



Outcomes of the activities of the *IAU/IAG Joint Working Group on Theory of Earth Rotation and Validation.*

**José M. Ferrándiz (1), Richard S. Gross (2), Alberto Escapa(1,3),
Juan Getino (4), Aleksander Brzezinski (5), Robert Heinkelmann (6)**

(1) University of Alicante VLBI Analysis Centre, UAVAC, Alicante, Spain,

(2) Jet Propulsion Laboratory, California Institute of Technology, Pasadena, USA

(3) University of Leon, Spain

(4) University of Valladolid, Spain

(5) Space Research Centre, Polish Academy of Sciences, Poland

(6) Helmholtz Centre Potsdam, GFZ German Research Centre for Geosciences, Potsdam, Germany

Introductory remarks

- This report is based mainly on:
 - *the IAU/IAG JWG TERV report to the 2018 IAU General Assembly,*
 - *the end-of-term report published in the IAG Travaux 2015-2019, and the references and bibliography therein*
 - *The reports of its three SWGs, particularly those presented at the IAG G04 Symposium during the IUGG 2019 GA*
- It gathers the main JWG TERV outcomes and finding.
- We distinguish between:
 - *Advances or findings on topics that can be considered scientifically solved (acknowledged by the 2019 IAG Resolution 5)*
 - *Other advances showing remarkable improvement of knowledge but still on progress*

Main Outcomes

The first part of the outcomes cited in this report is based mostly on papers published in recent years, which unveil that a noticeable part of the unexplained variance of the determined EOP series can be attributed mainly to:

- systematic errors (*e.g. in models*),
- inconsistencies (*internal to theories or among components of them*), and
- need of updating some specific components *after 20 years of their derivation*

Selected Findings (1)

1. The amplitudes of the nutation theory IAU2000 need to be updated.

The number of periodic terms that can be determined from VLBI observations is much larger than in 1999 and the errors bars much smaller, according to several papers.

The differences of amplitude for some periods can exceed some tens of micro arc seconds (μas)

Determinations of nutation amplitudes have been performed by different authors in the last years (Koot et al, Malkin, Gattano et al, Belda et al, etc.)

In the next slides, a few details taken from the SWG 3 report to IUGG 2019 GA are presented

Example from SWG3 report by Robert Heinkelmann et al,
presented at the 2019 IAG General Assembly

2. Lunisolar nutations - VLBI

- GLB17
- **Nutation** terms of IAU2000A (Mathews et al. 2002) revised (μas)
- Revised 21 terms
- Table: 10 terms
- Different IVS AC and IERS combined solutions ("Range")

Period (days)	Median amplitude		Range		Median error	
	Real	Imag	Real	Imag	Real	Imag
−6798.38	35.5	−28.7	47.4	17.6	2.2	2.2
6798.38	22.4	−32.8	18.9	18.3	2.2	2.2
−3399.19	4.5	−9.4	15.8	11.7	2.1	2.1
3399.19	11.3	−5.2	10.8	24.0	2.1	2.1
−1615.75	−2.0	−8.1	17.8	9.3	2.1	2.1
1615.75	0.9	−9.1	12.2	6.7	2.1	2.1
−1305.48	0.6	8.2	13.4	10.0	2.1	2.1
1305.48	0.0	4.4	11.8	12.8	2.1	2.1
−1095.18	−1.1	1.1	10.7	10.5	2.0	2.0
1095.18	−3.3	−1.2	9.5	9.2	2.0	2.0
−386.00	−1.2	−1.4	12.5	7.7	2.0	2.0
386.00	−4.3	1.9	10.6	11.7	2.0	2.0
−365.26	27.5	5.6	9.4	18.6	2.1	2.1
365.26	−3.8	−0.9	15.8	21.6	2.1	2.1
−346.64	−13.3	4.7	16.6	10.6	2.2	2.2
346.64	−2.2	2.2	6.8	8.7	2.2	2.2
−182.62	−15.2	4.2	8.5	6.6	2.0	2.0
182.62	7.5	−2.8	14.0	12.5	2.0	2.0
−121.75	−3.7	2.2	4.8	5.6	2.0	2.0

