



Testing the improvement of IAU precession using

different J_2 variation with time

Jia-Cheng Liu¹ (jcliu@nju.edu.cn), Nicole Capitaine², Yu-Ting Cheng¹

¹Nanjing University, CHINA ² Observatoire de Paris, FRANCE

Journées 2019 -- Astrometry, Earth Rotation and Reference systems in the Gaia era

7-9 October 2019 – PARIS, FRANCE



✓ The feature of the PO3 precession model

✓ Improvement in our 2017 paper (A&A, 597, A83)

✓ Effect of considering different J_2 long term variations

✓ Conclusion and recommendation





The IAU2006 precession

✓ Precession of the ecliptic based on the EMB position of DE406 over 2000 years

- Precession of the equator used
 - The IAU 2000 precession rates in longitude and obliquity
 - The value $\varepsilon_0 = 84381.406''$ for the mean obliquity of the ecliptic at J2000
 - The precession rates r_{ψ} , r_{ε} of non-rigid earth model

(Williams 1994; Mathews 2002)

- "observed" effect resulting from changing reference frames
- $A J_2$ rate value of $dJ_2/dt = -2.7774 \times 10^{-9} \text{ cy}^{-1}$



- ✓ Method: similar to P03 (semi-analytical)
- \checkmark Progress in observations and theories after 2003
 - Solar system Ephemerides (the accuracy of VSOP2013 is about 100 times better than VSOP87)
 - Observational material: VLBI, SLR, LLR (see Y.-T. Cheng's Poster in Session II) ...
 - Progresses in theories
- ✓ Two aspects of precession:
 - 1. Development of the precession of the ecliptic
 - 2. Integration of the precession of the equator

More tests on the J_2 long-term variation



Updating the precession of the ecliptic

- New analytical planetary theory VSOP2013 (Simon et al. 2013)
 - Chebyshev ephemerides
 - Poisson series elliptic elements
 - Secular terms of elliptic elements EMB *p*, *q* for precession of the ecliptic
- DE422 and INPOP10a ephemerides over long time interval
 - observational constrains
 - used to eliminate very long period nutation and improve the precession of the ecliptic



Results on the Precession of the ecliptic

		unit	t^1	t^2	t ³	t^4	t ⁵
	$P_{\rm A}$	"	4.19903	0.19401	-2.23533×10^{-4}	-1.03944×10^{-6}	2.15694×10^{-9}
Fit to $DE422$	$\Delta P_{\rm A}$	μ as	-65	18	1	-0.1	0.01
Fit to DE422	$Q_{\rm A}$	"	-46.81099	0.05102	5.21368×10^{-4}	-5.5808×10^{-7}	-1.2059×10^{-9}
	$\Delta Q_{\rm A}$	μ as	28	-11	-3	0.1	0.02
	P_{A}	"	4.19906	0.19400	-2.23575×10^{-4}	-1.02313×10^{-6}	2.26088×10^{-9}
Fit to INPOP10a	$\Delta P_{\rm A}$	μ as	-27	17	1	-0.1	-0.001
	$Q_{\rm A}$	"	-46.81102	0.05102	5.21248×10^{-4}	-5.64478×10^{-7}	5.5757×10^{-10}
	$\Delta Q_{\rm A}$	μ as	-10	-12	-3	0.1	0.02

Effect of using different ecliptics in developing the precession of the equator

$$\begin{split} \Delta\psi_{\rm A} &= -0.003\,t + 0.9\,t^2 + 0.02\,t^3\\ \Delta\omega_{\rm A} &= -0.4\,t^2 + 0.006\,t^3. \end{split}$$

Not very sensitive to the update of the ecliptic precession



Updating the precession of the equator

 Consideration of relativistic aspect of precession: treat geodetic precession as an additional torque (Gerlach, Klioner, Soffel 2012, arxiv:1202.5870v1)

– contribute ~ 100 μ as cy⁻¹ to precession rate in longitude

✓ Tidal poisson term contribution to precession rate in obliquity (Folgueira et al. 2007, A&A 469, 1197)

