RECENT IMPROVEMENTS IN THE DETERMINATION OF TIME TRANSFER FUNCTIONS

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ABSTRACT. A few years ago, a new approach allowed to determine in an exact form the time transfer functions in Schwarzschild-like spacetimes within the weak-field, linearized approximation. We give a brief survey of the main results thus obtained and we indicate how the new procedure can be used to compute the contributions to the travel time of light rays due to the mass and spin multipoles of a rotating axisymmetric body.

1. INTRODUCTION

In the area of highly precise astrometry as well in a lot of experiments performed to test the metric theories of gravity in the Solar System, it is of crucial interest to know the travel time of a light ray between an emitter A and a receiver B as a function of the position of the emitter, the position of the receiver and the instant of reception. We call such a function "a reception time transfer function". To be more precise, consider a light ray propagating through a vacuum in a region of spacetime covered by a coordinate system $x^\mu = (x^0, \boldsymbol{x})$, with $x^0 = ct$ and $\boldsymbol{x} = (x^i)$, $i = 1, 2, 3$, t being supposed to have the dimension of a time, and the x^i the dimension of a length for the sake of simplicity. Denote by $(x_A^0, \boldsymbol{x}_A)$ the point-event where the ray is emitted and by $(x_B^0, \boldsymbol{x}_B)$ the point-event where it is observed. The light ray joining $(x_A^0, \boldsymbol{x}_A)$ and $(x_B^0, \boldsymbol{x}_B)$ is a null geodesic of spacetime, which implies that the coordinate light travel time $(x_B^0 - x_A^0)/c = t_B - t_A$ is a function of $\boldsymbol{x}_A$, $\boldsymbol{x}_B$ and t_B, so that we can write

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B). \tag{1}$$

Knowing the reception time transfer function (TTF) $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B)$ associated with a light ray enables us to model the time delay and the Doppler tracking along this ray. It also enables us to compute the propagation direction of the ray at the point of observation (Le Poncin-Lafitte et al 2004), which explains the relevance of the notion of TTF in relativistic astrometry (see, e.g., Hees et al 2014, Bertone et al 2017). Nevertheless, in spite of a large amount of works, the explicit computation of all the possible TTFs in a given spacetime remains an unsolved problem, even in the special case of static, spherically symmetric spacetimes. Only partial results have been obtained, using several methods which are in fact equivalent. We summarize here the procedure presented in Teyssandier and Le Poncin-Lafitte 2008.

It is supposed that the metric of spacetime may be expanded in a power series of the Newtonian gravitational constant G as follows:

$$g_{\mu\nu}(x; G) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} G^n g_{\mu\nu}^{(n)}(x), \tag{2}$$

where $\eta_{\mu\nu}$ is the Minkowski metric: $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The procedure is based on the assumption that there exists at least a null geodesic linking the emitter and the receiver such that the corresponding reception TTF can be expanded in a power series of G as follows:

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B; G) = \frac{R_{AB}}{c} + \sum_{n=1}^{\infty} G^n \mathcal{T}_r^{(n)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B), \tag{3}$$

where R_{AB} is the 'Euclidean' distance between the positions $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$, that is

$$R_{AB} = |\boldsymbol{x}_B - \boldsymbol{x}_A| = \left[(x_B^1 - x_A^1)^2 + (x_B^2 - x_A^2)^2 + (x_B^3 - x_A^3)^2\right]^{1/2}. \tag{4}$$

Under these assumptions, it may be shown that each perturbation term $\mathcal{T}_r^{(n)}$ is given by an integral involving only the metric perturbations $g_{\mu\nu}^{(1)}, \dots, g_{\mu\nu}^{(n)}$ and the functions $\mathcal{T}_r^{(1)}, \dots, \mathcal{T}_r^{(n-1)}$ taken on a Minkowskian null straight line passing through the spatial positions $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$.

However, two major problems are raised by the iterative procedure outlined above. First, the method yields a single TTF, thus excluding the possibility to model the gravitational lensing, where multiple images appear. Moreover, the perturbation functions $\mathcal{T}_r^{(n)}$ involve 'enhanced terms', namely terms which are infinite for some positions of the emitter and the receiver (for the pioneer work, see Klioner and Zschocke 2010; see also Teyssandier 2012). An example of this pathology is furnished by the TTF calculated with this procedure in a static spherically symmetric spacetime, as it may be seen in the next section.