Example from SWG3 report by Robert Heinkelmann et al,
presented at the 2019 IAG General Assembly

2. Lunisolar nutations - VLBI (contd.)

- BHF17
- **Nutation** terms of IAU2000A (Mathews et al. 2002) revised (μas)
- Revised 197 terms
- Table: 14 terms, where correction $> 3 * \text{median error}$
- 7 out of 14 with $>10 \mu\text{as}$ in at least one component

Period (days)	CPO	Median		Range		Median Error	
		As	Ac	As	Ac	As	Ac
6798.383	<i>dX</i>	-6.7	63.7	2.0	10.4	2.9	3.7
	<i>dY</i>	20.8	-63.0	4.3	6.3	2.9	3.7
3399.192	<i>dX</i>	-7.1	5.6	2.1	1.0	3.0	2.8
	<i>dY</i>	-6.3	-11.4	4.6	2.6	3.0	2.8
1615.748	<i>dX</i>	-1.4	-6.8	3.4	1.7	2.7	2.8
	<i>dY</i>	4.0	-8.7	3.5	2.9	2.7	2.8
1305.479	<i>dX</i>	0.1	5.6	1.5	1.4	2.8	2.8
	<i>dY</i>	7.2	12.2	1.6	1.1	2.8	2.8
1095.175	<i>dX</i>	2.7	-15.1	3.0	0.6	2.7	2.8
	<i>dY</i>	9.3	2.1	2.5	1.8	2.7	2.8
182.621	<i>dX</i>	10.6	-12.7	4.3	5.5	2.7	2.8
	<i>dY</i>	24.6	10.2	1.6	2.9	2.7	2.8
169.002	<i>dX</i>	8.6	0.4	3.3	2.3	2.7	2.7
	<i>dY</i>	4.2	4.6	5.0	2.5	2.7	2.7
29.531	<i>dX</i>	4.0	-0.3	4.2	2.7	2.8	2.8
	<i>dY</i>	6.4	8.7	4.2	1.8	2.8	2.8
27.555	<i>dX</i>	-3.5	-16.4	3.0	3.6	2.7	2.7
	<i>dY</i>	10.3	-5.9	6.1	4.7	2.7	2.7
27.093	<i>dX</i>	3.2	-6.0	3.0	2.0	2.7	2.7
	<i>dY</i>	3.9	11.9	3.4	1.9	2.7	2.7
26.985	<i>dX</i>	-3.1	4.3	1.9	3.3	2.8	2.8
	<i>dY</i>	1.9	-9.9	4.2	5.4	2.8	2.8
25.325	<i>dX</i>	-2.1	-2.9	3.2	1.6	2.7	2.8
	<i>dY</i>	8.3	-0.4	2.9	2.8	2.7	2.8
13.749	<i>dX</i>	1.6	3.0	3.0	2.2	2.6	2.7
	<i>dY</i>	0.4	9.2	4.6	1.2	2.6	2.7
13.661	<i>dX</i>	-22.4	-12.2	12.7	3.8	2.7	2.7
	<i>dY</i>	-0.9	5.9	7.2	6.7	2.7	2.7

Units micro as

Selected Findings (2)

2. The offsets and trends of the determined CPO series deviates from 0 at the μas and $\mu\text{as/y}$ levels – *i.e. the current precession model may be not 100% accurate*

The tables shown in the next slides, are two examples taken from Gattano, Lambert and Bizouard (2017) and Belda et al(2017).

In general, the offsets of dX and dY are $> 30 \mu\text{as}$, the target accuracy recommended by the IAG Global Geodetic Observing System (GGOS) and *adopted as reference value by the JWG TERV*

Rates seems to be compliant with the GGOS goals.

