Second-order effects in IAU2000 nutation model

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Introduction

- Second order effects considered here are of a mathematical nature
- They emerge as a consequence of a more approximate solution of differential equations

$$\ddot{u} + u = \varepsilon \left(1 - u^2\right) \dot{u} \text{ (van der Pol's)}$$
$$u\left(t;\varepsilon\right) = u_0\left(t\right) + \varepsilon u_1\left(t\right) + \varepsilon^2 u_2\left(t\right) + \cdots$$

- The names second order effect in the sense of perturbation theories, nutation-nutation coupling, crossed-nutation effect, etc. belong to this kind of contributions and are equivalent
- They are different from second order effects associated to the physical modelling of features having a small magnitude (that can be incorporated through a first order theory)

Our main objective is to explain how these terms are currently incorporated in IAU2000 nutation model and why their treatment is not consistent and incomplete

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2nd order terms in IAU2000

- IAU2000 nutation model (MHB 2002) relies mainly on the rigid Earth Hamiltonian nutation series REN2000
- A Transfer Function (TF) is applied to the rigid nutation series of the figure axis to get the non-rigid nutations —lunisolar—

$$\tilde{\eta}(\sigma) = -\frac{\tilde{m}(\sigma)}{1+\sigma}$$

$$T(\sigma; e) \equiv \frac{\tilde{\eta}(\sigma)}{\tilde{\eta}_R(\sigma)} = \frac{\tilde{m}(\sigma)}{\tilde{m}_R(\sigma)} = \frac{e-\sigma}{e}\left[M^{-1}(\sigma)\, y(\sigma)\right]_1$$

- These relations are derived assuming a first order theory of the rigid Earth, polar motion $\tilde{m}_R(\sigma)$ being proportional to the gravitational potential $\tilde{\phi}(\sigma)$—the constant of proportionality is e, equivalently H_d or k (k_{moon}, k_{sun}, Kinoshita 1977)

$$\Delta_S(J \sin l) = k \sum_{\nu} \sum_{\varepsilon=\pm 1} \frac{C_\nu(\varepsilon)}{n_g - \varepsilon N_\nu} \sin(g + l - \varepsilon \Theta_\nu)$$

$$\Delta_S(J \cos l) = k \sum_{\nu} \sum_{\varepsilon=\pm 1} \frac{C_\nu(\varepsilon)}{n_g - \varepsilon N_\nu} \cos(g + l - \varepsilon \Theta_\nu)$$

$$\tilde{m}_R(\sigma) = \frac{e}{e-\sigma}\tilde{\phi}(\sigma)$$

2nd order terms in REN2000

- The second order terms constructed in REN2000 (Souchay et al. 1999) are given for the nutations of the angular momentum axis (Poisson terms)
- They consider two kind of second order effects:
  - Crossed-nutation: the influence of the nutation itself on the torque exerted by the moon and the sun (the most important, intrinsically associated to the rotation)
  - Spin-orbit coupling: interaction between the orbital motion of the moon and the J_2 component of the Earth geopotential (mainly related with the way in which ELP-2000 —Chapront-Touzé & Chapront 1988 — is constructed)
- As it can be seen in their analytical formulation, they depend on the orbital characteristic of the perturbers
- The only way in which the Earth structure enters into these expressions is through to H_d^2 —no proper modes dependency, like the Eulerian frequency.

2nd order terms in REN2000

- This can be neatly appreciated from some REN2000 formulae

$$\Delta\psi_{\mathrm{cr}}^{W2} = -\Delta h = -\frac{\partial W_2^{\mathrm{cr}}}{\partial H} = \left[\frac{1}{G\sin I}\right]\frac{\partial W_2^{\mathrm{cr}}}{\partial I} \propto H_d^2$$

$$W_2^{\mathrm{cr.}} = \frac{1}{2}\int [\mathbf{A}]\mathrm{d}t \tag{6}$$

with:

$$[\mathbf{A}] = (\sin I\cos^2 I)A_1 + \left(\frac{\sin^2 I\cos I}{2}\right)A_2 + (\cos I\cos 2I)A_3 + \left(\frac{\sin I\cos 2I}{2}\right)A_4 + \left(\frac{\cos^2 I\sin I}{2}\right)A_5 + \left(\frac{\sin^2 I\cos I}{4}\right)A_6 \tag{7}$$

