ON THE VARIABILITY OF THE CHANDLER WOBBLE

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ABSTRACT. The works that we carried out during the last ten years lead to significant progress in the knowledge of the Chandler wobble (CW) of the Earth polar motion (PM). In Zotov and Bizouard (2012) we reconstructed the excitation of this resonant mode by using Panteleev's corrective filtering. Now we are sure, that this filtering is a regularizing algorithm as far as its parameters are consistently selected with the uncertainty affecting the resonance parameters and the observations. The excitation demonstrated a quasi 20-year amplitude modulation. In the framework of the first order differential linear equation describing the polar motion, it is easy to show that this modulation accounts for the 40-year change of the CW amplitude as well as the splitting of the CW spectra. A simple model of the CW, composed of 80 and 40-year harmonics, accounts for the present minimum in Chandler wobble amplitude, like in 1930s, and also predicts that its amplitude will start to increase in the nearest future with a phase shift of π. On the other hand, geodetic excitation of the CW well matches the ocean-atmospheric excitation (Bizouard, 2020) over recent 50 years with a dominant role of the ocean, producing the 20-year modulation. Thus, the physical cause of it could stem from climatic or tidal process influencing the oceanic circulation.

1. INTRODUCTION

What is the Chandler wobble? For answering this questions two centuries of theoretical and observational studies should be matched. Euler published equations of the rigid body precession in 1765. Given the polar flattening of the Earth, the period of its free precession if it is assumed rigid would be 305 days. Chandler published his discovery in 1891, succeeding the series of attempts of the XIX-century astronomers to find the motion of latitudes. The two main wobbles found by Chandler had periods around 435 and 365 days. Nobody considered the annual component as the free Euler wobble, because it was attributed to mass redistribution in the Earth system, that are prominent at seasonal time scale. Therefore the Euler free motion was linked with the unexpected 435 day wobble, now known as a Chandler wobble. The mismatch of 435-305=135 days had to be explained. Earlier works of Liouville (1858) allowed to write the homogeneous Euler equations for the Earth, whose tensor of inertia undergoes small changes. That allowed to incorporate small changes of the Earth inertia tensor and, after linearisation to move them to the right-hand side of equations as the so-called excitations. In the commonly-used form of Euler-Liouville equations:

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt} + p(t) = \chi(t) \tag{1}$$

complex polar motion $p = p_1 + ip_2 \approx x_p - iy_p$ can be obtained from the observed pole coordinates x_p, y_p. Input χ is the excitation. By σ_c we denote the frequency of resonance.

The equations were still required to be adjusted to the observed frequency. The works of Newcomb (1892), helped to bring it closer to the observed one, 435 (more precise 433) days, by introduction of the elastic response of the Earth and oceans, trying to adapt their shape to the changed rotational axis position. The corresponding polar tide, raised by the additional centrifugal potential, as a kind of back-propagation effect, formalised by Love (1909) numbers, allowes to adjust the theoretical period of the free mode to about 482 days, as we can calculate now (Bizouard, 2020, Bizouard C., Zotov L., 2013). To reduce it closer to 433 days the theory should take into account the Earth's core.

The rotation theory of the Earth modelled as a solid ellipsoidal shell containing a fluid core was treated by Poincare (1910), and some results were obtained by Zhukovsky (1904). This theory was further developed by Molodensky (1961), Sasao et al. (1980), Mathews at al. (1991). Contemporary theory includes outer core stratification and inner solid core, see Dehant and Mathews (2015) book. Accounting for the electro-magnetic coupling, triaxiality and other second-order effects is also possible, but it does not change the Chandler period by more than several days (Guo and Shen, 2020), though it helps to build precise theory of precession and nutation.