Linear part of precession, from the SWG3 report at the IUGG 2019 General Assembly, by Heikelmann et al (1)

1. Precession - VLBI

- GLB17: Gattano, Lambert, Bizouard (2017)
- Comparison of IVS ACs and IERS combined solutions
- Corrections to **precession** offset b (mas) and rate p (mas/century)

	b_X	b_Y	p_X	p_Y
aus00007	0.068	−0.111	0.152	−0.512
bkg00014	0.029	−0.060	0.149	−0.103
cgs2014a	0.025	−0.065	0.018	0.469
gsf2014a	0.033	−0.087	0.115	−0.102
iaa2007a	0.027	−0.065	0.583	−0.501
opa2015a	0.025	−0.078	0.166	−0.092
spu00004	0.072	−0.105	0.058	−0.714
usn2015a	0.026	−0.078	0.128	−0.054
vieceop13	−0.005	−0.065	0.135	0.067
ivs14q2X	0.048	−0.076	0.073	−0.197
eop08c04	0.041	−0.084	0.131	−0.008
eopbullA	−0.026	−0.093	0.424	−0.452

Notice that units here are not the usual of GGOS: $3 \mu\text{as/y} = 0.3 \text{ mas/cy}$

Linear part of precession, from the SWG3 report at the IUGG 2019 General Assembly, by Heikermann et al (2)

1. Precession – VLBI (contd.)

- BHF17: Belda, Heinkelmann, Ferrandiz et al. (2017)
- Station coordinates (estimated) a priori from various ITRF-VLBI solutions, different a priori TRF and EOP tested
- Corrections to **precession** offset (mas) and rate (mas/century) of final solution:

$$b_X = 0.0410, b_Y = -0.0730; p_X = 0.1781, p_Y = -0.2083$$

- 1σ -errors: $b_{X,Y} 0.0021, p_{X,Y} 0.0272$
- Range: $b_X 0.0386 - 0.0410, b_Y -0.0689 - -0.0732$
 $p_X -0.0666 - 0.1781, p_Y -0.2083 - -0.0831$

Notice that units here are not the usual of GGOS: $3 \mu\text{as/y} = 0.3 \text{ mas/cy}$

Selected Findings (3)

3. The nutation theory IAU2000 and the precession theory IAU2006 are not consistent with each other

Inconsistencies arise from the fact that IAU2006 uses a (constant) J_2 rate unlike IAU2000, as well as different values for the obliquity and the “precession constant” (rate of longitude) than those of IAU2000

That issue can be solved with corrections to the nutations:

- Capitaine et al A&A 2005 (only geometrical)
- Escapa et al A&A 2017 (geometrical + dynamical)
- Their magnitude is small, but secular mixed terms appear.
(units μas & cy)

$$\begin{aligned}(-d\Delta\psi) &= -15.6 \sin \Omega - 1.4 \cos \Omega - 0.5 \cos I_S \\ &\quad + 39.8 t \sin \Omega - 0.6 t \sin 2\Omega \\ &\quad + 3.5 t \sin (2F - 2D + 2\Omega) + 0.6 t \sin (2F + 2\Omega), \\ (-d\Delta\varepsilon) &= +0.8 \cos \Omega - 0.8 \sin \Omega - 25.1 t \cos \Omega \\ &\quad - 1.7 t \cos (2F - 2D + 2\Omega),\end{aligned}\tag{50}$$