– contribute 88 μ as cy⁻¹to precession rate in obliquity

✓ Second order torque on tidal redistribution (Lambert, Mathews 2008, A&A 481, 883)

- -124 μas cy⁻¹ to precession rate in longitude (non-linear term)

 \checkmark New determination of J_2 variation from SLR observations (*)

✓ Effect from global rotation of the ICRS induced by the Galactic aberration

- About 10 uas cy⁻¹ ing precession rate in longitude



$\checkmark J_2$: gravitational degree-2 harmonic, Earth's dynamical oblateness



SLR observations (1976-2002)

$\dot{J}_2 (10^{-9} \mathrm{cy}^{-1})$	Reference
-3.0	Yoder et al. 1983
-2.6	Nerem et al. 1993
-3.0 ± 0.5	Stephenson & Morrison 1995
-3.0	adopted in the IAU 2006 precession
$+3.8\pm0.2$	Loomis et al. 2019 (2002-2019 data)



✓ Add more data (2002-2019): SLR derived C_{20} estimates





Figure credit:

https://neptune.gsfc.nasa.gov/gngphys/index.p hp?section=519



Long-term varition of the Earth's J_2 (c.f. Loomis et al. 2019)



 J_2 appears to be more quadratic than linear in nature.

 J_2 variation is an important factor in the modern precession model, but it is the main source of uncertainties (Capitaine et al. 2009).

Weighted fits of SLR J_2 observations to straight lines and parabolas.

	Time interval	t^0	t^1	t^2
Linear fit	1976-1996	$1.08263570 \times 10^{-3}$	$(-3.0 \pm 0.3) \times 10^{-9}$	_
Linear fit	1976-2011	$1.08263587 \times 10^{-3}$	$(-0.7 \pm 0.2) \times 10^{-9}$	_
Parabola fit	1976-2011	$1.08263582 \times 10^{-3}$	$(-0.5 \pm 0.2) \times 10^{-9}$	$(1.1 \pm 0.2) \times 10^{-8}$
Parabola fit	1976-2019	$1.08263579 \times 10^{-3}$	$(-0.3 \pm 0.1) \times 10^{-9}$	$(1.7 \pm 0.1) \times 10^{-8}$
Linear fit	2002-2019	$1.08263586 \times 10^{-3}$	$(+0.4 \pm 0.1) \times 10^{-9}$	_



Contribution of J_2 to precession rates

The dynamical equations for precession in equator:

$$\sin \omega_{A} \frac{d\psi_{A}}{dt} = (r_{\psi} \sin \epsilon_{A}) \cos \chi_{A} - r_{\epsilon} \sin \chi_{A}$$
$$\frac{d\omega_{A}}{dt} = r_{\epsilon} \cos \chi_{A} + (r_{\psi} \sin \epsilon_{A}) \sin \chi_{A}.$$

$$\sin\chi_{\rm A}\sin\omega_{\rm A} = P_{\rm A}\cos p_{\rm A} + Q_{\rm A}\sin p_{\rm A}$$

 $P_{\rm A}$ and $Q_{\rm A}$: precession of the ecliptic $p_{\rm A}$: general precession

Theoretical expressions for precession rate in longitude $r_{\psi} = r_0 + r_1 t + r_2 t^2 + r_3 t^3 + \cdots$

		t^2	t^3
IAU 2006	$r_{\psi}(J_2)$	-12.7 ± 3.0	0
LC17 paper	$r_{\psi}(J_2)$	-2.5 ± 1.2	$+50.6 \pm 9$
This work	$r_{\psi}(J_2)$	-1.4 ± 1.1	$+87.0 \pm 9$
J ₂ data (2002-2019)	$r_{\psi}(J_2)$	$+17.7\pm1.0$	0
No J_2	$r_{\psi}(J_2)$	0	0

Note: J_2 has no effect on the precession rate in obliquity.