Here we should briefly remind the reader, that the precession and nutation, as the solution of non-homogeneous Euler equations originate from the astronomical tides causing an external momentum of force on the Earth. On the contrary, the polar motion (PM), including the free wobble, is commonly treated as the homogeneous solution of eq. (1) resulting of geophysical excitations only. But it is quite difficult to separate astronomical and geophysical causes. For example, Earth geophysical properties define the eigen-frequencies of the transfer function, by which the amplitudes of solid-Earth nutations are multiplied. PM is also not free from the astronomically-induced changes. To make notions more distinctive, the precession-nutation was conventionally separated from the polar motion by a frequency criterium. So the first one is inside the band [-0.5, 0.5] cycles per day in celestial reference frame (CRF), while the PM is outside it.

Going back to the main question – why the Chandler wobble, theoretically the eigen mode, adjusted to the observational 433 days by implementation of nonrigid effects, is quite enigmatic, – we face the problem of its changing amplitude and phase at decadal time scale. It is well seen in Figure 1 up, where CW is shown, extracted from EOP C01 PM time series, covering 1846-2020 interval with 0.05 year sampling (the 0.1 year original sampling before 1900 required an interpolation) by Panteleev's filtering locating resonance at 0.843 cycles per year.

It was already proposed by Kant (1754) that the Earth decelerates because of tidal energy dissipation. Dissipative properties were also introduced through the imaginary parts of rheological coefficients into the Chandler wobble frequency. The complex Chandler frequency $\sigma_c = 2\pi f_c(1 + i/2Q)$ in eq. (1) now incorporates real frequency f_c and quality factor Q. The characteristic time of dissipation, estimated from observations themselves, is about 50-100 years. Not going into details the Q value estimates vary from tenth to hundreds. Some authors suppose, that the intervals where CW decreases rapidly, correspond to its free decay, while other scientits, estimating the input excitations χ from geophysical data, try to adjust the parameters of the eq. (1), given the observed output p. All the methods have their drawbacks and are based on the idea, posed already by Jeffreys (1916), that the energy is somehow supplied to maintain CW. Thus, the "free" mode, thanks to dissipation, becomes actually not free. And not only the adjustment of the observable frequency and quality factor, but also the search of the energy source is required.

R. Gross, A. Brzezinsky and their followers found the source of CW excitation in the ocean and atmosphere. It is done by means of cross-correlation and cross-coherence analysis of the so-called geophysical excitation with the geodetic excitation, computed from polar motion. To do this, much affords were undertaken to collect geophysical data over the ocean and atmosphere and run global circulation models, assimilating observation. The angular momentum functions χ, representing geophysical excitations, obtained from such reanalysis, are used, assuming that the ocean-atmosphere-solid Earth system is closed and no external momentum is supplied. In result, the excitation budget at Chandler frequency was closed by the ocean and, atmosphere, whereas the hydrological input seems to be negligible for now. The amplitude and phase of excitations found to be in good agreement, explaining the energy supply for the Chandler wobble. But the reliable geophysical excitations are available only since 1950s for atmosphere and since the 1990s for the

