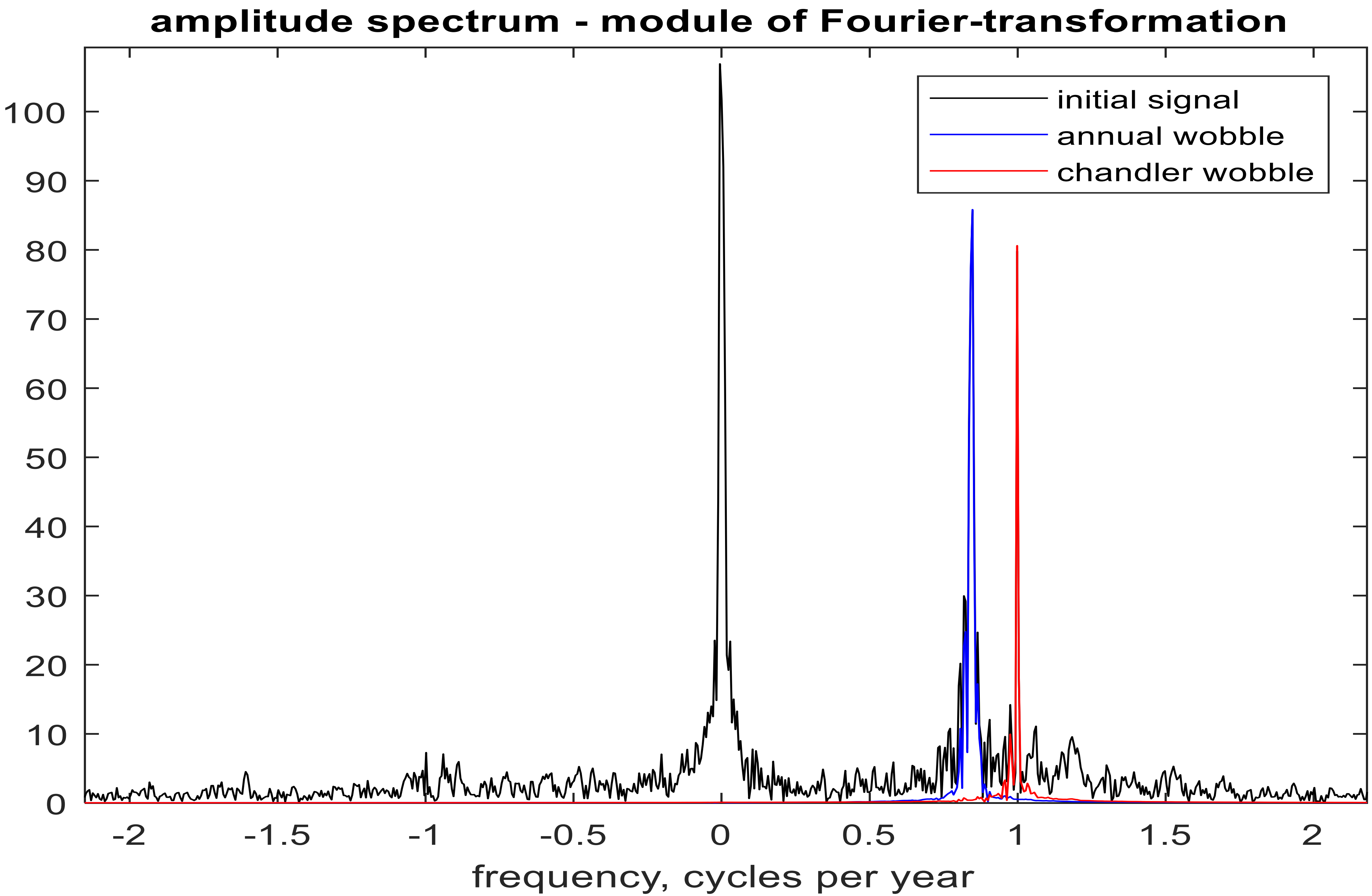




Polar motion principal components trend, annual and Chandler (433 days) wobble



Polar motion spectrum



Variable Chandler and Annual Wobbles in Earth's Polar Motion During 1900–2015

Surv Geophys (2016) 37:1075–1093
DOI 10.1007/s10712-016-9384-0

Guocheng Wang[1] · Lintao Liu[1] · Xiaoqing Su[2] · Xinghui Liang[1] · Haoming Yan[1] · Yi Tu[3] · Zhonghua Li[1] · Wenping Li[4]

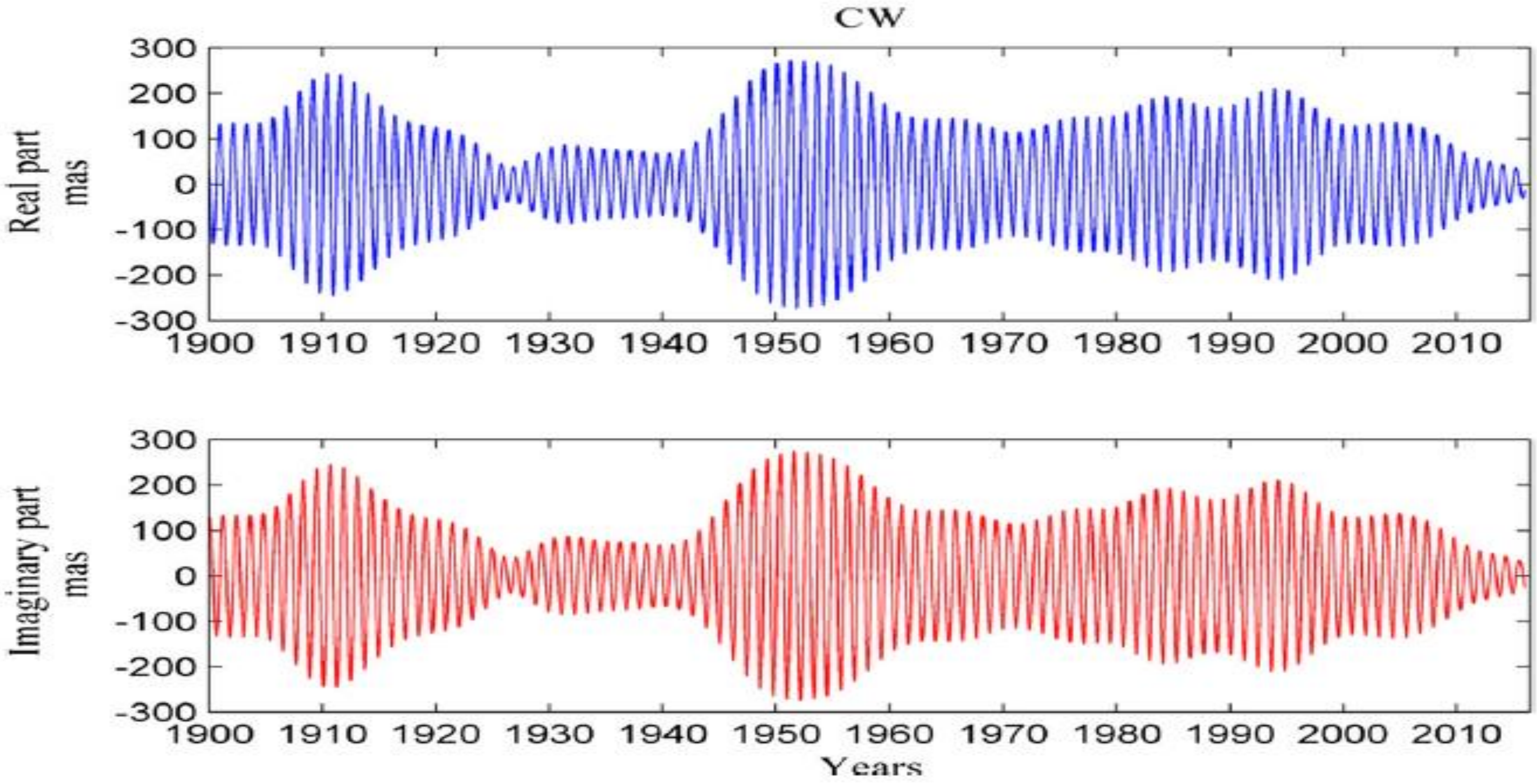


Fig. 8 Reconstruction of the CW using the FBPBPF method

Amplitude and phase variations of Earth's Chandler wobble under continual excitation

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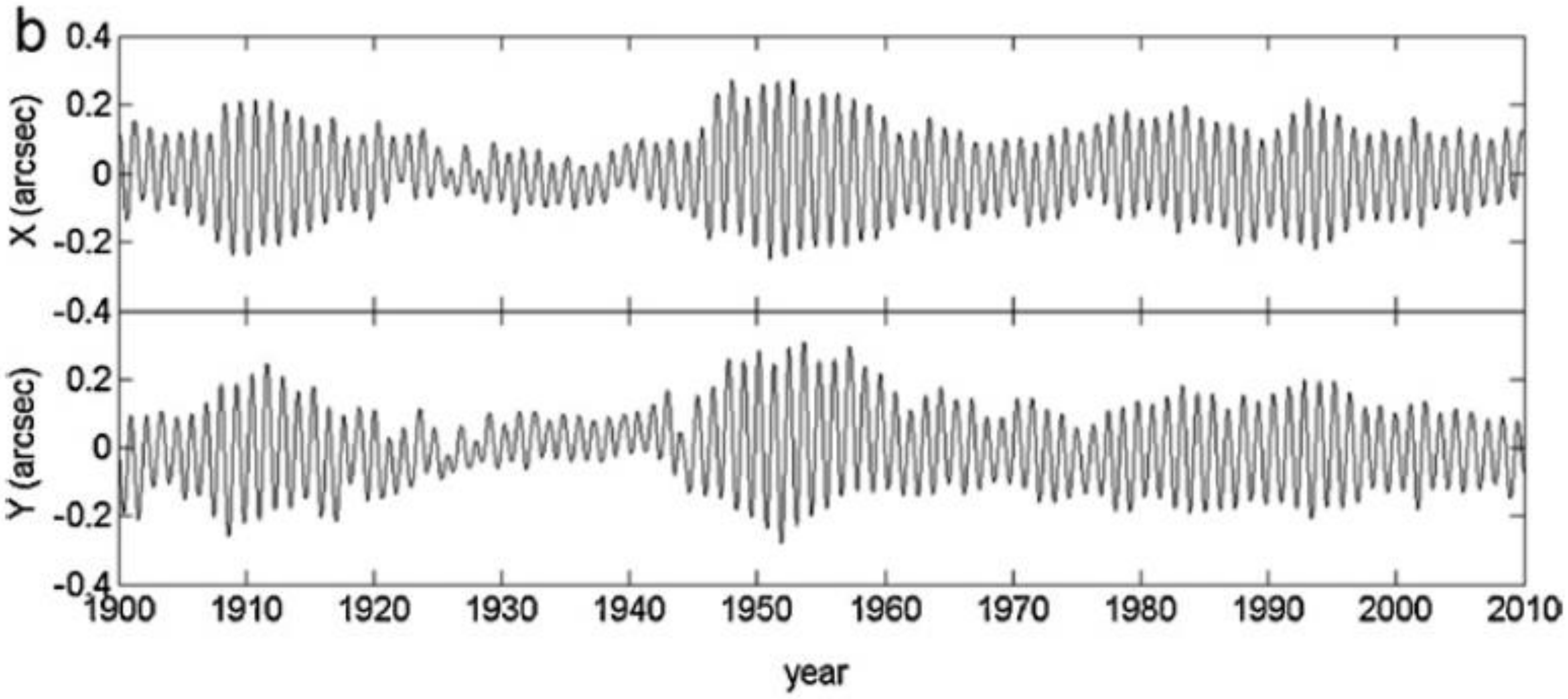
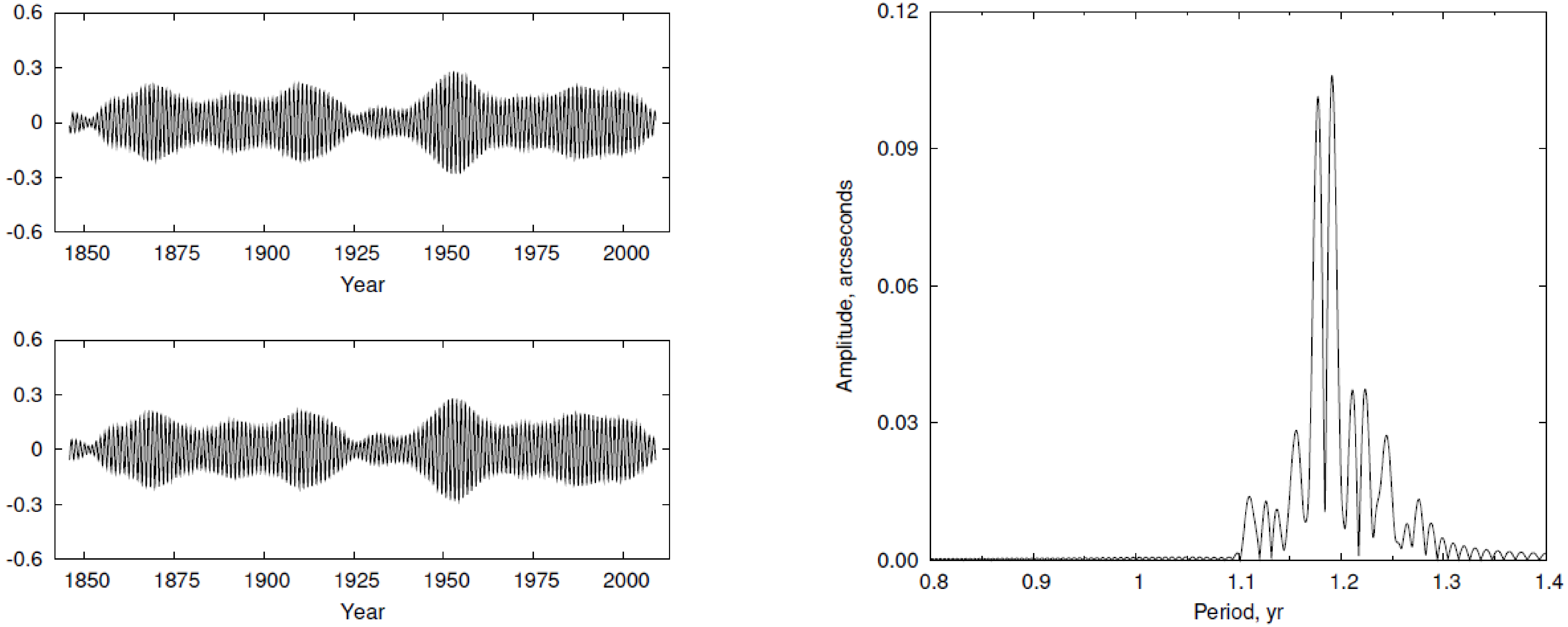