and:

$$A_1 = B_1^{\mathrm{M}} \times (C_0^{\mathrm{M}} + C_0^{\mathrm{S}}) - C_1^{\mathrm{M}} \times (B_0^{\mathrm{M}} + B_0^{\mathrm{S}}) \tag{8.1}$$

$$B_1^{\mathrm{M}} = k_{\mathrm{M}}\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^3 \sin\beta_{\mathrm{M}}\cos\beta_{\mathrm{M}}\cos(\lambda_{\mathrm{M}} - h) \qquad C_0^{\mathrm{M}} = \int B_0^{\mathrm{M}}\mathrm{d}t \qquad k_M = H_d c_M$$

$$B_0^{\mathrm{S}} = -\left(\frac{k_{\mathrm{S}}}{2}\right)\left(\frac{a_{\mathrm{S}}}{r_{\mathrm{S}}}\right)^3 \qquad C_0^{\mathrm{S}} = \int B_0^{\mathrm{S}}\mathrm{d}t \qquad k_S = H_d c_S$$

2nd order terms in IAU2000

- The Transfer Function (TF) is applied to the whole rigid nutation (first and second order)
- If we focus in the second order part, we face several problems:
  - First order TF cannot be applied to second order terms, since $\tilde{m}_R(\sigma)$ is proportional to e^2, as it is shown in Getino et al. (2010)
  - Even if it were correct, it cannot be applied to REN 2000 second order terms, because they do not depend on Earth structure (not consistent)
  - They just require a scaling of the form $H_d^2/H_{\mathrm{R}d}^2$, to take into account the changes in the dynamical ellipticity of the model
- IAU2000 lacks from the effect of Earth's structure on the second order terms, simply because it is the case of REN2000 —for the rigid Earth they are very small (Getino et al. 2010)— (incomplete)
  - 2nd order Poisson terms: a part is absent
  - 2nd order Oppolzer terms : all is absent

2nd order terms in IAU2000

Second order terms	REN2000	IAU2000
Poisson	Present	Incorrect modeled (**not consistent**)
	Absent	Absent (**incomplete**)
Oppolzer	Absent	Absent (**incomplete**)

Independent on Earth structure (just H_d^2)

Dependent on Earth structure (normal modes)

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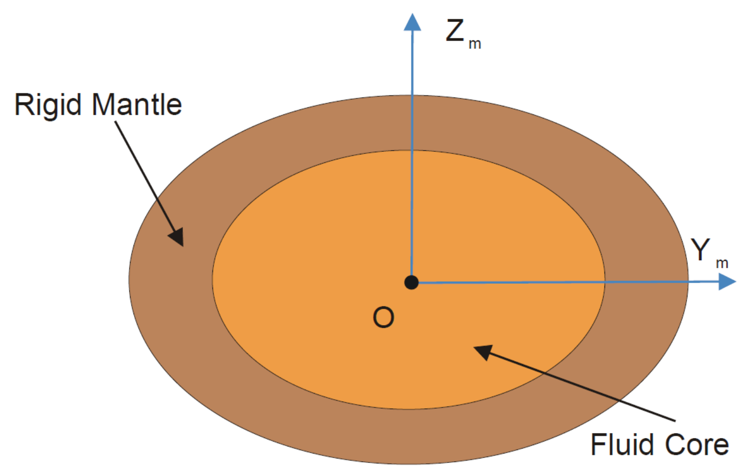
Poincaré Earth model

- We are going to determine the influence of the Earth model (normal modes) in second order Poisson terms
- We will follow a Hamitonian approach, since this formalism is naturally fitted to construct analytical approximate solutions of the second order by means of perturbation theories
- Even with the use of symbolic software, the procedure is quite cumbersome, hence we start by considering those contributions for a Poincaré Earth model (Getino et al. 2019)