The new precession of the equator is based on:

- ♦ ecliptic precession derived from VSOP2013 and DE422/INPOP10a ephemerides
- updated integration constants* (PO3 and PO4 approaches to deal with the "observed effects" and frame bias)
- Various contributions to precession rates including J_2



The solution for the main precession quantities:

- $\psi_{A} = 5038''.482041 t 1''.072687 t^{2} + 0''.0278555 t^{3} + 0''.00012342 t^{4} 0''.0000001096 t^{5}$
- $\omega_{\rm A} = \epsilon_0 0^{\prime\prime}.025754 t + 0^{\prime\prime}.0512626 t^2 0^{\prime\prime}.0077249 t^3$ $-0^{\prime\prime}.000000086 t^4 + 0^{\prime\prime}.000000221 t^5$

Precession expression ψ_A	minus t	the IAU 2	006 model.
$\Delta \psi_A$	t^1	t^2	t^3
IAU 2006	0	0	0
LC17 paper	532	5767	16847
This work	534	6320	28995
J_2 data (2002-2019)	534	15862	-5
No J_2	534	7012	-2



CPO series for precession with quadratic J_2 variation



2019/10/8



Fits of empirical models to VLBI celestial pole offsets (OPA2018a, 1979-2018) corresponding to different precession models The unit is micro-arcsecond.

Model	t^0	t^1	t^2	$\sin \Omega$	$\cos \Omega$	WRMS _{pre}	WRMS _{post}
IAU 2006	13 ± 1	382 ± 12		36 ± 1	-18 ± 1	126	116
	2 ± 1	-90 ± 23	4436 ± 184	54 ± 1	-31 ± 1	126	115
LC17 paper	9 ± 1	-210 ± 12		48 ± 1	-31 ± 1	128	115
	6 ± 1	-353 ± 23	1355 ± 184	54 ± 1	-35 ± 1	128	115
This work	9 ± 1	-330 ± 12		52 ± 1	-36 ± 1	135	115
	8 ± 1	-391 ± 23	579 ± 184	54 ± 1	-38 ± 1	135	115
Positive J_2 rate	-2 ± 1	-503 ± 12		61 ± 1	-37 ± 1	152	115
	3 ± 1	-301 ± 23	-1884 ± 184	54 ± 1	-31 ± 1	152	115
No J ₂	6 ± 1	82 ± 12		47 ± 1	-26 ± 1	152	115
	3 ± 1	-92 ± 23	1635 ± 184	54 ± 1	-31 ± 1	152	115

	Correlation	s between	fitted	parameters
--	-------------	-----------	--------	------------

			-	
Term	t^0	t^1	t^2	$\sin \Omega$
t^1	-0.7			
$\sin \Omega$	+0.0	+0.4		
$\cos\Omega$	+0.2	-0.3		-0.0
t^1	-0.1			
t^2	-0.3	-0.8		
$\sin \Omega$	-0.2	-0.3	+0.5	
$\cos \Omega$	+0.2	+0.3	-0.4	-0.2

Two functions to interpret the long wavelength curvature in the VLBI residuals:

- (1) Straight line + 18.6-year nutation
- (2) Parabola + 18.6-year nutation

We have noted the difficulty of interpreting the fit of a linear term or a parabola, plus a term with a 18.6-yr period over using only 28yr of accurate VLBI data.



- ✓ In this study we have investigated the possibility of improving the IAU 2006 precession model (Capitaine et al. 2003) by updats in our 2017 work (Liu & Capitaine 2017), with:
 - (i) introducing an empirical expression for J_2 based on the most recent and accurate determination of the J_2 variations before integrating the precession equations for the equator
 - (ii) adding 2 years of VLBI data for comparing the fits of the precession models to observations.
- ✓ A serious study of the latest results on the J_2 variations with time is very interesting and would be required before introducing a so different representation in a precession model.
- ✓ To retain IAU 2006 as standard model is a good choice before these corrections are significant and robust.



Thank you for attention!