Fig. 1. (a) The x and y components of the Hipparcos polar motion series (for the period 1899.7–1992.0) concatenated with EOP-C01 (for the period 1992.0–2010.0) for the total span of 110 years at 5-day intervals. (b) The Chandler wobble series $m(t)$ obtained from (a) after removing the least-squares estimates of the annual wobble and a linear trend.

Chandler wobble: two more large phase jumps revealed

Zinovy Malkin and Natalia Miller

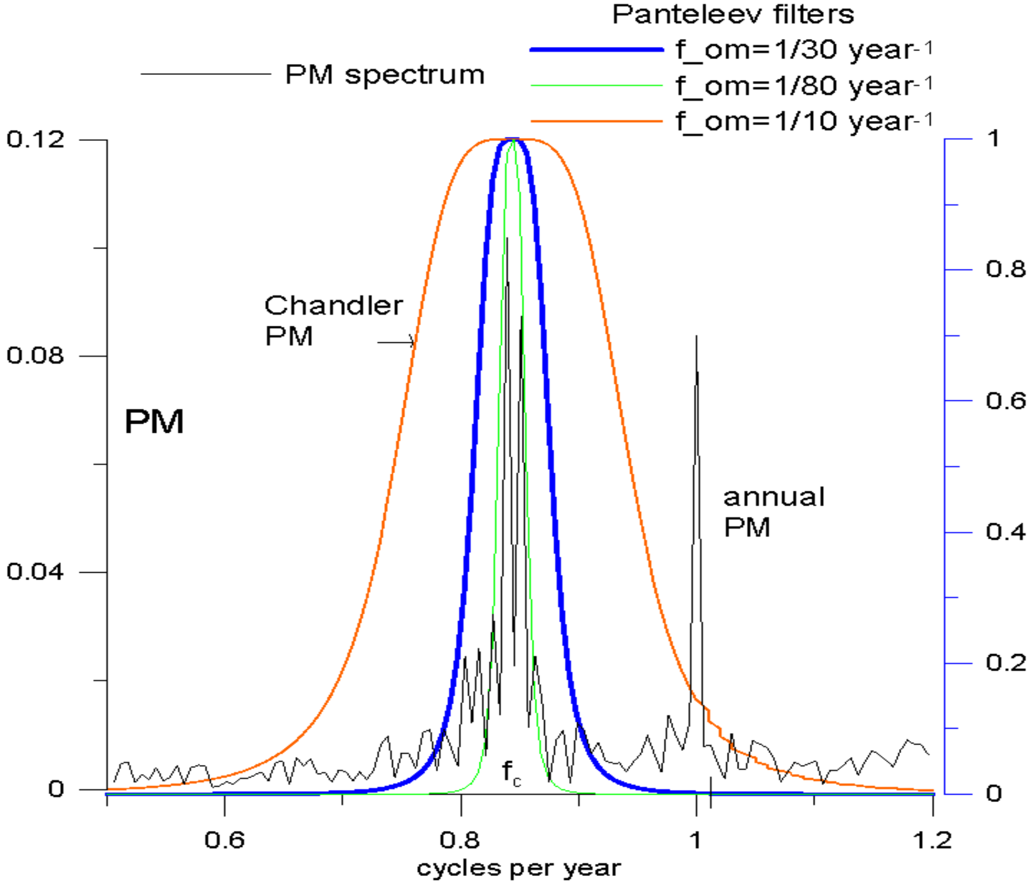
FT-filtered PM series



Panteleev's filtering in the Chandler band

V.L. Panteleev

Several harmonics incorporate information about changes of Chandler wobble instantaneous amplitude and phase



Filtering in time domain – convolution

Filtering in frequency domain – spectra multiplication

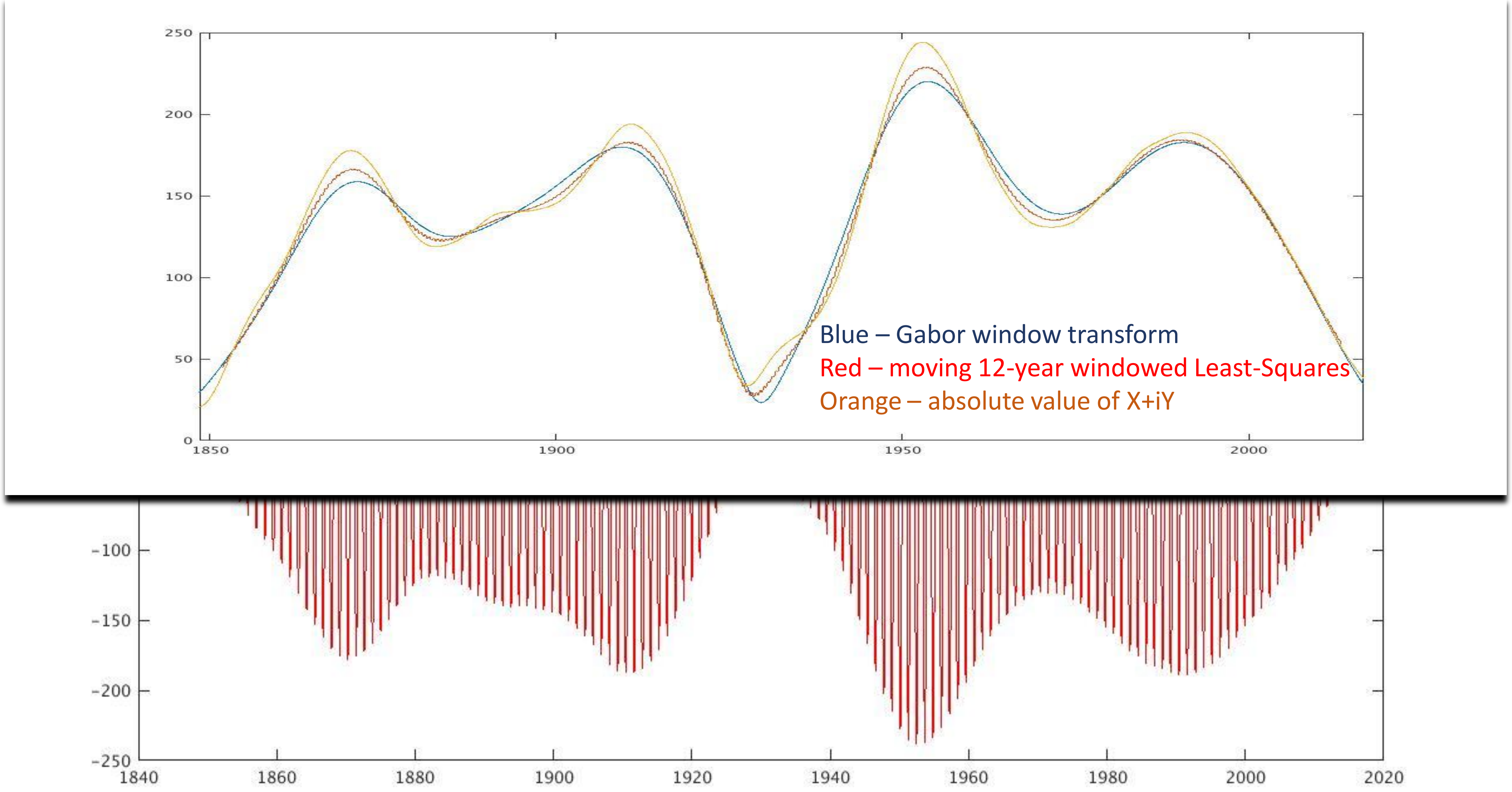
Panteleev's band-pass filter for Chandler wobble extraction

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\left(\frac{\omega_0|t|}{\sqrt{2}} - i2\pi f_c t\right)} \left(\cos\frac{\omega_0 t}{\sqrt{2}} + \sin\frac{\omega_0 |t|}{\sqrt{2}}\right), \qquad L_h(f) = \frac{f_0^4}{(f - f_c)^4 + f_0^4}.$$

$f_c = 0.8435$ years $^{-1}$

$f_0 = 0.04$ years $^{-1}$

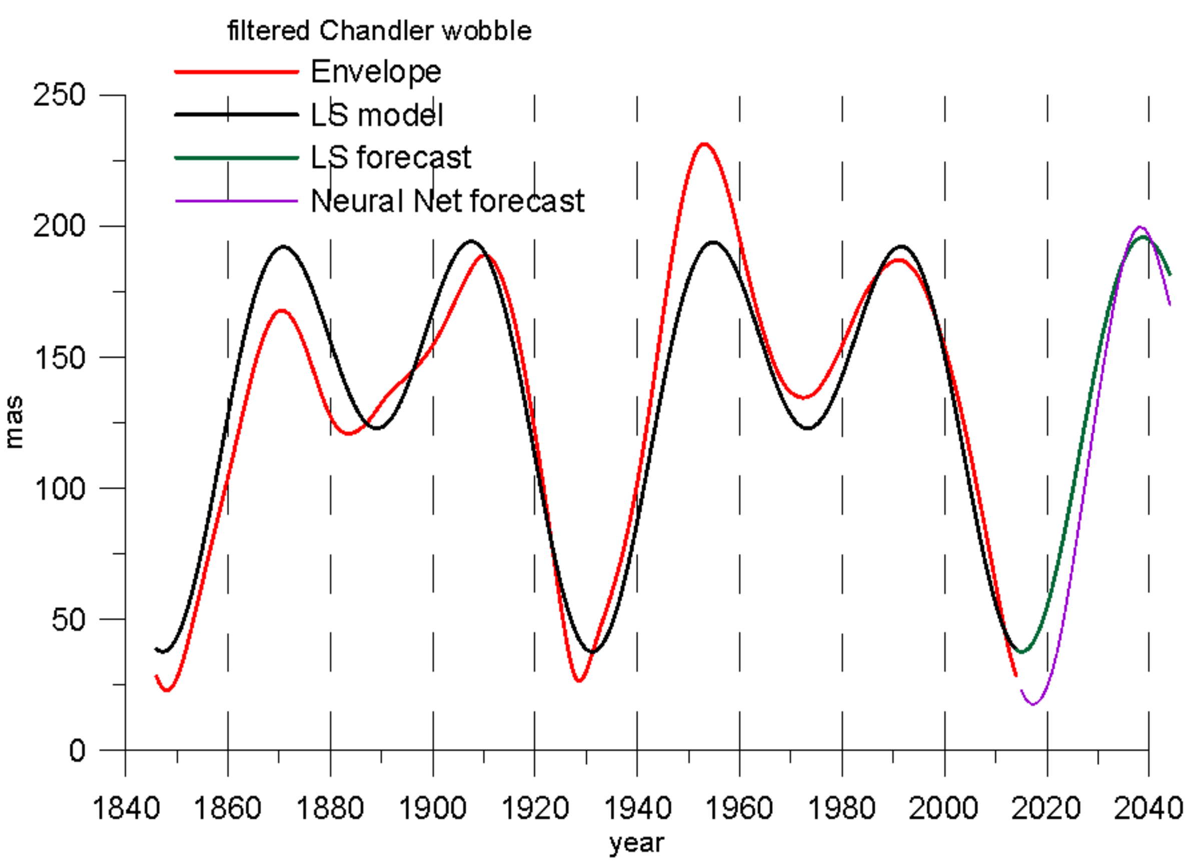
Filtered Chandler wobble and its envelope