2 CASE OF STATIC SPHERICALLY SYMMETRIC SPACETIMES

For an isolated spherically symmetric body of mass M, the coordinates $(x^0, \boldsymbol{x})$ may be chosen so that the metric takes the form

$$ds^2 = \left(1 - \frac{2m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \cdots\right)(dx^0)^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\gamma_2\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \cdots\right)\boldsymbol{dx}^2, \tag{5}$$

where $m = GM/c^2$, $r = |\boldsymbol{x}| = [\sum_{i=1}^3 (x^i)^2]^{1/2}$, $\boldsymbol{dx}^2 = \sum_{i=1}^3 (dx^i)^2$, and β, β_3, γ, γ_2, γ_3 are post-Newtonian parameters equal to 1 in general relativity.

Owing to the static character of spacetime, each TTF for the metric (**??**) is independent of the reception time. So we shall henceforth omit the index r standing for 'reception'. The procedure outlined in the Introduction yields a TTF as follows (for the terms in m^2, see Le Poncin-Lafitte et al 2004 and Klioner and Zschocke 2010; for the terms in m^3, see Linet and Teyssandier 2013):

$$\begin{aligned}\mathcal{T}_{\mathrm{Spher}}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \mathcal{T}_{\mathrm{Shap}}(\boldsymbol{x}_A, \boldsymbol{x}_B) - \frac{m^2 R_{AB}}{c r_A r_B}\left[\frac{(\gamma+1)^2}{1+\cos\psi_{AB}} - \kappa_2\frac{\psi_{AB}}{\sin\psi_{AB}}\right] \\ + \frac{m^3(r_A + r_B)R_{AB}}{c r_A^2 r_B^2 (1+\cos\psi_{AB})}\left[\frac{(\gamma+1)^3}{1+\cos\psi_{AB}} - \kappa_2(\gamma+1)\frac{\psi_{AB}}{\sin\psi_{AB}} + \kappa_3\right] + \mathcal{O}(m^4),\end{aligned} \tag{6}$$

where $r_A = |\boldsymbol{x}_A|$, $r_B = |\boldsymbol{x}_B|$, R_{AB} is defined by (**??**), ψ_{AB} is the measure of the angle between $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ laid down by

$$\cos\psi_{AB} = \frac{\boldsymbol{x}_A}{r_A} \cdot \frac{\boldsymbol{x}_B}{r_B}, \qquad 0 \le \psi_{AB} \le \pi, \tag{7}$$

$\mathcal{T}_{\mathrm{Shap}}$ is the Shapiro time delay, namely (see, e.g., Blanchet et al 2001 and refs. therein)

$$\mathcal{T}_{\mathrm{Shap}}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{(\gamma+1)m}{c}\ln\left(\frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}}\right) \tag{8}$$

and κ_2 and κ_3 are constants defined by

$$\kappa_2 = 2(\gamma+1) - \beta + \frac{3}{4}\gamma_2, \tag{9}$$

$$\kappa_3 = 4(\gamma - \beta + 1) - 2\beta\gamma + \frac{3}{2}\gamma_2 + \frac{1}{4}(3\beta_3 + \gamma_3). \tag{10}$$

An enhanced effect is ensured since each term in Eq. (**??**) – except R_{AB} – tends to infinity when $\psi_{AB} \to \pi$. This divergent behaviour is not surprising. If ψ_{AB} is sufficiently close to π, the straight line joining $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ is passing through a region of strong field. Consequently, a convergent expansion in a series of powers of G cannot be reasonably expected for a TTF. Clearly, the second assumption is not realist in this case. Moreover, it is obvious that if r_A and r_B are sufficiently large and ψ_{AB} is sufficiently close to π, there exist at least two distinct light rays joining the emitter and the receiver and confined to the zone of weak field: this means that the usual theory of the time transfer functions does not work in a configuration of gravitational lensing. So, the theory summarized in the Introduction must be revised.

3. EXACT SOLUTIONS FOR LINEARIZED SPHERICALLY SYMMETRIC METRICS

3.1 The complete set of time transfer functions

In order to overcome the difficulties raised in the previous section, we can reason differently. The dominant divergent terms in Eq. (**??**) are manifestly linked to the linear part of the metric since they only involve the post-Newtonian parameter γ. So, in a first approach, we may be content with treating the problem in the weak-field, linearized approximation provided that we use the rigorous solutions to the null geodesic equations. This program has been successfully carried through for the Schwarzschild-like metrics in Linet and Teyssandier 2016. The main results of this paper are summarized in this section.