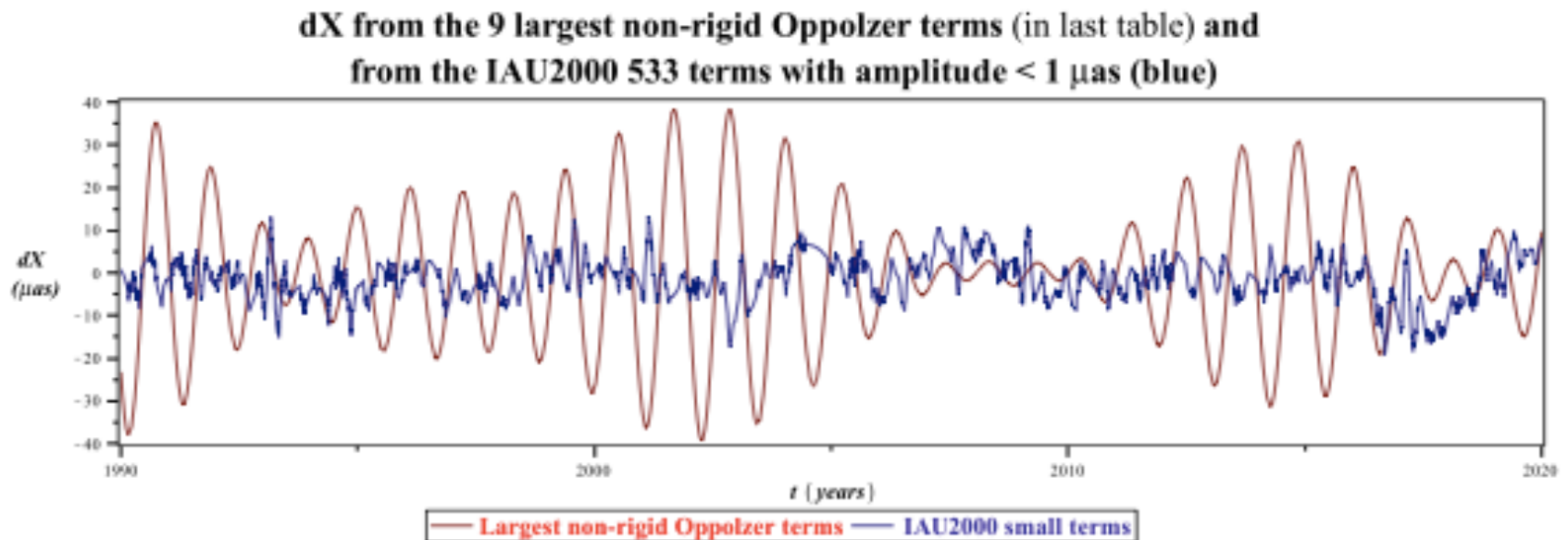
Selected Findings (4)

4. The IAU2000 nutation model is not internally consistent, because:

- A half of the nutation series (namely the 678 “lunisolar” terms) provides the nutations of the **axis of figure for a non-rigid Earth** (3-layer + oceans + dissipations at CMB)
- The other half (made of 687 “planetary” terms) provides the nutations of the **axis of the angular momentum for a rigid Earth**
- Besides, the accumulated **contribution of the main neglected planetary Oppolzer terms for the non-rigid Earth** (*e.g. only 9 of them*) is larger than that of the sum of all more than *500 minor planetary terms* of IAU2000

Selected Findings (4+)

First, we present a quick view of the accumulated effects of the largest of the missing non-rigid Oppolzer terms and the hundreds of small planetary terms of IAU2000



(Ferrandiz et al, Journées 2017)

Selected Findings (4++)

Largest Oppolzer terms of planetary origin for the Earth's figure axis.

Units: amplitudes in μas , periods in mean solar days.

L_{Ve}	L_E	L_{Ma}	L_J	p_A	Period (days)	dX_{in} (sin)	dX_{out} (cos)	dY_{in} (cos)	dY_{out} (sin)	Origin code
0	1	0	-1	0	398.884	0.2	0	0.2	0	0
2	-4	0	0	2	-487.638	-1.8	-0.1	1.7	-0.1	1
0	1	0	-2	0	439.332	1.1	-14	1.3	14.3	1
0	3	-4	0	0	418.266	-3.2	0.5	-3.1	-0.4	1
3	-4	0	0	0	416.688	-2.9	8.2	-2.9	-7.9	1
0	1	0	-1	0	398.884	-18.8	-3	-17.2	3	1
0	2	-2	0	0	389.968	-4.3	-0.4	-3.8	0.4	1
2	-4	0	0	2	-487.638	-5.6	-0.5	5.6	-0.5	2
0	1	0	-1	0	398.884	1.6	0.2	1.5	-0.2	5

(a) Effect origin code: 0 indirect Moon; 1 indirect Sun; 2 direct Venus; 5 direct Jupiter

(Ferrandiz et al A&A 2018)

Selected Findings (5)

5. The reference value of the dynamical ellipticity H_d has to be changed up to some parts-per-million

- The reason is that some minor contributions to the rate of the precession in longitude of the equator were first derived not enough accurately (*Baenas et al 2017,2019*)
- Main consequences are the change of H_d and the indirect effects on nutations (rescaling) up to almost 100 μas for some periodic components