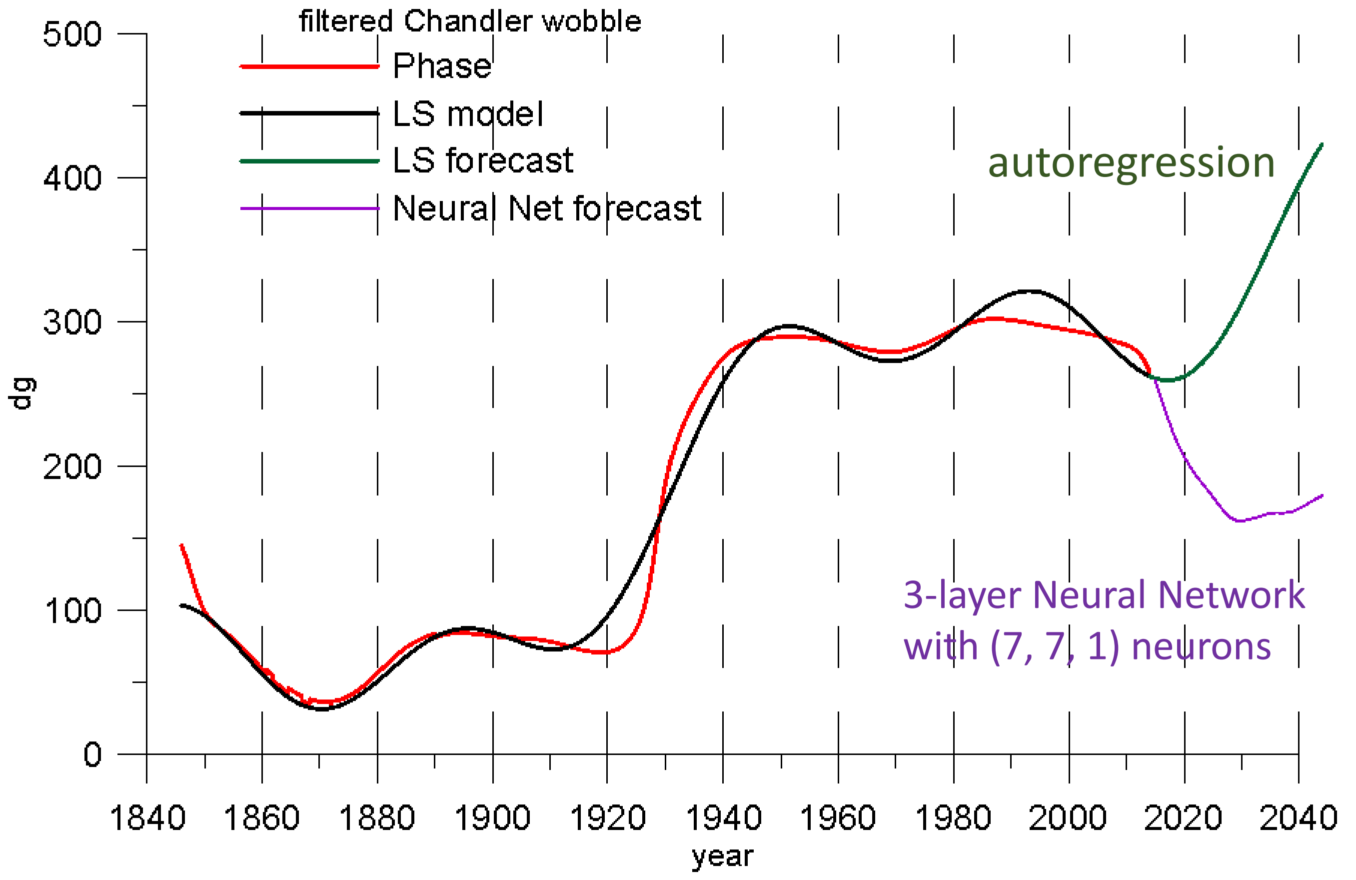


CW amplitude and phase model and prediction

~Chandler wobble amplitude NLSM fit			
	Period, years	Amplitude	Phase (1880)
~80-year component	83.44	42.6 mas	40.8°
~40-year component	42.0	54.6 mas	-101.5 °
mean		134.8 mas	

~Chandler wobble phase NLSM fit			
	Period, years	Amplitude	Phase (1880)
~100-year component	117.8	59 dg	-118°
~50-year component	50.9	34 dg	95 °
1-order trend	2dg/year		





Comparison of left and right parts of Euler-Liouville equation for polar motion

Geodetic excitation – left –hand part

Geophysical excitation – right hand part

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt} + p(t) = \chi^{tot}(t),$$

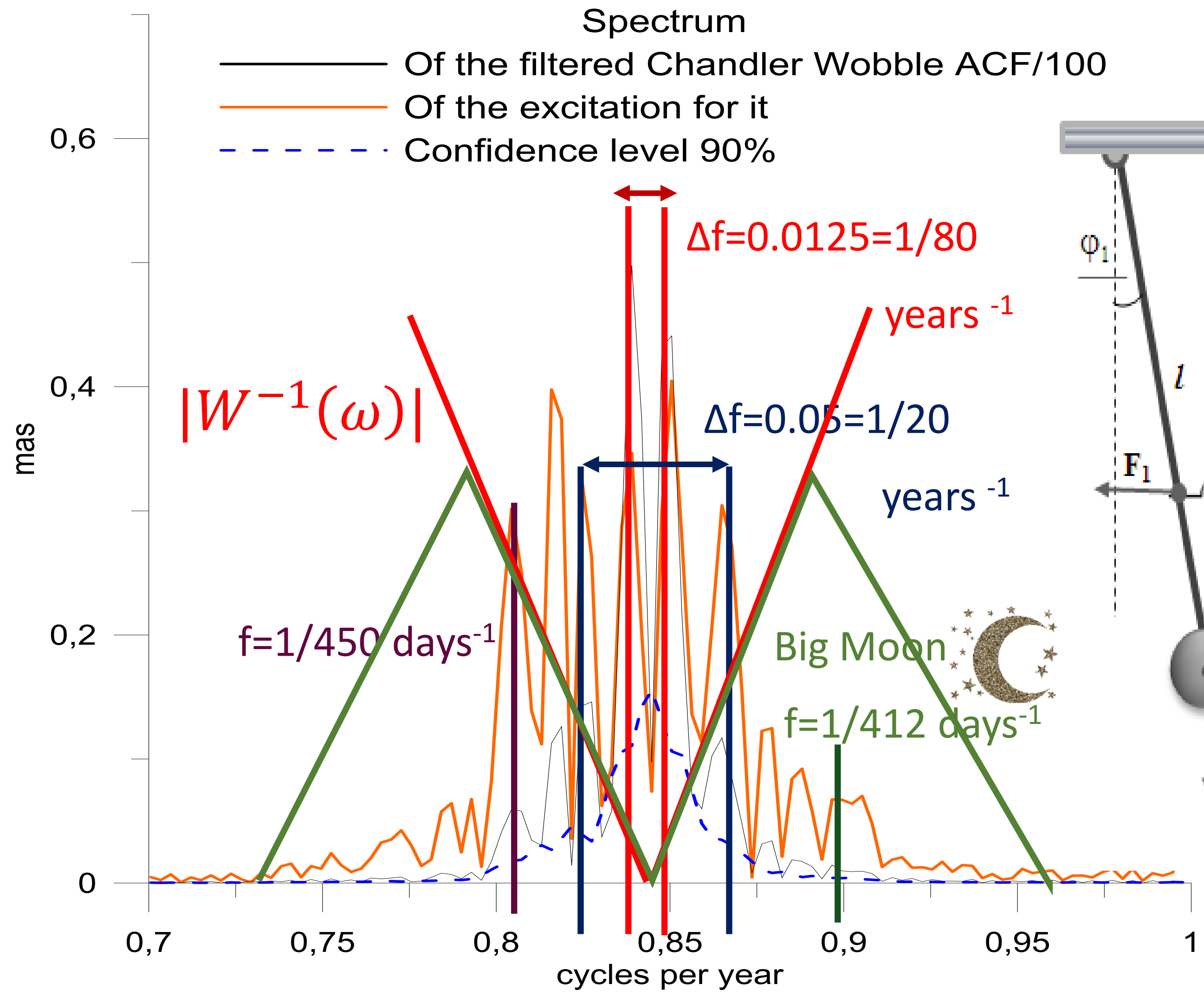
p(t) – complex polar motion trajectory

$\chi(t)$ – excitation

$$\sigma_c = \sigma_e \frac{\left(1 - \frac{\tilde{k}_2}{k_s}\right)}{\left(1 + e\frac{\tilde{k}_2}{k_s}\right)} = 2\pi f_c \left(1 + \frac{i}{2Q}\right).$$

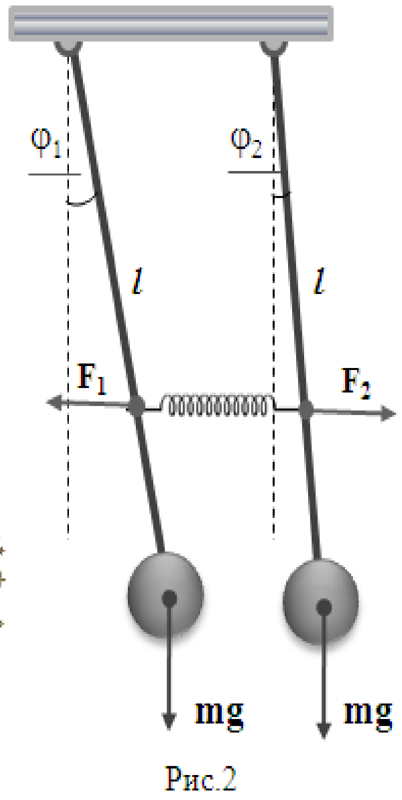
$$\chi_e^{mass} = \frac{1 + k_2'}{1 - \tilde{k}_2/k_s}\chi^{mass}, \qquad \chi_e^{motion} = \frac{1}{1 - \tilde{k}_2/k_s}\chi^{motion}.$$

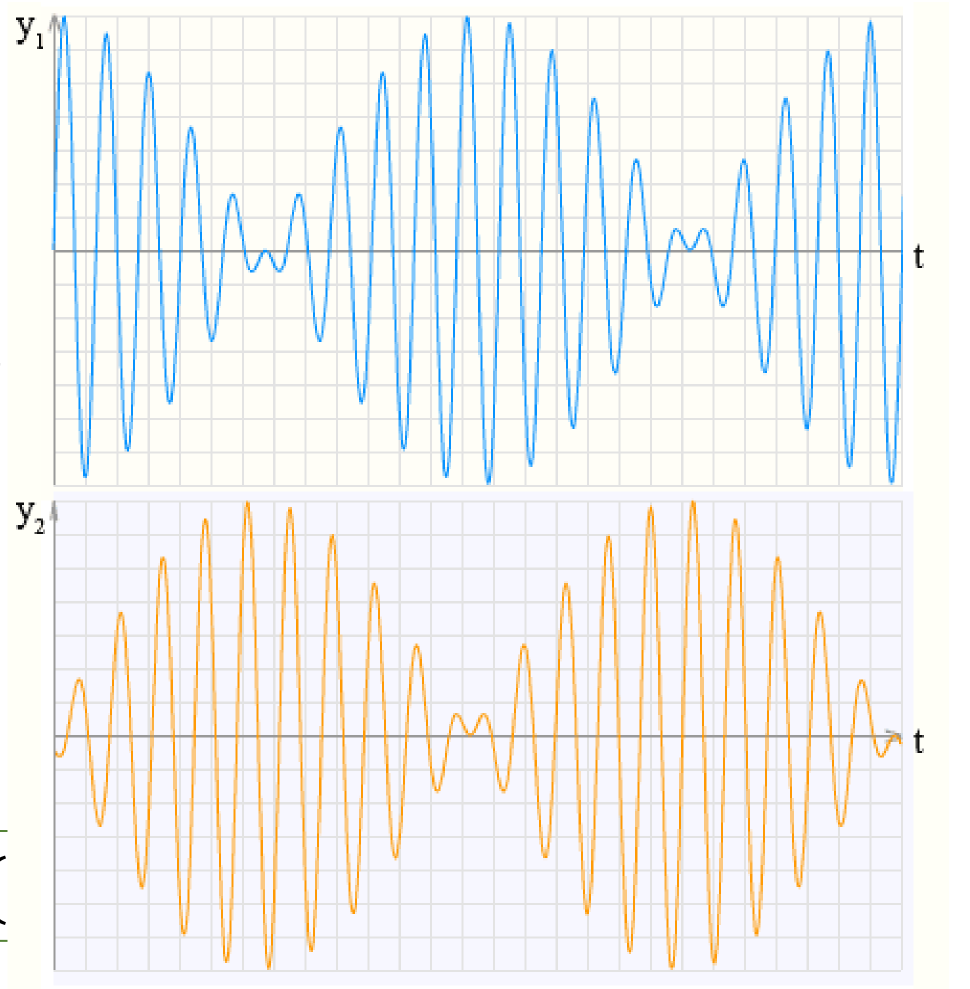
Splitted Chandler wobble spectral line



Coupled oscillators near resonance?