Since the terms of order m^2/r^2 in the metric are ignored in the linearized approximation, the initial metric (**??**) may be replaced by the metric

$$ds^2 = \left(1 - \frac{2m}{r}\right)\left\{(dx^0)^2 - \left[1 + \frac{2(\gamma+1)m}{r}\right]\boldsymbol{dx}^2\right\}. \tag{11}$$

However, it is well known that the null geodesics considered as points sets are identical for two conformal metrics (see, e.g. Joshi 2007). So the time transfer functions we are searching for coincide with the time transfer functions of the metric

$$d\tilde{s}^2 = (dx^0)^2 - \left[1 + \frac{2(\gamma+1)m}{r}\right]\boldsymbol{dx}^2. \tag{12}$$

Owing to the spherical symmetry of spacetime, each light ray joining $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ is confined to a plane passing through $\boldsymbol{x}_A$, $\boldsymbol{x}_B$ and the origin O of the spatial coordinates (this plane is unique when the position vectors of the emitter and the receiver are not colinear). We adopt spherical coordinates (r, ϑ, φ) such that $\vartheta = \pi/2$ for the plane containing the light ray and $\varphi = 0$ for the point of emission.

A rigorous integration of the null geodesic equations of the metric (**??**) is easy to perform. It is shown that there exist two and only two light rays joining A and B, provided that $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ are not aligned with the origin O. We denote by $\Gamma_{AB}^{(0)+}$ (resp. $\Gamma_{AB}^{(0)-}$) the light ray joining $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$ along which φ increases from 0 to ψ_{AB} (resp. decreases from 0 to $\psi_{AB} - 2\pi$). $\Gamma_{AB}^{(0)+}$ and $\Gamma_{AB}^{(0)-}$ are Keplerian hyperbolas having the origin O as a focus and respective impact parameters given by

$$b_{\pm} = \frac{r_A r_B\sqrt{1-\cos\psi_{AB}}}{2R_{AB}}\left[\sqrt{1+\cos\psi_{AB} + \frac{2(\gamma+1)m(r_A+r_B-R_{AB})}{r_A r_B}}\right.$$
$$\left.\pm\sqrt{1+\cos\psi_{AB} + \frac{2(\gamma+1)m(r_A+r_B+R_{AB})}{r_A r_B}}\right]. \tag{13}$$

The time transfer functions which correspond to the light rays $\Gamma_{AB}^{(0)+}$ and $\Gamma_{AB}^{(0)-}$ will be denoted by $\mathcal{T}^{+}$ and $\mathcal{T}^{-}$, respectively. The full expression of these functions can be deduced in a closed form from Eq. (**??**):

$$T^{\pm}(\boldsymbol{x}_A,\boldsymbol{x}_B)=\frac{1}{2c}\sqrt{r_A+r_B+R_{AB}}\sqrt{r_A+r_B+R_{AB}+4(\gamma+1)m}$$
$$\mp\frac{1}{2c}\sqrt{r_A+r_B-R_{AB}}\sqrt{r_A+r_B-R_{AB}+4(\gamma+1)m}$$
$$+\frac{2(\gamma+1)m}{c}\ln\left(\frac{\sqrt{r_A+r_B+R_{AB}+4(\gamma+1)m}+\sqrt{r_A+r_B+R_{AB}}}{\sqrt{r_A+r_B-R_{AB}+4(\gamma+1)m}\pm\sqrt{r_A+r_B-R_{AB}}}\right). \tag{14}$$

These TTFs are regular everywhere: there is no appearance of any enhanced term.

3.2 Time transfer function relevant for the missions in the Solar System

The time transfer function relevant for the missions in the Solar System is T^+, which corresponds to $0\leq\varphi\leq\psi_{AB}\leq\pi$. This function may be expanded in a convergent series of powers of m if and only if

$$1+\cos\psi_{AB}\geq\frac{4(\gamma+1)m[r_A+r_B-2(\gamma+1)m]}{r_Ar_B}. \tag{15}$$

Then:

$$T^{+}(\boldsymbol{x}_A,\boldsymbol{x}_B)=\frac{R_{AB}}{c}+\mathcal{T}_{\text{Shap}}(\boldsymbol{x}_A,\boldsymbol{x}_B)-\frac{(\gamma+1)^2m^2R_{AB}}{cr_Ar_B(1+\cos\psi_{AB})}$$
$$+\frac{(\gamma+1)^3m^3(r_A+r_B)R_{AB}}{cr_A^2r_B^2(1+\cos\psi_{AB})^2}+\mathcal{O}(m^4). \tag{16}$$

Thus we formally recover the dominant enhanced terms of orders m^2 and m^3 in expansion (**??**), but now we know that these terms cannot yield the correct behaviour of cT^+ when ψ_{AB} is very close or equal to π since the domain of validity of the expansion (**??**) is delimited by the condition (**??**). However, in the present state of the art, the differences $\pi-\psi_{AB}$ in the missions confined in the Solar System are not sufficiently close to zero to invalidate the use of (**??**) for estimating the gravitational time delay with the required accuracy. See, e.g., the discussion of the measurement of γ with the Cassini 2002 experiment given in Ashby and Bertotti 2010.