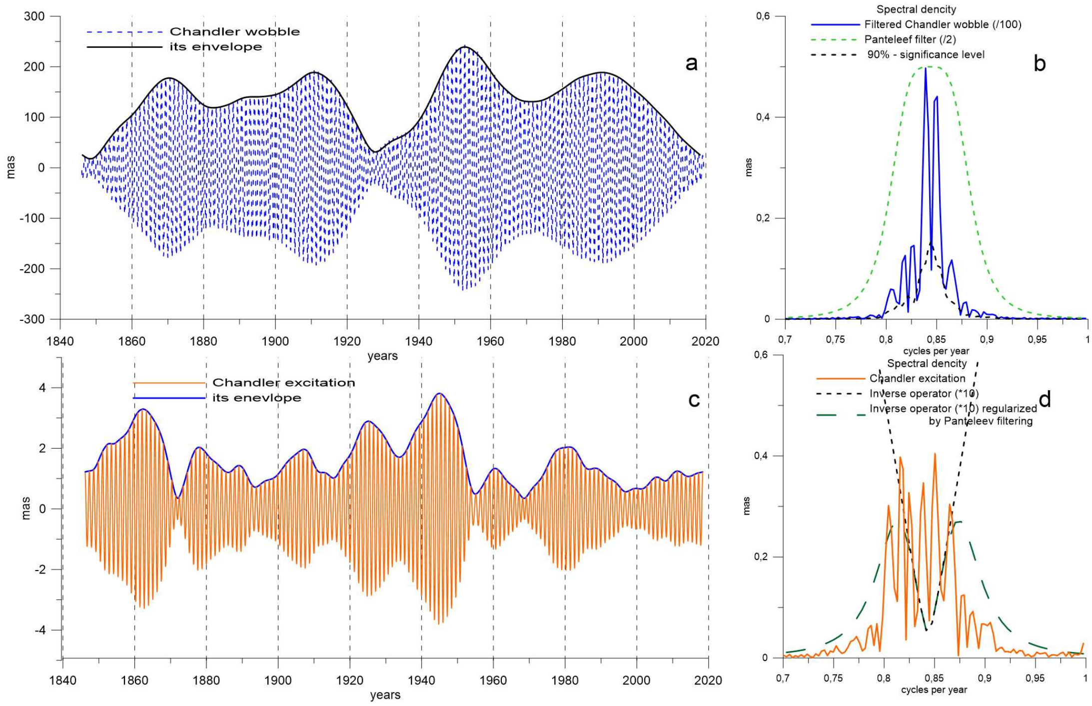

Figure 1: a) Chandler wobble extracted by the Panteleev filter, b) its spectra, c) Chandler excitation, and d) its spectra with superimposed inverse operator (black) and its regularized version (green) (Zotov, Bizouard, 2015).

ocean. The major amplitude changes of the CW, observed over more than a century, including decay happened in the 1930s, were not covered. To overcome this difficulty, B. Chao (2012) run modelling covering several hundreds of years pointing out, that the amplitude changes of CW can result from the integration of random white noises and occur occasionally over centuries. Still what we observe for the CW today was not predicted by any model. And we cannot be fully satisfied with a general statistical hypothesis, saying that noises in the atmospheric and oceanic excitations are responsible for modulations of the Chandler wobble observed in Figure 1a.

2. VARIATIONS OF CW AND ITS EXCITATION

Looking at the Chandler wobble spectrum, Figure 1b, obtained through the Fourier transform of the auto-covariance function for the EOP C01 time interval, we see, that Chandler wobble pike is splitted and has multiple side-harmonics. This reflects the to CW amplitude modulations. We consider here only the prograde Chandler band, not accounting for ellipticity and effects of triaxiality and not discussing different ways of filtering, developed by Gross, Malkin, Wang, Chao and others. We just use Panteleev's narrow-band filter (Zotov, Bizouard, 2012). The amplitude changes of the Chandler wobble, emphasised in Figure 1a by the solid-line envelope, reveal some regularities, which can be reduced to 80- and 40-year quasi-periodical changes. Many sceptical scientists pose question – to what extent can we trust observations of the 19-th and early 20-th century? Taking into account drastic change of precision starting from 0.1 arcsec and reaching tenth of microseconds today, the question is reasonable. But similar increase of observational precision happens in many fields of science. Does it mean we should through away old observations? Of cause not!. In least squares procedure the weights, inversely proportional to squared standard error and changed by 1000 times, would make recent observations million times more influential, what would pull the adjustment to the recent 30 years. But the simple regularisation of weights while estimating amplitudes and phases of 80 and 40-year harmonics in envelope gives results not very different form the unweighed non-linear least squares. Based on these results we predicted the increase of the CW amplitude in the decade 2020-2030 after the present decrease with possible phase jump by π, as in the 1930s.