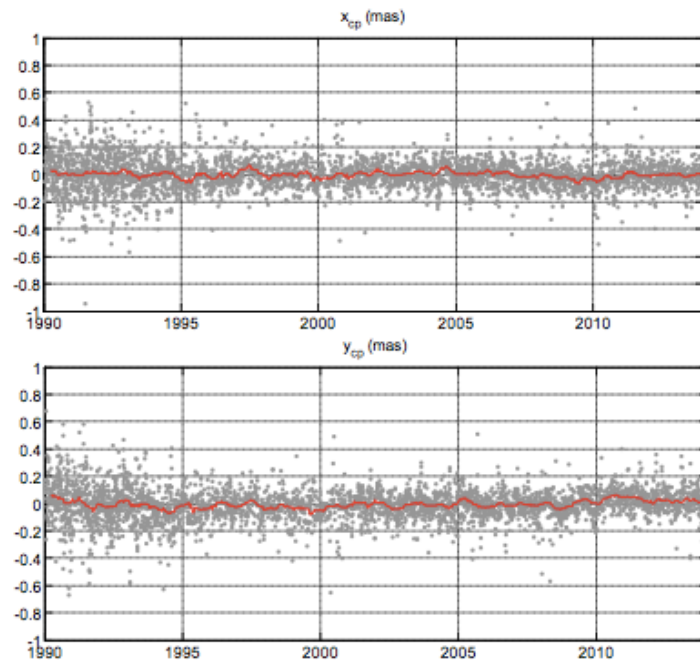
Period (days)	Indirect effects on nutation (μas)					
	$\Delta\psi$ (sin)	$\Delta\epsilon$ (cos)	$\Delta\psi$ (sin)	$\Delta\epsilon$ (cos)	$\Delta\psi$ (sin)	$\Delta\epsilon$ (cos)
−6798.36	−146.74	78.51	−29.06	15.55	−175.80	94.06
−3399.18	1.77	−0.77	0.35	−0.15	2.12	−0.92
+365.26	1.25	0.06	0.25	0.01	1.51	0.07
+182.62	−11.23	4.89	−2.22	0.97	−13.46	5.85
+13.66	−1.94	0.83	−0.38	0.17	−2.33	1.00

Selected Findings (6)

6. The use of suitable, improved FCN models allows a remarkable reduction of the unexplained CPO variance

New FCN models have been published by Malkin, Belda et al, Xu et al, and there are forthcoming (Bizouard, etc.)

Example from Heinkelmann et al (2018) at IAU GA

IAU2006/2000A+FCN⁽¹⁾ - EOP finals differences

From 1993.0 on	Shift (μ as)	Drift (μ as/yr)	Rms (μ as)
dX	1.5	- 0.6	97.7

From 1993.0 on	Shift (μ as)	Drift (μ as/yr)	Rms (μ as)
dY	- 21.1	3.0	102.2

Selected Findings (7, 8)

7. Suitable FCN models also helps to reduce the error and error growth of CPO predictions (*Ferrándiz et al 2018*)
8. The results of the determinations of all the EOPs are affected to different but detectable extents for the use of different:
 - TRFs of reference and associated elements (e.g. definitions and input data of geocenter and station motions)
 - Processing strategies in data analysis
 - Models of geophysical excitation functions (e.g. angular momentum time series)
 - Geomagnetic jerks (GMJ), etc.

Selected research in progress and advanced stage of development

A. Theories for the rotation of two- and three-layer, triaxial Earth models

The free frequencies are sensitive to the extension of the Earth model

B. Improved geophysical excitation functions are providing valuable further insight into polar motion (PM) and UT1 modeling and prediction – free frequencies, forced motion at different frequency bands, or prediction/forecasting at various time scales

Selected research topics requiring further efforts

- a. **Second order contributions to the mathematical solution for nutations – limited to a simplified two-layer Earth model**
- b. **Several models embedded in the IAU2000 theory are outdated- it was really approved as a fixed “numerical” series and the possibility of updating it by re-fitting parameters to new nutations amplitudes or switching to newer geophysical models was not envisaged**
- c. **Getting further insight into the theoretical effects of the implications of the fine details of the xTRF definitions on the determined EOP**

Selected research topics requiring further efforts

- d. Explaining better the observed EOP behavior at different time scales and improve models – decadal, fortnightly, subdaily, etc.
- e. Improvement of the Earth's interior modeling
 - evaluation of the ellipticity of the inner layers and full explanation of the observed free periods
 - modelling of CMB (core-mantle boundary) topography and its effects
 - Effects of lateral heterogeneities of the mantle
 - Modelling of the Earth's core, its electromagnetic processes and dynamical behaviour (*including e.g. changes of velocity and pressure at the CMB*), etc.