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$





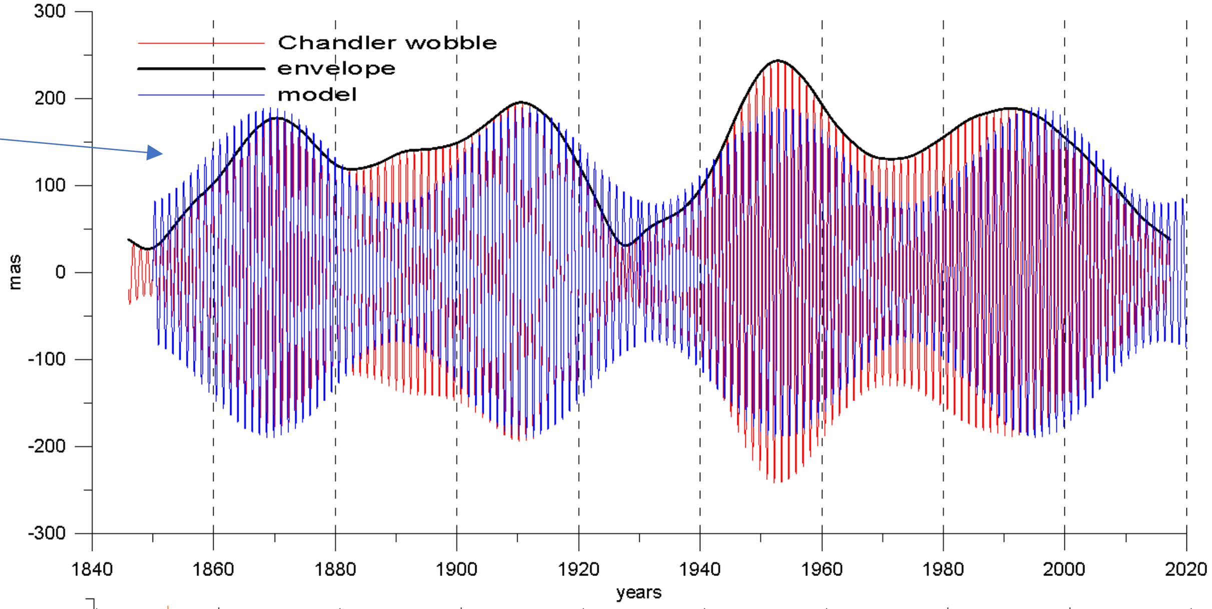
$\hat{\chi}($

Modelling amplitude changes in the Chandler Wobble excitation

Model of the Chandler wobble

$$p(t) = [a_0 + a_1 \cos(2\pi f_m t)] \exp(i2\pi f_c t + \phi_\pi).$$

a_0=135 mas
a_1=55 mas
w_c=2π/42(yrs)

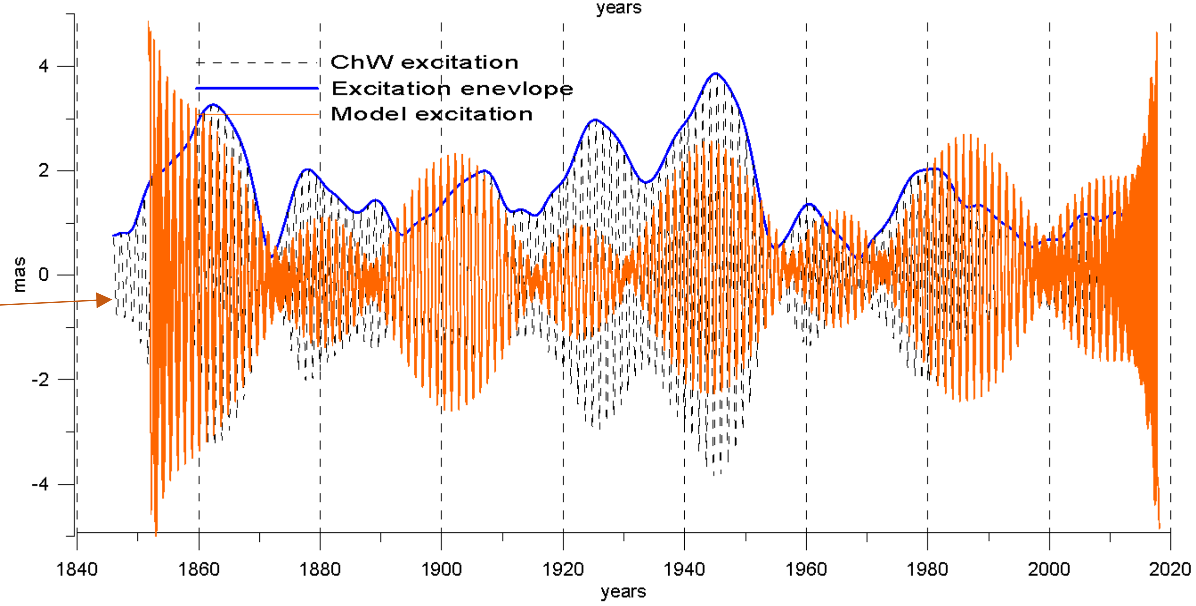


Model substitution into Euler-Liouville equation gives

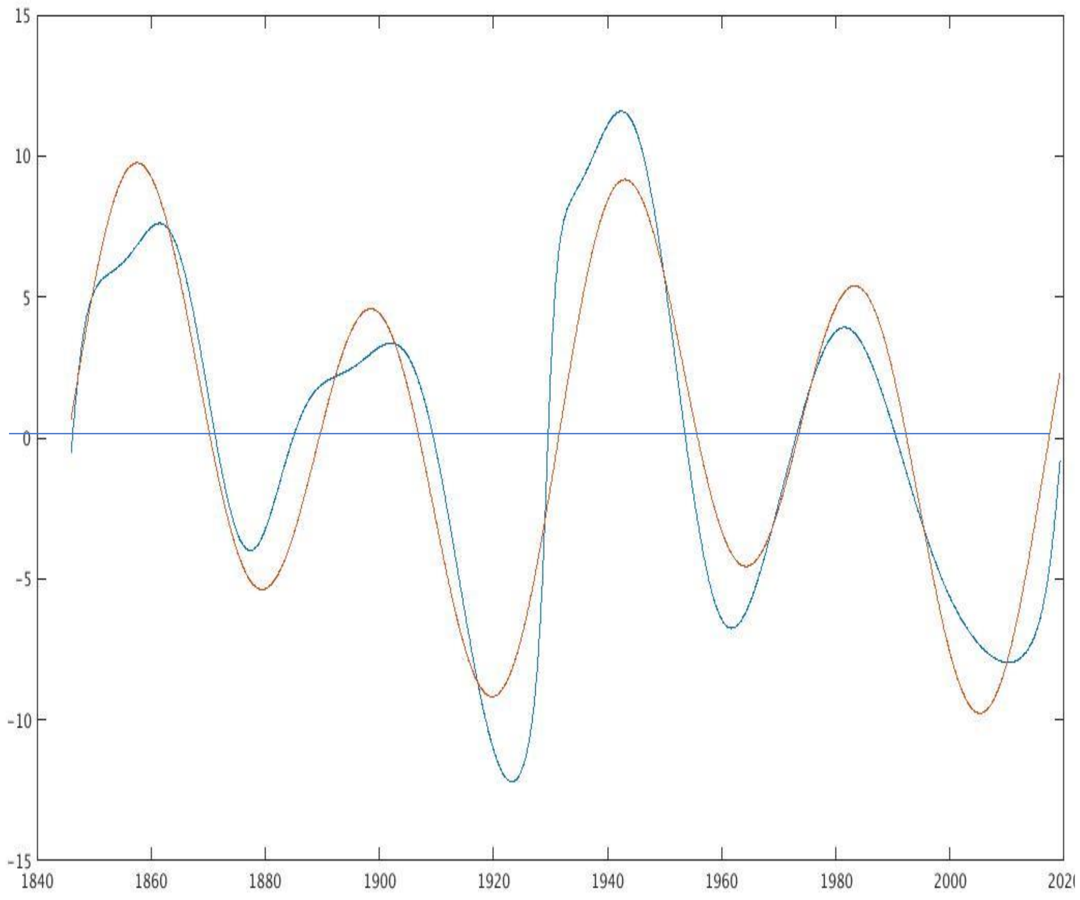
$$\sigma_c \chi(t) = -\omega_c p(t) + \sigma_c p(t) + i\frac{d|p(t)|}{dt} \exp(i\omega_c t + \phi_\pi).$$

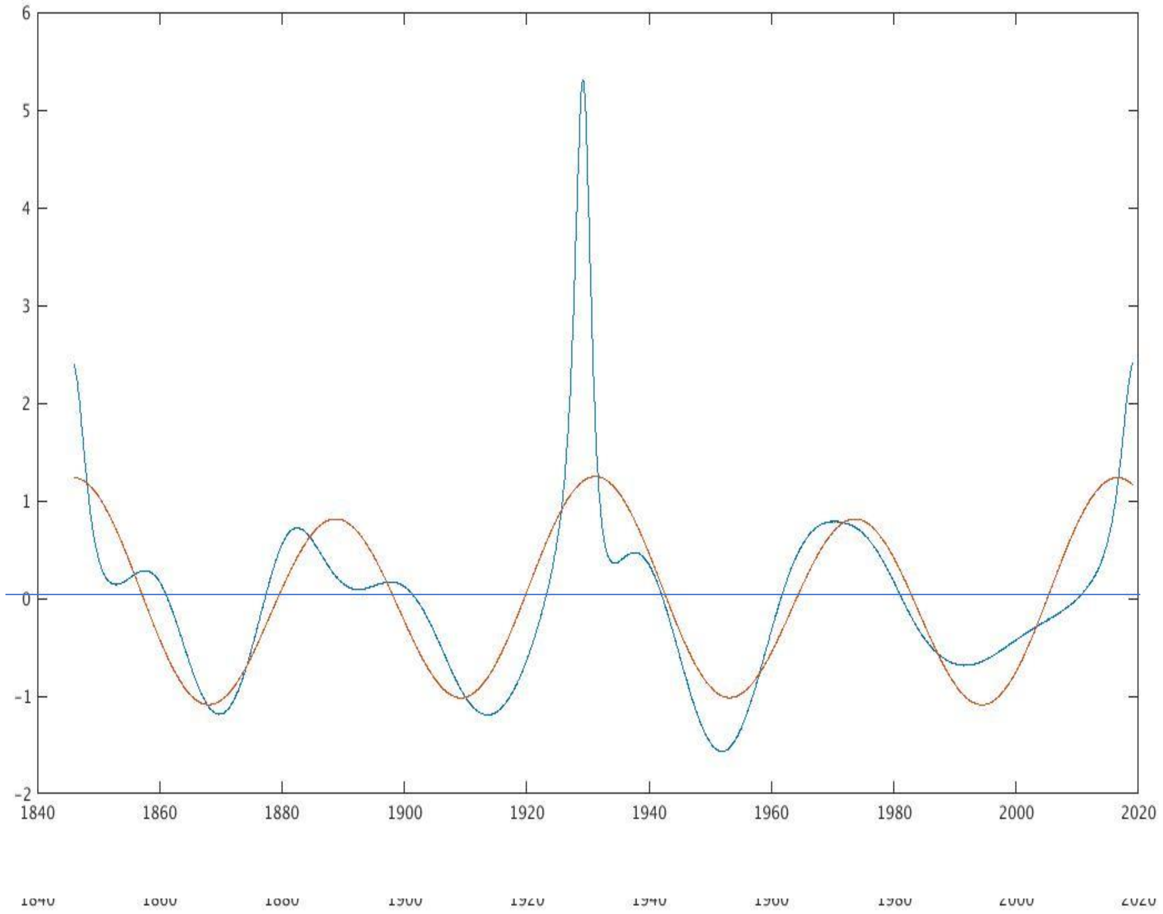
Excitation :

$$\chi = \frac{i}{\sigma_c}\left(\frac{\omega_c}{2Q}[a_0 + a_1 \cos(\omega_m t)] - \omega_m a_1 \sin(\omega_m t)\right) \exp(i\omega_c t + \phi_\pi).$$