A simple simulation, with CW harmonic modelled by 150 mas, annual harmonic by 100 mas amplitude oscillation, and white noises of $a_n = 10, 30, 60, 90, 120$ mas variance, result in standard errors of CW extraction by Panteleev's filter $\sigma = 2.4, 2.6, 4.5, 6.4, 8.0$ mas, correspondingly. That means that the difference between simulated CW and that, extracted from the mix of signals, is less than 8% of the total CW amplitude.

So the obtained results, Figure 1a and b can be accepted as a matter of fact. An investigation comparing the geodetic and geophysical excitations on the left and right hand sides of Euler-Liouville equation (1) requires the estimation of the geodetic excitation, which is an inverse problem. Differentiation of observations p can amplify the noises and real cause of CW will be lost in errors. To regularize inverse problem we should get rid of annual oscillation and noises, what actually is done by implementation of the Panteleev filter. The fact, that its frequency transfer band is very narrow, requires the filter-window to be long in time and causes border effects of 20 years. To compare geodetic and geophysical excitations we can use only time interval of mutual data availability, let's say since 1940th for ECCO oceanic (OAM) and NCEP atmospheric angular momentum (AAM) χ. Cutting out 20 years from the edges, or developing the techniques of edge-effects reduction, we would have only about four decades for intercomparision, 1960-2000. Comparison made in Zotov, Bizouard (2016) revealed that the ocean and atmosphere together can explain more than 90% of CW excitation almost on equal share, in agreement with other published results.

But can the ocean and atmosphere be responsible for 80 and 40-year quasi-regular variations in the CW amplitude? The preliminarily analysis made in Zotov, Bizouard (2016), over 1960-2000 interval showed, that the ocean could be responsible.

We did reconstruction of geodetic excitation χ for CW in Zotov et al. (2016) and obtained results shown in Figure 1c. Initially our attention was attracted by quasy 20-year regular envelope, more or less in phase with 18-year lunar nodes precession. As further study revealed, the observed 20-year excitation amplitude changes can be explained by the 40-year CW amplitude changes (Zotov, 2019) superimposed on the mean amplitude of CW, assuming constant parameters of equation (1). Their cause can be interpreted as an alternation of 20-years periods of CW amplitude increase and 20-years periods of amplitude decrease. A 80-year variations also present in excitation amplitude, but they are less prominent and appears only if more narrow-band filter is applied. A frequency-dependent phase shift between excitation's and CW's envelops occures, since the first one depends on the derivative of the letter, eq. (1).

The excitation of 2-3 mas, provides Chandler wobble of almost 200-mas. It agrees with the resonant amplification of input, but makes the inverse reconstruction and research of the Chandler excitation very complicated. Can we estimate the error in geodetic excitation, introduced by noises, narrow-band filtering and operator inversion? Theory of inverse problems solutions states that such an error can not be accessed. It depends on our a-priory knowledge: errors of observations and operator, eq. (1), stability of its parameters f_c, Q. The only way to give an approximate error of inverse solutions is to estimate the diameter of the set, they are selected from. Shortly speaking, if a-priory knowledge was wrong and our filter was improperly designed, the solution could be wrong. But if we put some reasonable constraints on the noises and operator errors, we can estimate the error in the solution through the spread of the worst scenarios. From such simulations we testified our Panteleev's filter, finding its width 0.04 years^{-1} optimal, for the CW excitation error the upper limit obtained is 1 mas.

3. POSSIBLE CAUSES OF VARIABILITY

Since we still want to find the cause of CW amplitude changes and its spectral line split, we allow ourself to speculate, trying to choose the most probable pathway for further research. So we assume, that quasy 80 and 40-years variations of CW amplitude really exist in 19-20 cnt., and at the stage of CW filtering the noises in observations were treated properly.