Resolution 5: Improvement of the Earth's Rotation Theories and Models

The outcomes of the IAU/IAG JWG Terv served as a basis for that IAG Resolution approved in July 2019:

(extract of the contents)

*The International Association of Geodesy,
recognizing,*

...

noting,

- The results of the IAG Commission 3 Joint Working Group on Theory of Earth and validation, joint with the International Astronomical Union (IAU) Commission A2, summarized in its 2015-2019 report ([see note 1](#)),

(Resolution 5, continues)

...

resolves,

- To encourage a prompt improvement of the Earth rotation theory regarding its accuracy, consistency, and ability to model and predict the essential EOP,
- That the definition of all the EOP, and related theories, equations, and ancillary models governing their time evolution, must be consistent with the reference frames and the resolutions, conventional models, products, and standards adopted by the IAG and its components,
- That the new models should be closer to the dynamically time-varying, actual Earth, and adaptable as much as possible to future updating of the reference frames and standards.

A few selected references

Basic materials are taken from reports of the IAU/IAG JWG TERV and its SWGs, and papers like these:

- Baenas, T., Escapa, A., Ferrándiz, J.M. (2019) Precession of the non-rigid Earth: Effect of the mass redistribution. *Astronomy & Astrophysics* 626
- Baenas, T., Ferrándiz, J.M., Escapa, A., Getino, J., Navarro, J.F. (2017) Contributions of the Elasticity to the Precession of a Two-layer Earth Model. *The Astronomical Journal*, 153
- Belda, S., Heinkelmann, R., Ferrandiz, J. M., Karbon, M., Nilsson, T., Schuh, H. (2017) An Improved Empirical Harmonic Model of the Celestial Intermediate Pole Offsets from a Global VLBI Solution. *Astronomical J*
- Belda S, Ferrandiz JM, Heinkelmann R & Schuh H. (2018) A new method to improve the prediction of the celestial pole offsets. *Scientific Reports* Bizouard, C.; Lambert, S.; Gattano, C.; Becker, O.; Richard, J. (2019) The IERS EOP 14C04 solution for Earth orientation parameters consistent with ITRF 2014. *Journal of Geodesy*, 93
- Escapa, A., Getino, J, Ferrándiz J.M., Baenas T. (2017) Dynamical adjustments in IAU 2000A nutation series arising from IAU 2006 precession. *Astron. & Astroph*
- Ferrándiz J.M., Navarro J.F., Martínez-Belda M. C., Escapa, A., Getino, J. (2018) Limitations of the IAU2000 nutation model accuracy due to the lack of Oppolzer terms of planetary origin *A&A* 618
- Gattano C., Lambert S., Bizouard C. (2016) Observation of the Earth's nutation by the VLBI: how accurate is the geophysical signal, *J. Geod.*
- Guo, Z.; Shen, W. (2019) Formulation of a Triaxial Three-Layered Earth Rotation: I. Theory and Rotational Normal Mode Solutions. [2019arXiv190110066G](https://arxiv.org/abs/2019arXiv190110066G)
- Heinkelmann, R., S. Belda, T. Nilsson, J.M. Ferrandiz, H. Schuh (2018) The consistency of TRF, CRF and EOP - a VLBI perspective. Invited presentation at the XXXth IAU General Assembly
- Malkin Z. Joint analysis of celestial pole offset and free core nutation series. *Journal of Geodesy* 91 (2017)
- Watkins, A., Yuning, F., Gross, R. (2018) Earth's Subdecadal Angular Momentum Balance from Deformation and Rotation Data. *Scientific Reports* 8:13761

Acknowledgement.-JMF, AE & JG authors were supported in part by Spanish Project AYA2016-79775-P (AEI/FEDER, UE).