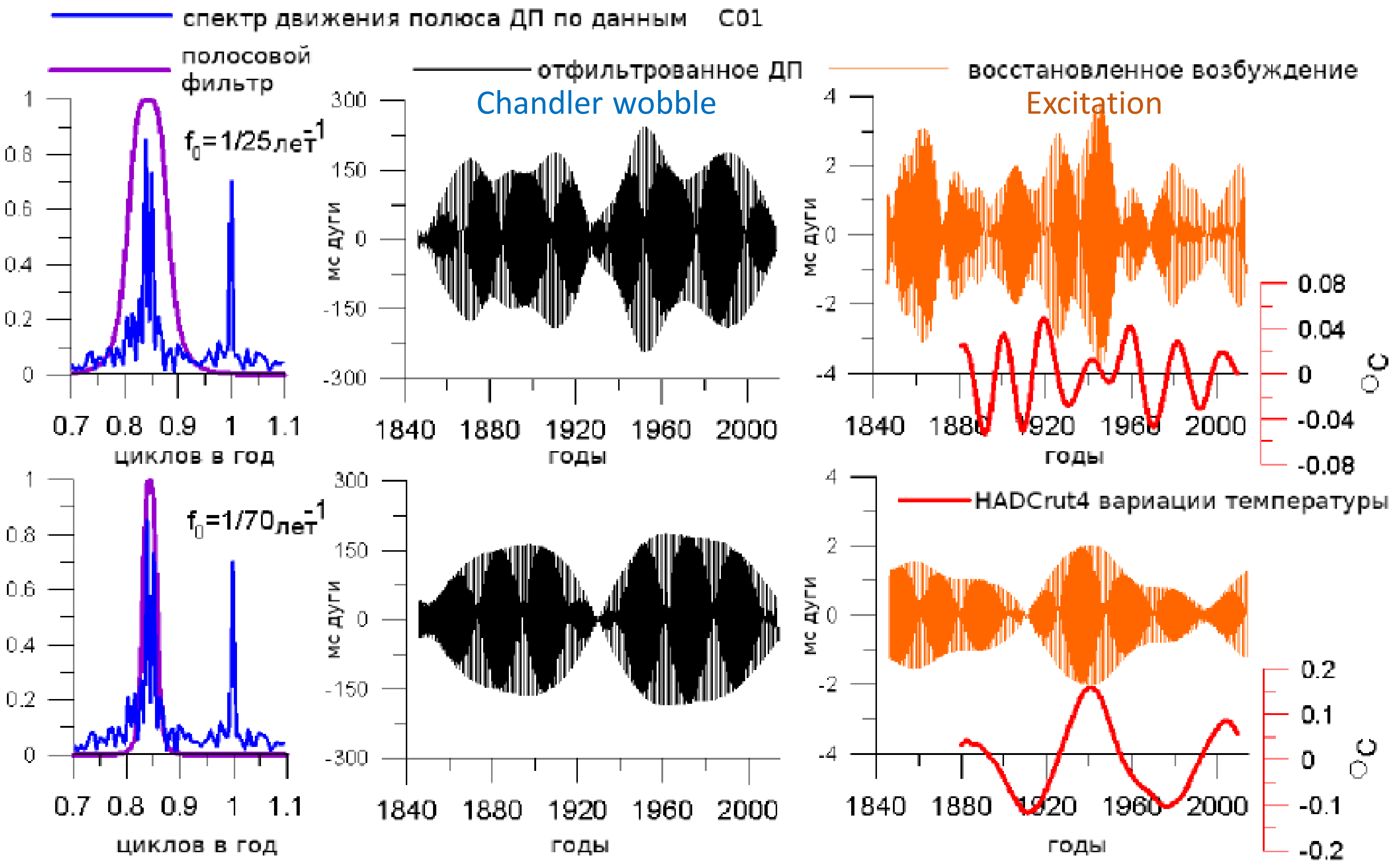
First derivative of envelope CW Second derivative of envelope





How to select filter width?

Different filter-width parameter f_0



Investigation of inverse problem solution uncertainty

$$\boxed{p(t) = A\chi(t) + \delta}$$

direct problem, δ – observational noise

$$\boxed{\breve{\chi}\,(t) = R\,p(t)}$$

solution of the inverse problem,
R - regularizing algorithm

$$\breve{\chi} = argmin\|A\breve{\chi} - p\|$$

there is error *h* in operator *A*
which depends on knowledge of CW frequency f_c and quality factor Q.

$$h\|\breve{\chi}\|$$

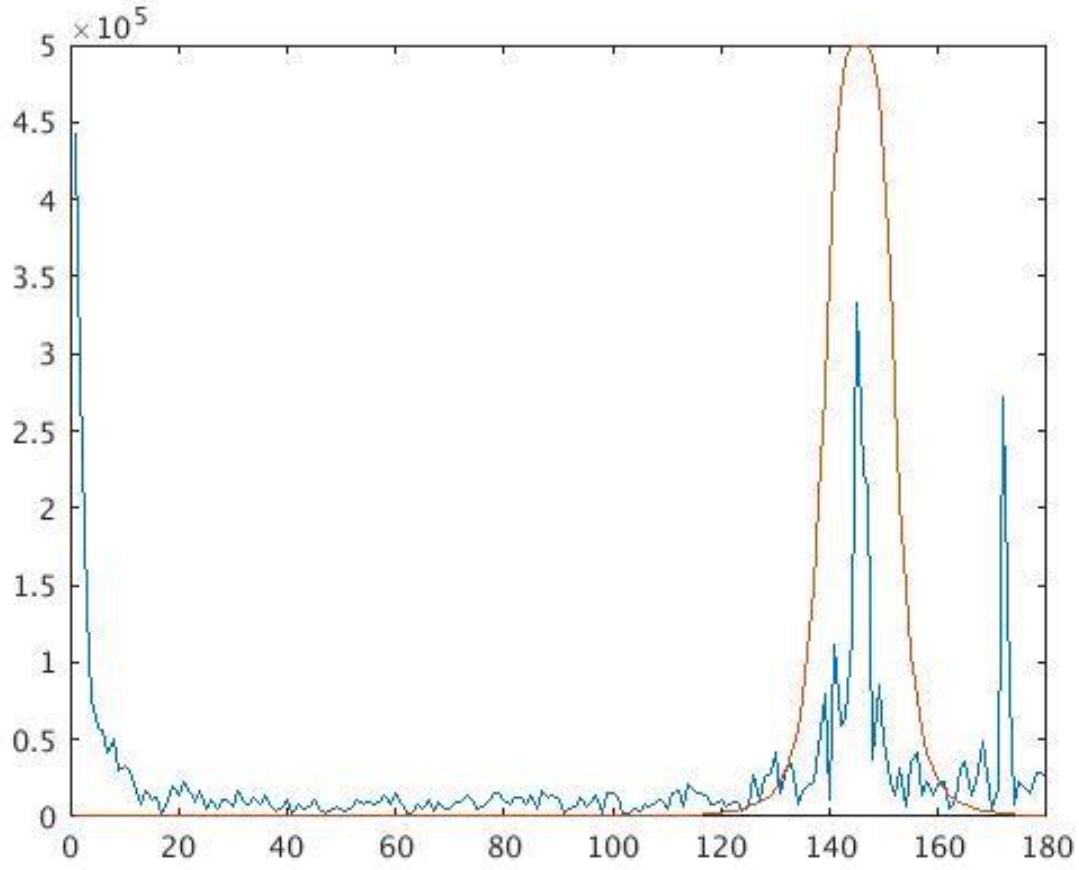
- error in *p* related to operator uncertainty

R also depends on CW frequency f_c and quality factor Q,
but the main parameter of the regularizing algorithm is the filter width parameter f_0

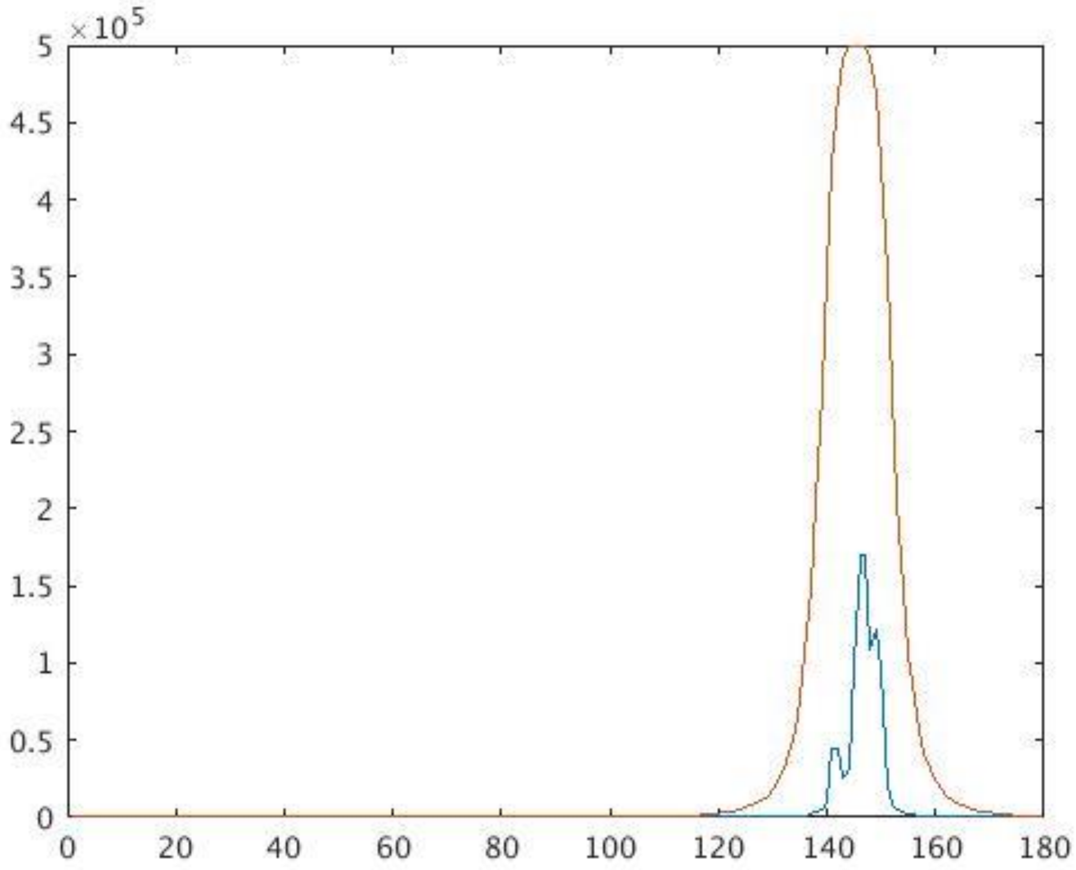
the only way to access the solution $\breve{\chi}$ uncertainty is to provide the diameter of the set, it is selected from, consistened with uncertainties *h*, δ and a priory conditions, used in *R*