4. APPLICATION TO A SPINNING AXISYMMETRIC BODY

Consider now a spacetime containing a single axisymmetric body slowly spinning around its axis of symmetry. The results set out above can be used to find the TTFs of this spacetime which can be regarded as first-order perturbations of the spherically symmetric TTFs given by Eq. (**??**). We denote by $\boldsymbol{S}$ the angular momentum of the body, we put

$$\boldsymbol{n}=\frac{\boldsymbol{x}}{r},\qquad \boldsymbol{s}=\frac{\boldsymbol{S}}{|\boldsymbol{S}|}$$

and we introduce the Kerr parameter a defined by

$$a=\frac{|\boldsymbol{S}|}{Mc}=\frac{G|\boldsymbol{S}|}{mc^3}. \tag{17}$$

Since the terms of order m^2/r^2 are neglected within the weak-field, linearized approximation, we start up from the metric

$$ds^2=\left(1-\frac{2W}{c^2}\right)\left\{(dx^0)^2+\frac{4(\gamma+1)}{c^3}(\boldsymbol{W}.\boldsymbol{dx})dx^0-\left[1+\frac{2(\gamma+1)W}{c^2}\right]\boldsymbol{dx}^2\right\}, \tag{18}$$

where W is the Newtonian potential of the body and $\boldsymbol{W}$ is the gravitomagnetic potential generated by the angular momentum. Outside any sphere of radius r_0 centered on the origin O and enclosing the central body, W and $\boldsymbol{W}$ can be expanded as

$$\frac{1}{c^2}W(\boldsymbol{x}) = \frac{m}{r}\left[1 - \sum_{n=1}^{\infty} J_n \left(\frac{r_0}{r}\right)^n P_n(\boldsymbol{s}.\boldsymbol{n})\right] \tag{19}$$

and (see Linet and Teyssandier 2002):

$$\frac{1}{c^3}\boldsymbol{W}(\boldsymbol{x}) = \frac{ma(\boldsymbol{s}\times\boldsymbol{x})}{2r^3}\left[1 - \sum_{n=1}^{\infty} K_n \left(\frac{r_0}{r}\right)^n P'_{n+1}(\boldsymbol{s}.\boldsymbol{n})\right], \tag{20}$$

where $P_n(x)$ is the Legendre polynomial of degree n and $P'_n(x)$ its derivative with respect to x; J_n and K_n are the mass-multipole and spin-multipole moments of order n, respectively.

According to a remark pointed out in subsect. 3.1, the problem comes down to determine TTFs of the conformal metric

$$d\tilde{s}^2 = (dx^0)^2 + \frac{4(\gamma+1)}{c^3}(\boldsymbol{W}.\boldsymbol{dx})dx^0 - \left[1 + \frac{2(\gamma+1)W}{c^2}\right]\boldsymbol{dx}^2. \tag{21}$$

The metric (**??**) is supposed to be a small perturbation of the static spherically symmetric metric (**??**). So it is natural to assume that there exist time transfer functions $\mathcal{T}^+$ and $\mathcal{T}^-$ which can be expanded as follows:

$$\begin{aligned}\mathcal{T}^{\pm}(\boldsymbol{x}_A, \boldsymbol{x}_B; J_n, \boldsymbol{S}, K_n) = T^{\pm}(\boldsymbol{x}_A, \boldsymbol{x}_B) + \sum_{n=1}^{\infty} J_n \Delta\mathcal{T}^{\pm}_{J_n}(\boldsymbol{x}_A, \boldsymbol{x}_B) + a\Delta\mathcal{T}^{\pm}_{\boldsymbol{S}}(\boldsymbol{x}_A, \boldsymbol{x}_B) \\ + \sum_{n=1}^{\infty} K_n \Delta\mathcal{T}^{\pm}_{K_n}(\boldsymbol{x}_A, \boldsymbol{x}_B) + \cdots,\end{aligned} \tag{22}$$

where the symbols $+\cdots$ stand for the second-order perturbation terms which are neglected.