Poincaré Earth model

Poincaré Earth model

- The general structure of these nutations are

$$\Delta_2\lambda = H_d^2 \sum_{p,q} c_p c_q \left[\sum_{\substack{i_p \neq 0,\, j_q \neq 0 \\ \tau,\, \rho = \pm 1}} \mathcal{L}^a_{i_p,\, j_q,\, \tau,\, \rho} + \sum_{\substack{i_p,\, j_q \\ \tau,\, \rho = \pm 1}} \mathcal{L}^b_{i_p,\, j_q,\, \tau,\, \rho} \right] \sin\left(\tau\Theta_{i_p} - \rho\Theta_{j_q}\right)$$

$$\Delta_2 I = H_d^2 \sum_{p,q} c_p c_q \left[\sum_{\substack{i_p \neq 0,\, j_q \neq 0 \\ \tau,\, \rho = \pm 1}} \mathcal{O}^a_{i_p,\, j_q,\, \tau,\, \rho} + \sum_{\substack{i_p,\, j_q \\ \tau,\, \rho = \pm 1}} \mathcal{O}^b_{i_p,\, j_q,\, \tau,\, \rho} \right] \cos\left(\tau\Theta_{i_p} - \rho\Theta_{j_q}\right)$$

- There are two kinds of amplitudes:
  - Independent of the Earth model —but H_d^2: superscript a

$$\text{Example: } \frac{1}{8} \frac{\tau m_{5i}}{\tau n_i - \rho n_j} \left(\frac{1}{\tau n_i} + \frac{1}{\rho n_j} \right) \left(\tau m_{5i} B_i B'_j + \rho m_{5j} B'_i B_j\right)$$

  - Dependent on the Earth model: superscript b

$$\text{Example : } \frac{\sin I}{2} \frac{1}{(\tau n_i - \rho n_j)} \frac{\omega_E - \tau n_i - r_3}{\prod_{k=1,2} (\omega_E - \tau n_i - \sigma_k)} \left(C'_{i,\tau} C_{j,\rho} + C_{i,\tau} C'_{j,\rho}\right)$$

$$r_3 = \omega_E \left(1 + r_{cm}\right)\left(1 + e_c\right), \sigma_{1,2} \longrightarrow \text{normal modes}$$

Poincaré Earth model

- For a particular parameters set of a Poincaré model, we have evaluated the former formulas, recovering as a limiting case the rigid values of REN (also Getino et al. 2010)
- We have also reproduced (not displayed) the second order contribution to the precession rate (Ferrándiz et al. 2004)
- Some of the numerical differences are relevant with current accuracy threshold (amplification at 182.62 days)

Second order Poisson terms: In–phase (Unit=μas)

Argument					Period	Poincaré Earth		Rigid Earth		Difference	
l_M	l_S	F	D	Ω	(days)	Lon.	Obl.	Lon.	Obl.	Lon.	Obl.
0	0	0	0	1	-6798.36	-27.2	72.0	-30.1	30.0	2.9	42.0
0	0	0	0	2	-3399.18	-1209.0	234.5	-1212.6	236.4	3.6	-1.9
0	1	0	0	0	365.26	0.4	-0.9	1.1	-0.1	-0.7	-0.8
0	0	2	-2	2	182.62	-7.4	3.7	-0.3	0.1	-7.2	3.9
0	0	2	-2	1	177.84	91.9	-72.5	92.6	-73.0	-0.8	0.6
0	0	2	0	2	13.66	-5.7	1.4	-4.9	1.0	-0.9	0.6

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Summary

- With respect to second order terms in the sense of perturbation theories, current IAU2000 nutation model:
  - Models incorrectly second order Poisson terms of REN (a few μas)
  - Lacks the influence of the Earth structure (normal modes) in Poisson terms and Oppolzer terms (not present in REN2000)
  - Even having the whole second order rigid part, obtaining the non-rigid contributions is not direct because current MHB2002 TF cannot be used
- Hamiltonian approach provides a suitable framework to derived the second order nutations of a non-rigid Earth
- For Poincaré model, we have shown that at the second order Poisson terms are affected by the Earth structure with non-negligible amplitudes
- This approach must be extended to compute Oppolzer terms and applied to other more realistic Earth models

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