Simulating amplitude changes in Chandler Wobble and excitation

Real polar motion spectrum

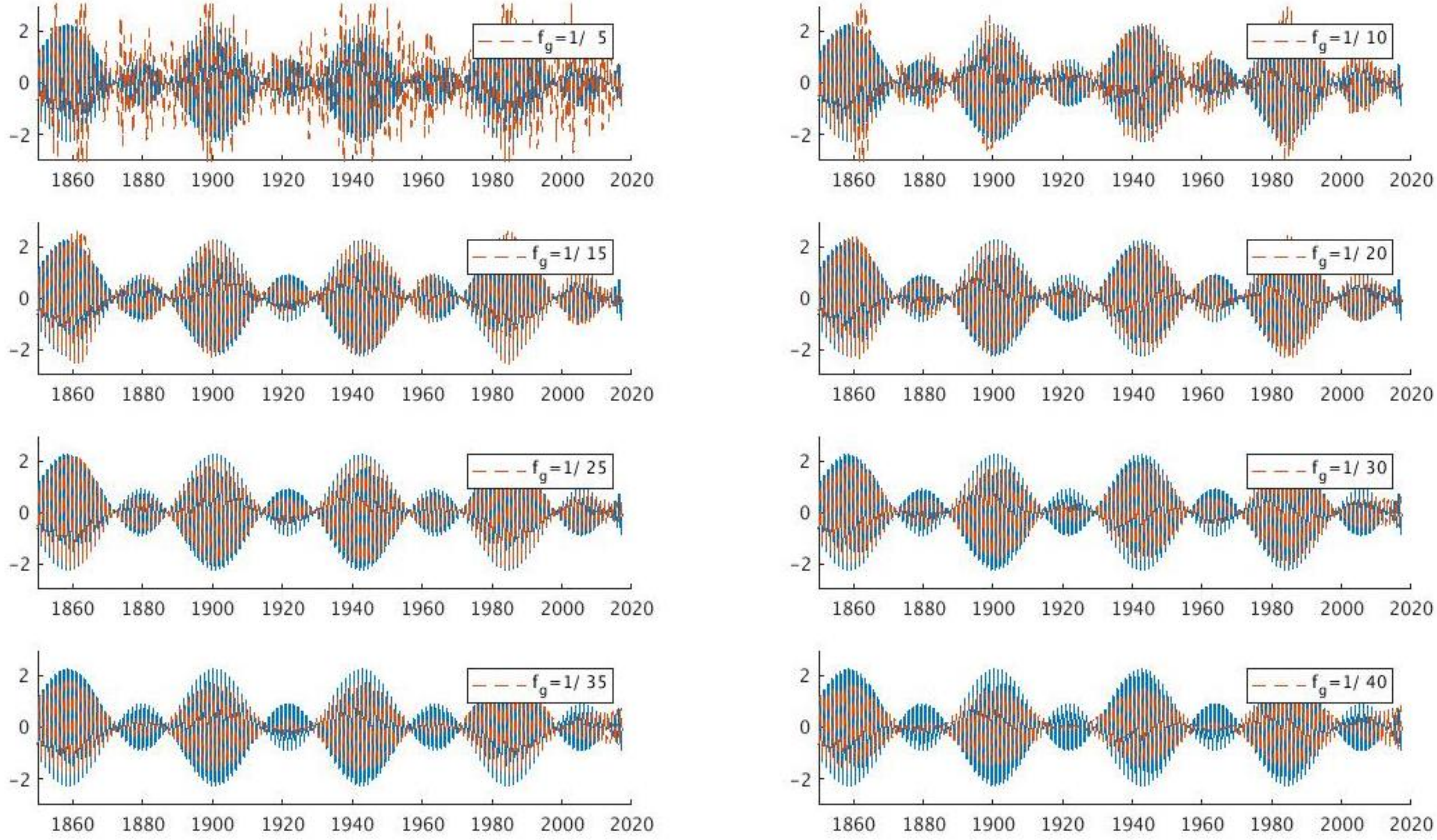


Spectrum of the modeled CW



filter width parameter $f_0=1/T_0$

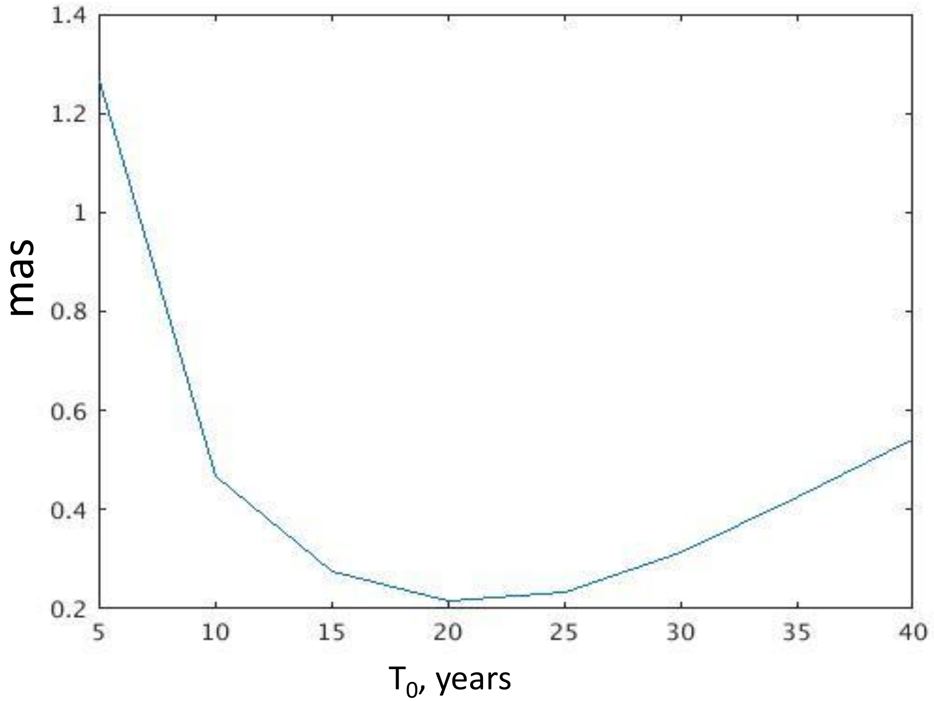
Comparison of exact modeled excitation
with what was reconstructed through corrective filtering (inversion+filter)
for different values of filter width parameter $f_0=1/T_0$



The discrepancy valued $\|A\chi - p\|$ depending on the filter parameter selected

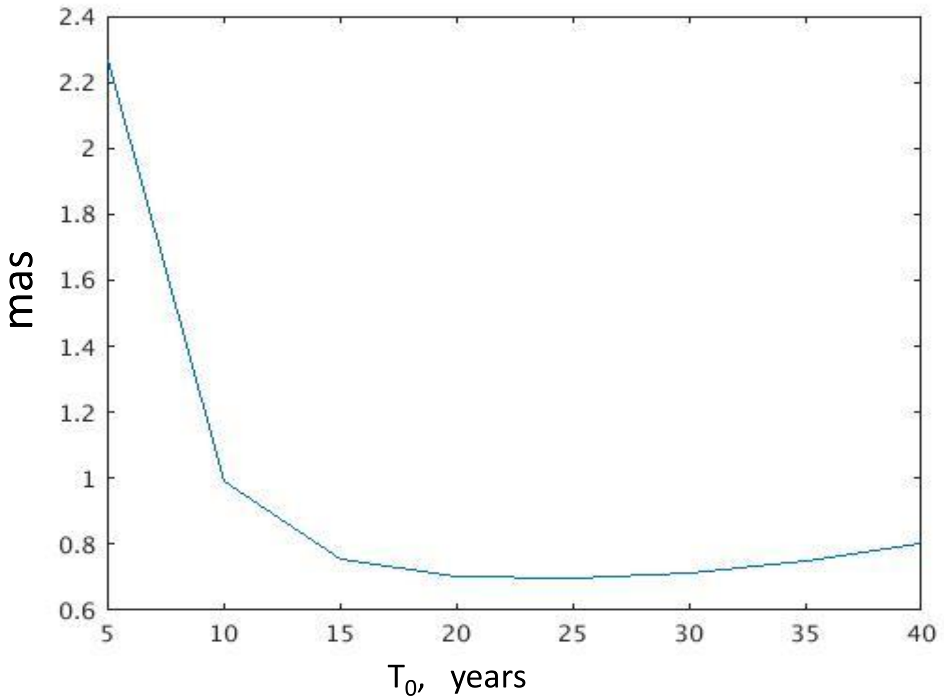
For exact operator
and noise only in observations

$h\|\chi\| \sim 130\ mas$



For both disturbed operator
and noises in observations

$h\|\chi\| \sim 130\ mas, \qquad \delta \sim 50\ mas$



$\mu = \|A\breve{\chi} - p\| + \delta + h\|\breve{\chi}\|$ - generalized uncertainty

This proves that the optimal half-width parameter of the filter$f_0=1/T_0$ is about
1/20 - 1/25 (in our previous work $f_0=1/25$ year $^{-1}$)

Angular momentum – geophysical excitations

Mass component

Motion component

$$\mathbf{H}(t) = \mathbf{H}^{mass}(t) + \mathbf{H}^{motion}(t) = \int \rho(r,t)\mathbf{r} \times [\mathbf{\Omega} \times \mathbf{r} + \mathbf{v}(r,t)]dV.$$

$$\mathbf{H}_x^{mass}(t) = -\Omega\frac{r^4}{g}\int\int p(\phi,\lambda)\sin\phi\cos^2\phi cos\lambda d\phi d\lambda,$$

$$\mathbf{H}_y^{mass}(t) = -\Omega\frac{r^4}{g}\int\int p(\phi,\lambda)\sin\phi\cos^2\phi sin\lambda d\phi d\lambda,$$

$$\mathbf{H}_z^{mass}(t) = \Omega\frac{r^4}{g}\int\int p(\phi,\lambda)\cos^3\phi d\phi d\lambda,$$

$$\mathbf{H}_x^{motion}(t) = \frac{r^3}{g}\int\int\int[\cos\phi\sin\lambda v(r,t) - \sin\phi\cos\phi\cos\lambda u(r,t)]d\phi d\lambda,$$

$$\mathbf{H}_y^{motion}(t) = \frac{r^3}{g}\int\int\int[-\cos\phi\cos\lambda v(r,t) - \sin\phi\cos\phi\sin\lambda u(r,t)]dpd\phi d\lambda,$$

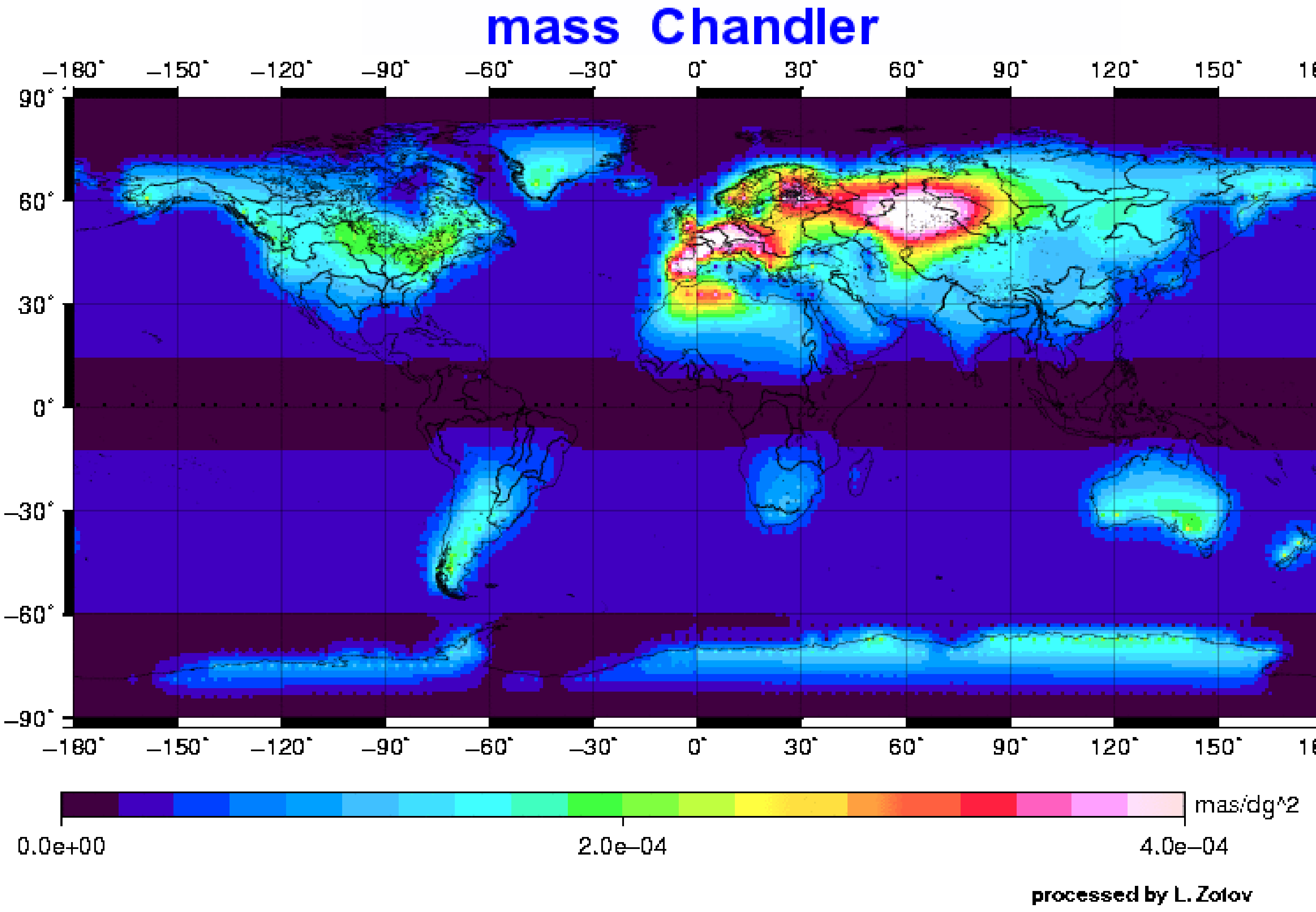
$$\mathbf{H}_z^{motion}(t) = \frac{r^3}{g}\int\int\int\cos^2\phi u(r,t)dpd\phi d\lambda,$$