Substituting the right-hand side of Eq. (**??**) for $\mathcal{T}^{\pm}$ into the eikonal equation satisfied by any TTF (cf. Teyssandier and Le Poncin 2008), and then separating the zeroth-order equation satisfied by $T^{\pm}$ and the first-order equation satisfied by each perturbation term, it can be shown that each $\Delta\mathcal{T}^+$ (resp. $\Delta\mathcal{T}^-$) can be expressed by an integral taken along the unperturbed light ray $\Gamma^{(0)+}_{AB}$ (resp. $\Gamma^{(0)-}_{AB}$). These integrals can be calculated with any symbolic computer program.

Computing the contribution of the quadrupole J_2 is easy when the unperturbed light rays are confined to the equatorial plane, i.e. when $\boldsymbol{s}.\boldsymbol{n}_A = \boldsymbol{s}.\boldsymbol{n}_B = 0$. We have in this case

$$\begin{aligned}J_2\Delta\mathcal{T}^{\pm}_{J_2}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{(\gamma+1)m}{2c} J_2 \left(\frac{r_0}{b_\pm}\right)^2 \Bigg[\left(\frac{b_\pm}{r_A} + \frac{b_\pm}{r_B}\right)\frac{1-\cos\psi_{AB}}{\sin\psi_{AB}} \\ + \frac{(\gamma+1)m}{b_\pm}\left(\psi^{\pm}_{AB} - 2\frac{1-\cos\psi_{AB}}{\sin\psi_{AB}}\right)\Bigg],\end{aligned} \tag{23}$$

where $b^{\pm}$ is given by Eq. (**??**) and $\psi^{\pm}_{AB}$ is defined by

$$\psi^+_{AB} = \psi_{AB}, \qquad \psi^-_{AB} = \psi_{AB} - 2\pi. \tag{24}$$

When $\psi_{AB} \ll \pi$, only the unperturbed ray $\Gamma^{(0)+}_{AB}$ is relevant since $\Gamma^{(0)-}_{AB}$ is not confined in the zone of weak field $r \gg 2m$ (see Linet and Teyssandier 2016). As a consequence, only $J_2\Delta\mathcal{T}^+_{J_2}$ has to be retained in this case. The impact parameter b_+ may be expanded as

$$b_+ = r_c\left[1 + \frac{(\gamma+1)m(r_A+r_B)}{r_Ar_B(1+\cos\psi_{AB})} + \mathcal{O}(m^2)\right], \tag{25}$$

where r_c is the 'Euclidean' distance between the origin O and the straight line passing through $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$, namely

$$r_c = \frac{r_A r_B \sin\psi_{AB}}{R_{AB}}. \tag{26}$$

Then, it follows from Eq. (**??**) that

$$J_2 \Delta\mathcal{T}^{+}_{J_2}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{(\gamma+1)m}{2c} J_2 \frac{r_0^2(r_A + r_B)R_{AB}}{r_A^2 r_B^2(1+\cos\psi_{AB})} + \mathcal{O}(m^2 J_2) \tag{27}$$

when ψ_{AB} is sufficiently far from π. The contribution of J_2 to the travel time of light given by Eq. (**??**) is equivalent to the one previously obtained in the litterature (Klioner 1991, Linet and Teyssandier 2002, Le Poncin-Lafitte and Teyssandier 2008, Zschocke and Klioner 2011). The progress is that henceforth the enhanced effect apparently predicted by Eq. (**??**) when $\psi_{AB} \to \pi$ must be regarded as fictitious. This divergence is just warning us that the right-hand side of Eq. (**??**) cannot be expanded in a convergent series in powers of m for any value of ψ_{AB}. This feature does not prevent $J_2\Delta\mathcal{T}^{\pm}_{J_2}$ from remaining bounded when $\psi_{AB} \to \pi$. Indeed, taking into account that

$$\lim_{\psi_{AB}\to\pi} b_{\pm} = \pm\sqrt{\frac{2(\gamma+1)\, m\, r_A r_B}{r_A + r_B}}, \tag{28}$$

it may be inferred from Eq. (**??**) that

$$\lim_{\psi_{AB}\to\pi}\left[J_2\Delta\mathcal{T}^{\pm}_{J_2}(\boldsymbol{x}_A, \boldsymbol{x}_B)\right] = \frac{J_2 r_0^2(r_A + r_B)}{2c r_A r_B}\left[\sqrt{1 + \frac{2(\gamma+1)m}{r_A + r_B}} + \frac{\pi}{2}\sqrt{\frac{(\gamma+1)m(r_A + r_B)}{2 r_A r_B}}\right]. \tag{29}$$

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