It seems that our contemporary knowledge of resonant period is correct since the theoretical CW eigenfrequency is now consistent with observations, Love numbers, precession and nutation model, etc. But what if we still miss-interpret 433-day wobble? Could it result from input at another frequency interplay with resonance at not yet quite known frequency? Linear theory says, that input and output should be at the same frequency, and it is the simplest assumption. The hypothesis of double Chandler resonance, which could split the pike, drifting parameters, and so on, make the model too complicated and contradicts the Occam razor principle. In fact, there is no notable spectral pike in OAM and AAM at Chandler frequency. We do not know any other geophysical process at 433-day frequency. The only planetary process, whose frequency is close to 433 days, was mentioned by Yu. Avsyuk (1996): it is a 412-day period of the full moon in perigee. This important periodic event, called "Big Moon" has astronomical origin, it modulates the tides, but we do not have evident spectral line in geophysical processes at this frequency. It might be in result of improper selection of reference system for the consideration of modulations. But since we deal with PM, related to geophysical causes, CW in particular, we try to look for small excitations hidden in noises of oceanic and atmospheric variability and suppose, that they are sufficient. So, low-frequency astronomical processes become out of scope. But can the processes on Earth be really so radically isolated?

Let us think about the problem in the framework of mechanics of several oscillators, where, if they are coupled together and synchronized, the changes of amplitude of particular oscillator is often observed in result of energy transfer from one pendulum to another. We can treat multiple Chandler wobble spectral pikes and side-lobes as an effect of such multiple oscillators, providing modulations. In Fig 1d we can observe spectral lines not only at 1/433, but also at 1/450, 1/440, 1/410 cycles per day, altogether responsible for 80-year, 40-year and 20-year modulations of excitation amplitude. The blue dashed line of 90% confidence level shows in Figure 1b, that the majority of these spectral lines are quite real. What if some of them appears at one interval of time, others – at another, then Fourier analysis would not be a proper technique. Wavelet and time-frequency analysis in a moving window do not alter the observed peaks. The average CW period seems to be stable. The only phase jump by π happened in 1930s during the amplitude minima can be modelled by a simple decay of the wobble amplitude with the envelope sign changing from "positive decrease" to "negative increase". If we would try to imagine, that the input excitation frequency crosses the eigen-frequency, let's say from left to right, then the phase jump would happen in result of phase response at resonance, but at the same time, the amplitude of response should increase, what is not observed. Thus, this phase-shift even remains unexplained.

Going back to mutual oscillators and synchronisation hypothesis, we need to identify them. Probably the correct way is to look for processes, which could interact and synchronise each other in the Earth shells and surrounding space not at the periods close to the CW carrying 433-day frequency or its side-lobes, but at the periods of CW envelops.

In this framework all the variations, including climatic modes, like El Niño, 35-year meteorological period, related with the beating between lunar (355 days) and solar year, 412-day period of the 'Big Moon' should not be ignored.

We expect critics, saying that the old "astrological" methods has nothing in common with contemporary physics. But we would only mention, that the properly chosen "reference system" has been always the crucial point in astronomy and mechanics. We also know many examples, when physical models and eigenfrequencies of multi-dimensional systems of differential equations were adjusted, to satisfy evident synchronisations and to explain what we observe in reality. On this way of combination of intuition, precise observations, new evidences uncovered from noises, and novel mathematical modelling, we sooner or later, will solve the detective story of Chandler wobble variability.

4. CONCLUSIONS

Despite the great progress of precession/nutation theory and its agreement with observations up to the 0.1 milliseconds order, the modelling of polar motion and its prediction remain unsatisfactory. Complex system of ocean, atmosphere, mantle and Earth interior shells, involving a great diversity of long and short-term geophysical processes, makes this field of scientific research comprehensive and less developed. Though PM parameters are needed for matrix transformations from celestial to terrestrial system as well as nutation parameters, being precisely estimated from GPS, VLBI and laser observations, they are hardly predictable. Chandler wobble, changing its amplitude from tens to hundreds of mas, introduces the major uncertainty into the polar motion predictions for horizons larger than one year. Explanation of its changes are crucial for geophysics and applications, including navigation, but requires a deep understanding of meteorological, oceanographic, climatological processes and global planetary geophysics.

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