Effective angular momentum

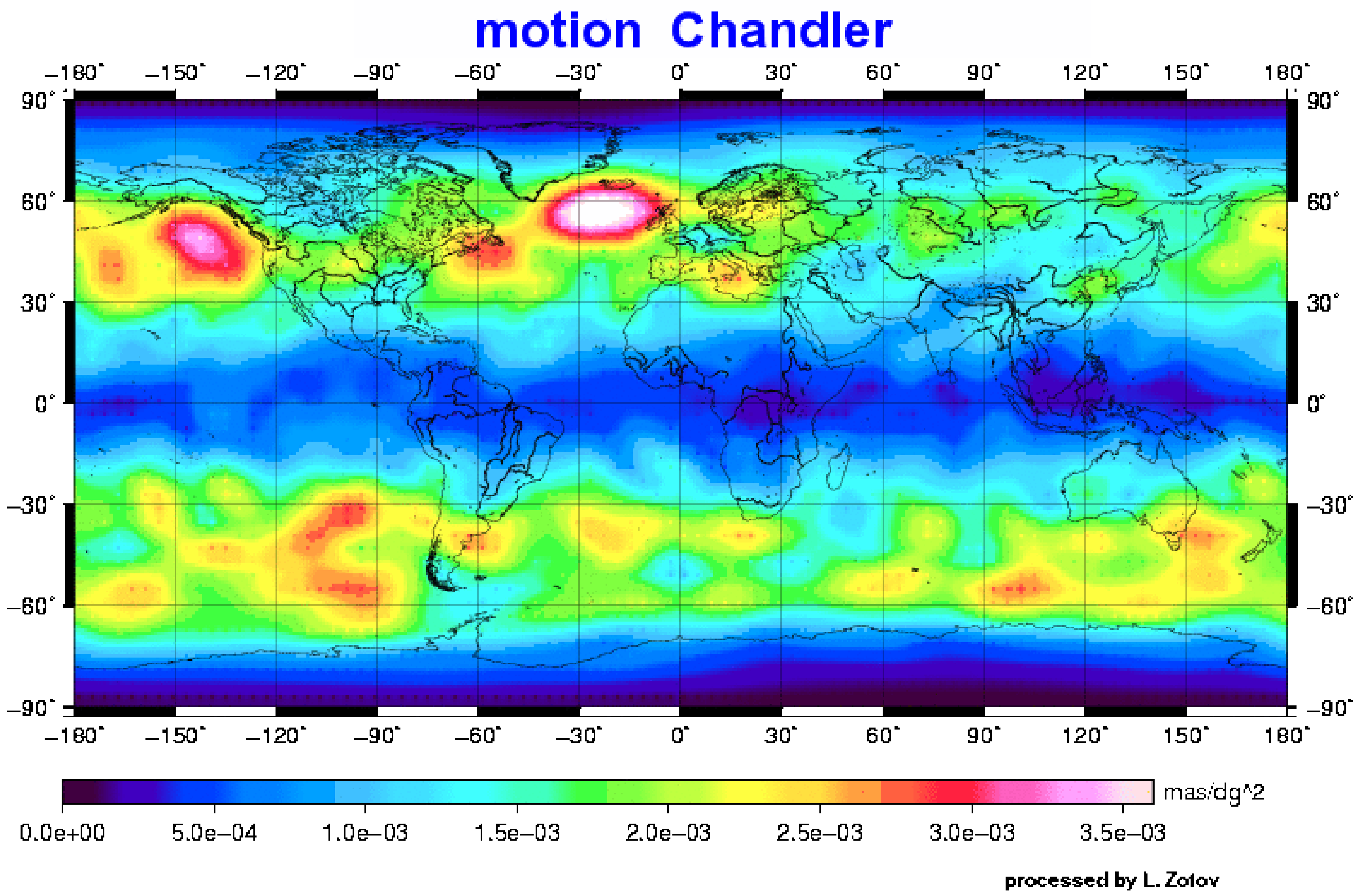
$$\chi_{x,y}^{motion} = \frac{1.5913}{\Omega(C-A)}\mathbf{H}_{x,y}^{motion}, \qquad \chi_z^{motion} = \frac{0.998}{\Omega C}\mathbf{H}_z^{motion},$$

$$\chi_{x,y}^{mass} = \frac{1.098}{\Omega(C-A)}\mathbf{H}_{x,y}^{mass}, \qquad \chi_z^{mass} = \frac{0.753}{\Omega C}\mathbf{H}_z^{mass}.$$

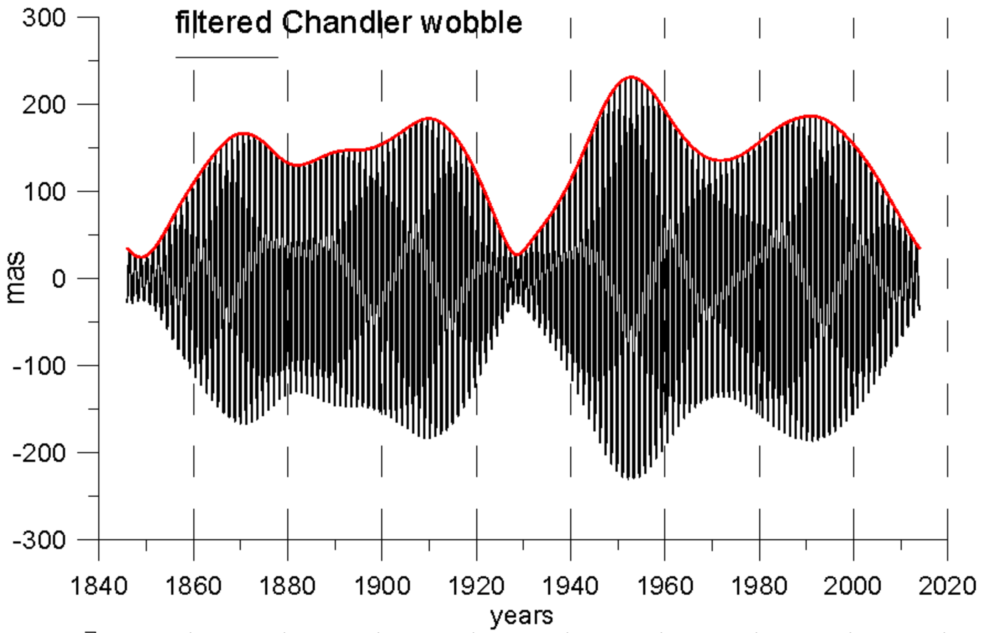
Absolute meridional AAM ECMWF changes filtered in the Chandler band, MASS term

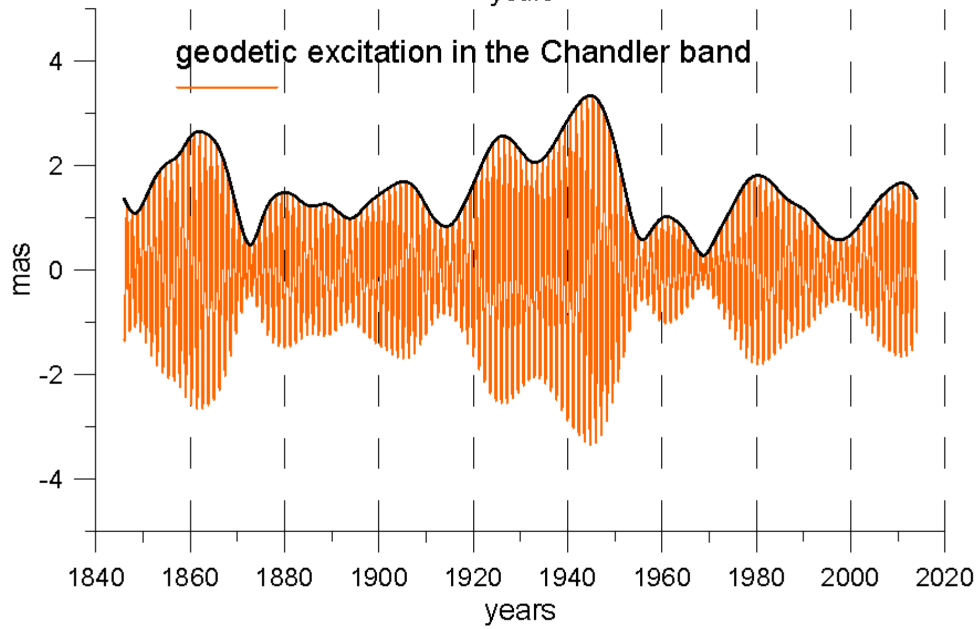


Absolute meridional AAM ECMWF changes filtered in the Chandler band, MOTION term



Chandler wobble, comparison of excitations





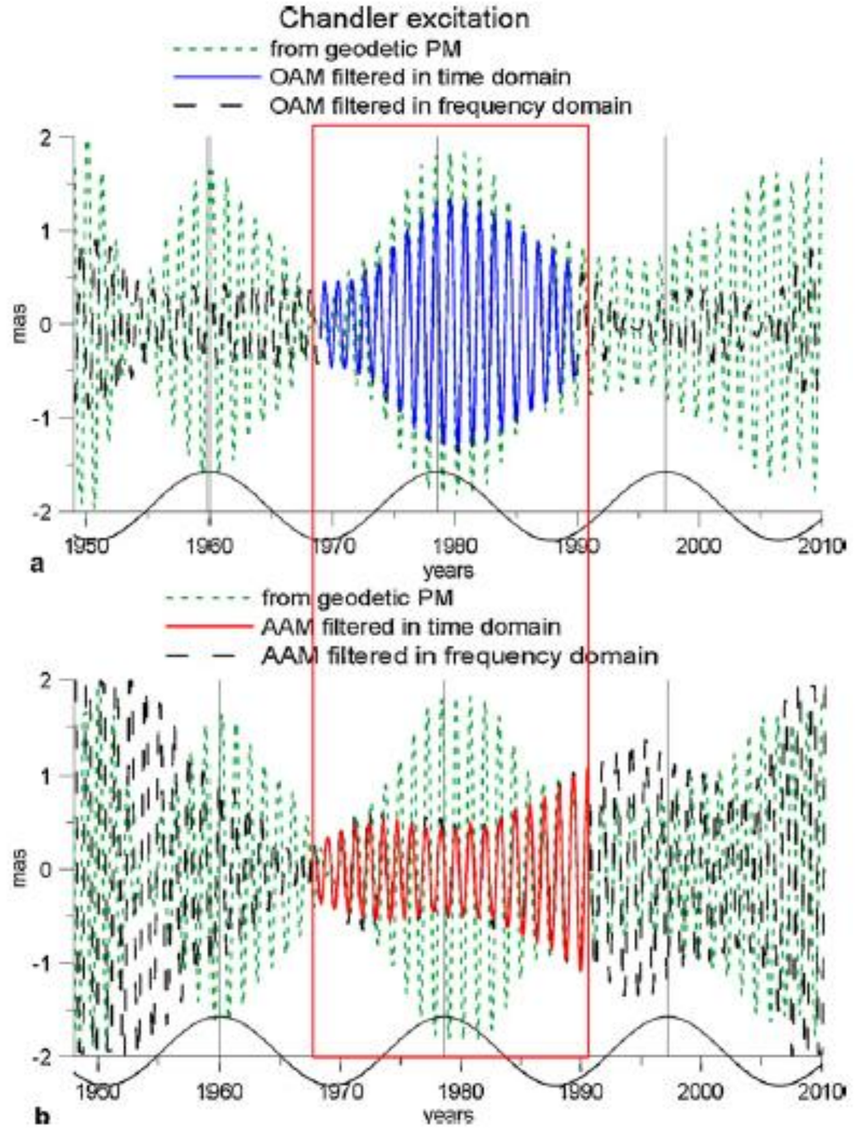
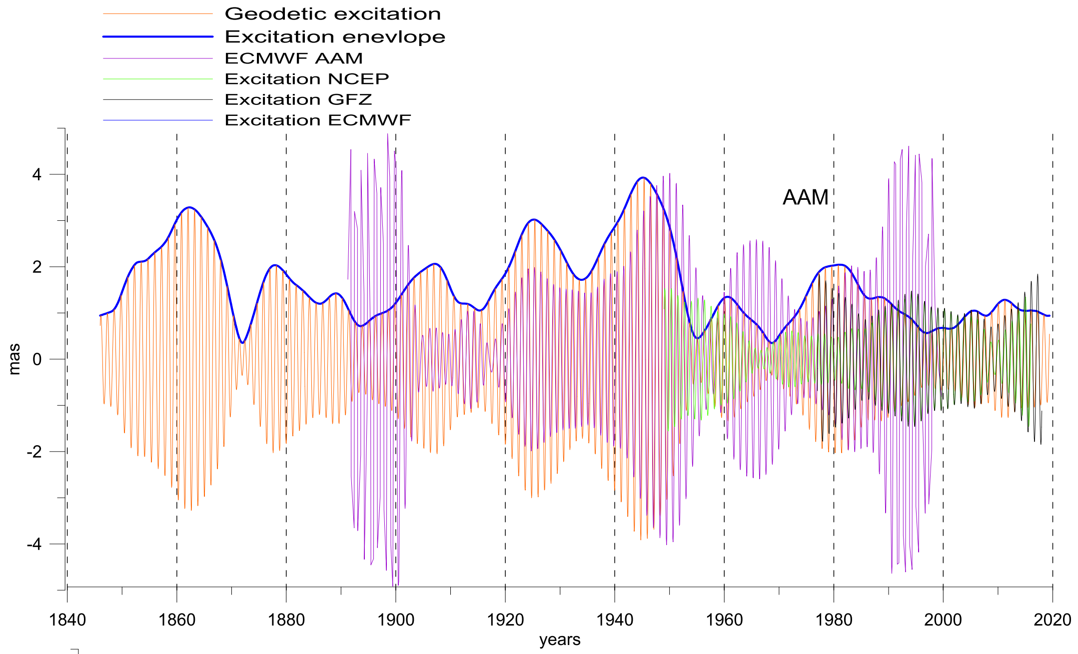


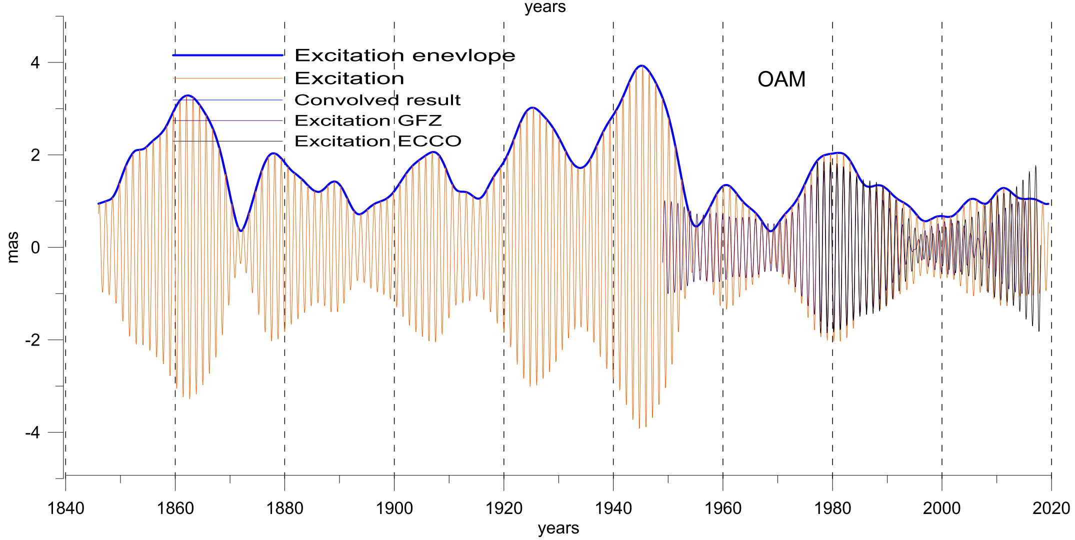
Fig. 4. The hydro-atmospheric Chandler excitation compared with the geodetic Chandler excitation: the oceanic (a) and atmospheric (b) contributions into the *x* component.

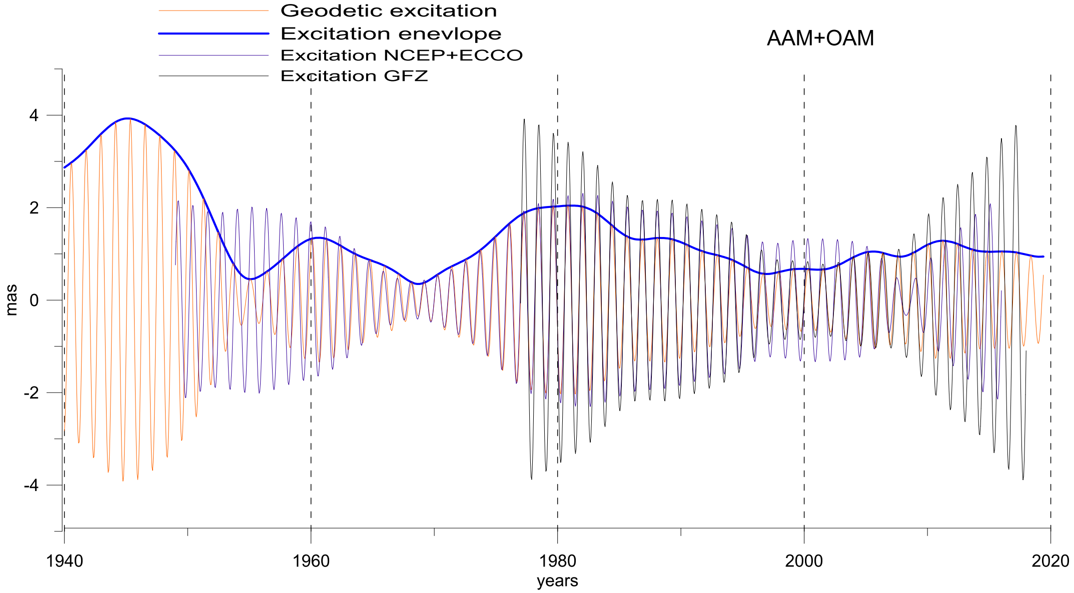
Oceanic 海洋 excitation

Atmospheric 大气 excitation

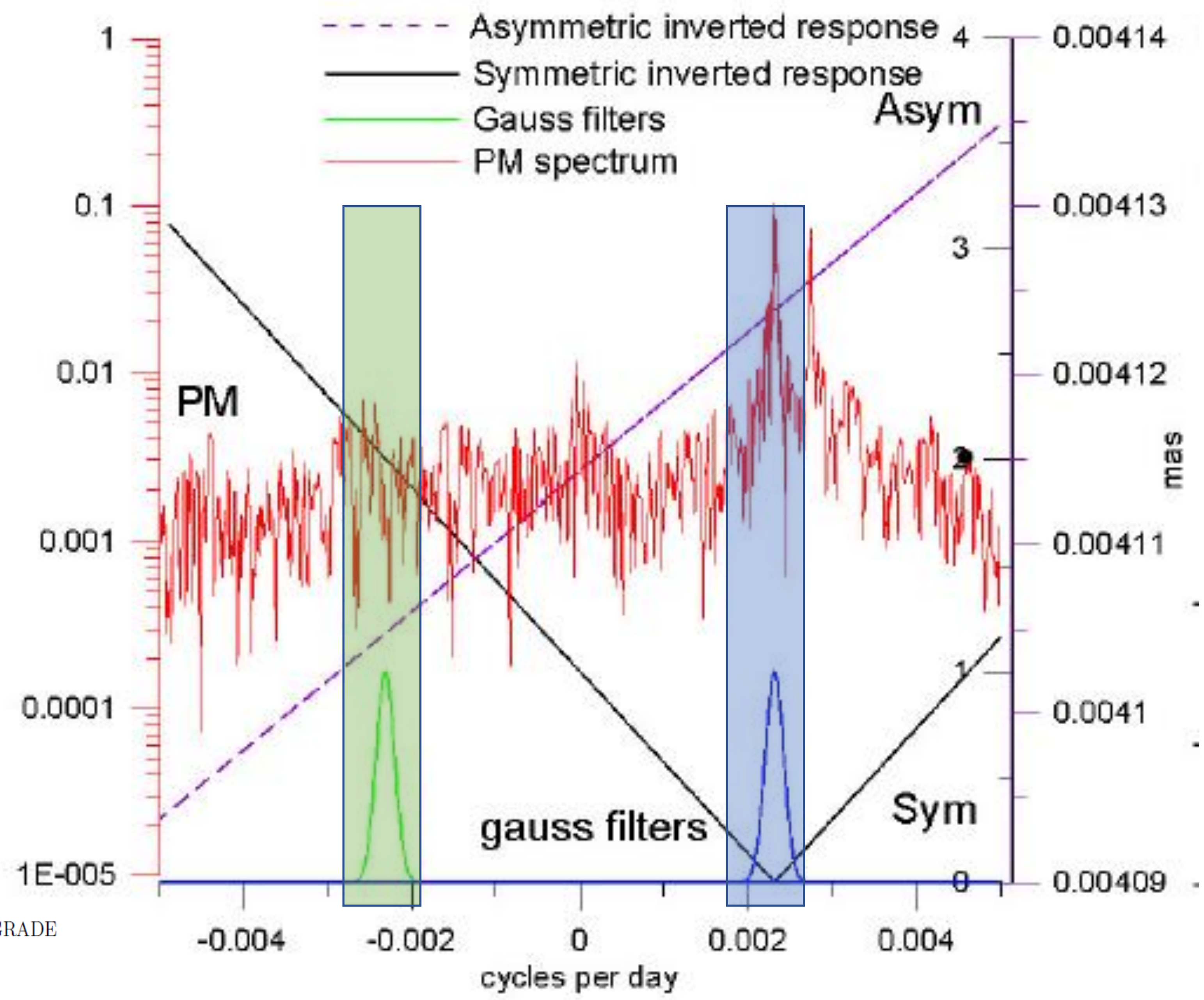
Chandler wobble, comparison of excitations







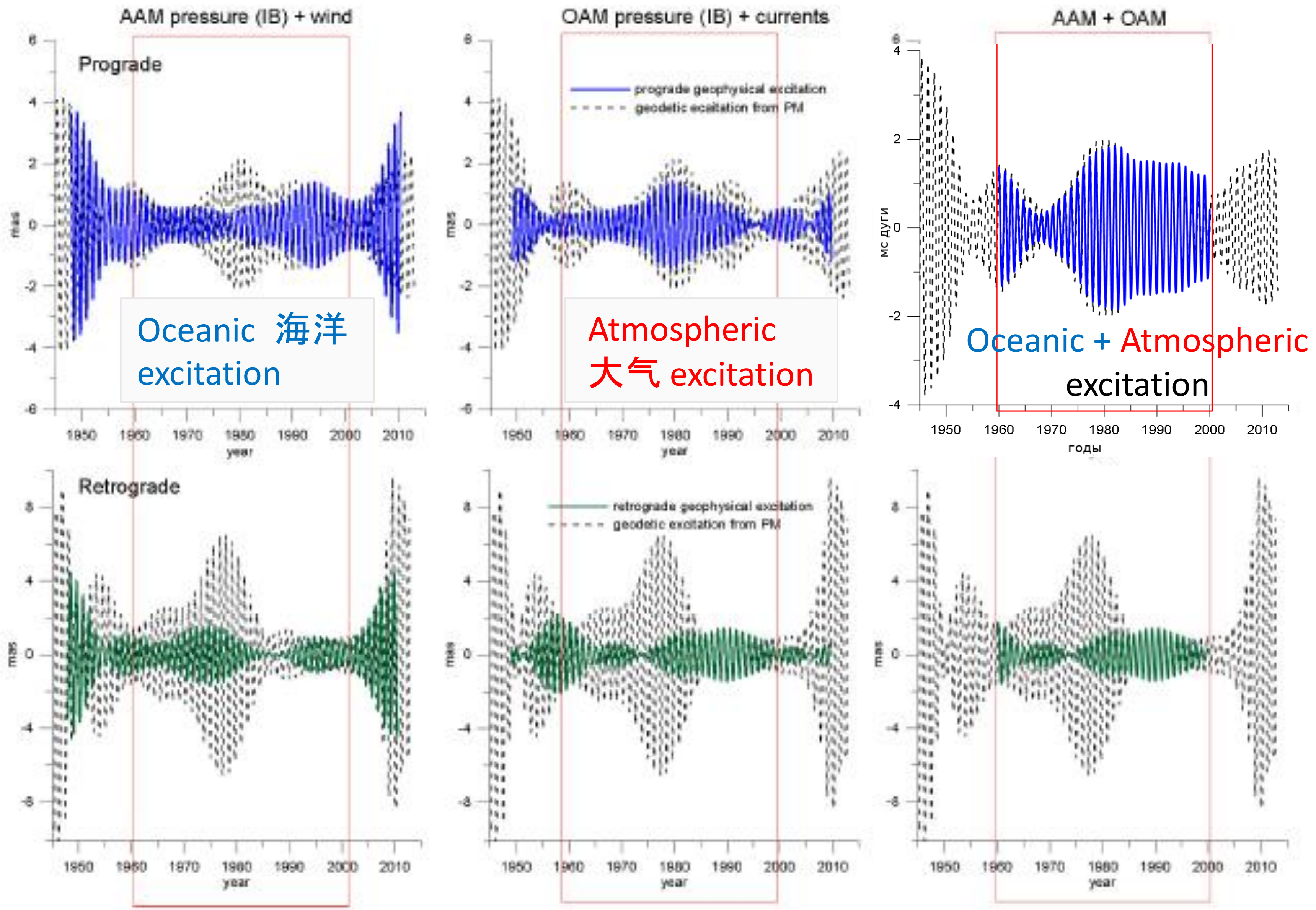
Planteleev’s filtering in prograde and retrograde Chandler band



STUDY OF THE PROGRADE AND RETROGRADE CHANDLER EXCITATION

L.V. ZOTOV[1,2], C. BIZOUARD[2],

Geodetic and geophysical excitations in prograde and retrograde Chandler bands



Conclusions

- Chandler wobble can be extracted by different types of filtering
- CW amplitude and phase changes observed in our epoch and over more than 100 years can be crucial for Chandler wobble understanding
- spectral line of CW is splitted, what manifests its variability in time, may be coupled oscillations in the Earth systems are responsible for regularities
- if 80 and 40-years quasi-periodic changes in CW amplitude are real, they would present in excitation (assuming stability of equation parameters)
- modelling proves, that ~40-year changes in CW amplitude are related to the repetitions of ~20-year excitation and ~20-year damping epochs
- the so-called “free” Chandler mode actually is not free, being provided by atmospheric and oceanic excitations with, possibly, hidden regularity

Thank you for attention!